Multi-order Vector Finite Element Modeling of 3D Magnetotelluric Data Including Complex Geometry and Anisotropy

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For the life.

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List of Symbols	This list presents the definition of most used symbols in this thesis.
(ξ, η, ζ)	Isoparametric coordinates
(x, y, z)	Cartesian coordinates in m or km
$\boldsymbol{\nu}$, $\boldsymbol{\kappa}$	General 3×3 EM model tensors
δ	Skin-depth in m
Γ	Surface boundary of a domain
$\langle \mathbf{a}, \mathbf{b} \rangle$	Vector inner product
$\hat{\mathbf{n}}$	Unit normal vector
$\tilde{\mathbf{s}}^F$	Modified source field with GPML
$\mathbf{B} = (B_x, B_y, B_z)$	Magnetic induction in T
$\mathbf{B}_0$	Magnetic induction on a reference point on the surface
$\mathbf{D}$	Displacement currents in Cm^{-2}
$\mathbf{E} = (E_x, E_y, E_z)$	Electric field in Vm^{-1}
$\mathbf{E}_0$	Electric field on a reference point on the surface
$\mathbf{F} = (F_x, F_y, F_z)$	General field
$\mathbf{F}_0$	General field on a reference point on the surface
$\mathbf{H} = (H_x, H_y, H_z)$	Magnetic field in Am^{-1}
$\mathbf{I}$	3×3 Identity matrix
$\mathbf{j}$	Current density in Am^{-2}
$\mathbf{r}$	$\mathbf{r} = (x, y, z)$ is a point with Cartesian coordinates
$\mathbf{s}$	General source vector
$\mathbf{v}_i(\xi, \eta, \zeta)$	Vector basis function (edge-element)

$\mathbf{w}$	Weighting vector function for the Galerkin method
$\mathcal{C}(\Omega)$	Differentiability class of continuous and differentiable functions
$\mathcal{H}(\Omega)$	Sobolev space
$\mathcal{H}_0(\Omega)$	Sobolev space in terms of the boundary of the domain
C	Schmucker-Weidelt transfer function in km
$\mathrm{D}(\mathbf{m})$	Differential operator with an EM model $\mathbf{m}$
$\mathrm{e}^{\mathrm{x}} = \exp(x)$	Exponential function
Ω	A 3D domain
$\omega = 2\pi f$	Angular frequency in Hz
Ω_e	Hexahedral element of the domain Ω
ϕ	MT phase in degrees
$\rho = 1/\sigma$	Resistivity in Ωm
ρ_a	Apparent resistivity in Ωm
a_0, b_0	GPML stretching factor parameters of Fang (1996)
$a_{min}, a_{max}, b_{min}, b_{max}$	GPML stretching factor parameters of Zhou et al. (2012)
$c(\omega)$	MT transfer function for layered Earth
C_Γ	Defines the boundary integration in the Galerkin method
$e(z)$	Plane wave propagation within layers
f	Frequency in Hz
$f(z)$	Plane wave characteristic of EM fields in a homogeneous Earth
$h_{x_m} = h_{x_m}(x_m)$	GPML stretching factor
$i = \sqrt{-1}$	Imaginary number
$k = \sqrt{i\omega\mu\sigma}$	Propagation constant or wave number
m_e	Number of edges in the hexahedral element
n_e	Number of nodes in the hexahedral element
$N_i(\xi, \eta, \zeta)$	Nodal basis function
t	Time in s
$\boldsymbol{\sigma}$	3×3 Conductivity tensor in Sm^{-1}
σ	Scalar conductivity in Sm^{-1}

