University of Adelaide
School of Chemistry and Physics

Doctor of Philosophy

Regularization and Renormalization of Non-Perturbative Quantum Electrodynamics via the Dyson-Schwinger Equations

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March 2014

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Abstract

We investigate the effect of using different regularization and renormalization schemes in studying numerical solutions of quenched quantum electrodynamics (QED) using the Dyson-Schwinger equations for the fermion propagator. A primary motivation is to explore dynamical chiral symmetry breaking (DCSB) and mass generation in the theory - typically, QED undergoes a phase transition to a DCSB state above a certain critical value of the interaction strength.

Previous studies have used a cut-off regulator to render the equations finite before applying off-shell renormalization. A major outcome of these studies has been to show the equivalence of dimensional regularization and the cut-off regulator incorporating the gauge covariance improvement which maintains translational invariance of the theory. This was tested numerically using both a minimal vertex and a more realistic Ansatz based on leading log expansion (the Curtis-Pennington vertex). A recasting of the equations for the cut-off regulator in favour of using renormalized quantities exclusively (the regularization-independent method) allowed for much higher precision in extrapolating the ultraviolet end of the fermion propagator, provided one knows its functional form. These are well known by now: for example, the mass function is real power-law behaved below criticality and complex power-law behaved above criticality. The power-law exponents and location of the critical interaction strength, which depend on the choice of vertex, are theoretically calculable, and should take into account the gauge covariance improvement.

Nevertheless, undesirable features remain: for example, the presence of oscillations in the mass function above criticality, and a lack in understanding how to take the chiral limit in a well-defined manner. Whether these defects are fundamental or merely artifacts of the quenched approximation must await a fuller exploration of the unquenched theory which solves the photon equation as well.

Statement of Originality

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Tom Sizer

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Acknowledgements

I would like to thank my supervisors, Professor Tony Williams and Dr. Ayse Kızılersü, the former for igniting my interest in this subject, the latter for re-igniting it. Thanks also to Dr. Andreas Schreiber for his inspiration