$\mu = \mu_0\mu_r$	Scalar magnetic permeability in Hm^{-1}
$\mu_0 = 4\pi \times 10^{-7}$	Magnetic permeability of a vacuum in Hm^{-1}
$\mu_r \approx 1$	Relative magnetic permeability of a medium
$\varepsilon = \varepsilon_0\varepsilon_r$	Electric permittivity in Fm^{-1}
$\varepsilon_0 = 8.854 \times 10^{-12}$	Electric permittivity of a vacuum in Fm^{-1}
ε_r	Relative electric permittivity of a medium
$\nabla = (\partial_x, \partial_y, \partial_z)$	Nabla operator
$\partial_x = \frac{\partial}{\partial x}$	Partial derivative with respect to x variable
$\mathbf{Z}$	2×2 Impedance tensor in VA^{-1}
Z	1D MT impedance in VA^{-1}
$\partial_{\tilde{x}_i} = \frac{1}{h_{x_i}}\frac{\partial}{\partial x_i}$	GPML modified partial derivative
$\tilde{\nabla} = (\partial_{\tilde{x}_1}, \partial_{\tilde{x}_2}, \partial_{\tilde{x}_3})$	Modified differential operator with GPML
q	Electric charge
$\Im$	Imaginary part
$\Re$	Real part
Acronyms	This list presents the definition of most used acronyms in this thesis.
1D	One-Dimensional
2D	Two-Dimensional
3D	Three-Dimensional
EM	Electromagnetic
FD	Finite Difference
FEM	Finite Element Method
GPML	Generalized Perfect Matched Layers
IE	Integral Equation
MoVFEM	Multi-order Vector Finite Element Method Algorithm
MT	Magnetotelluric
PDE	Partial Differential Equation
PML	Perfect Matched Layer

TE	Transverse Electric
TM	Transverse Magnetic
VFEM	Vector Finite Element Method

Abstract

This thesis presents the development of a computational algorithm in Fortran, to model 3D magnetotelluric (MT) data using a Multi-order Vector Finite Element Method (MoVFEM) to include complex geometry (such as topography, and subsurface interfaces). All the modules in MoVFEM have been programmed from the beginning, unless specified by referencing the libraries used. The governing equations to be solved are the decoupled electromagnetic (EM) partial differential equations for the secondary electric field, or the secondary magnetic field, with a symmetric conductivity tensor to include anisotropy. The primary fields are the solution of a plane-wave within the air domain.

Two boundary conditions are implemented, namely the Generalized Perfect Matched Layers method (GPML) and Dirichlet boundary conditions. Three Dirichlet boundary schemes are applied, first considering zero EM fields at the boundaries of the computational domain; secondly, considering the boundaries as homogeneous Earth; and finally, considering the boundaries as a layered Earth. Two formulations of GPML are implemented in this algorithm, firstly the original GPML formulation and secondly, the GPML parameters are modified for the MT and Controlled Source Electromagnetic (CSEM) problem.

High-order edge-elements are defined based on covariant projections, and mixed-order edge-elements for hexahedra. The vector basis functions are defined for linear elements (12 edge-elements), quadratic elements (36 edge-elements), and Lagrangian elements (54 edge-elements). By this definition, the vector basis will have zero divergence in the case of rectangular elements and relatively small divergence in the case of distorted elements.

The validation of this computational algorithm is performed with a homogeneous Earth, where the analytic solution of the MT problem is known. In the validation, the convergence of the solution is analyzed for different grid spacing and for different element-orders with Dirichlet boundary conditions. High-order elements produce accurate solutions with larger spacing than the fine grid needed for linear-order elements.

After the convergence analysis, the solution obtained with all the proposed boundary conditions, and edge-element orders are compared for one frequency, and for a frequency range. In the homogeneous Earth, Dirichlet boundary condition presents backward reflections from the boundaries of the computational domain to the center of the model. Both GPML formulations produce more stable solutions, where no boundary reflections are present. However the MT responses fluctuate within a small range close to the values for the homogeneous Earth. The GPML formulation for MT and CSEM produce more accurate results and stabilize the MT responses over a frequency range.

This algorithm is applied to synthetic examples with complex conductivity structures. Some of these synthetic examples have been published previously, thus the results of this algorithm are compared qualitatively. In the case of anisotropy and complex geometry, the proposed synthetic examples have not been published, and a discussion of how the MT responses behave for these Earth examples is presented.

This computational algorithm could be extended with the use of an adaptive method, and it could be implemented in an algorithm for 3D inversion of MT data.

Statement of Originality

I certify that the work presented in this thesis is original and has not been accepted for the award of any other degree or diploma in any university or other tertiary institution and, contains no material previously published or written by another person, except where due reference has been made in the text.

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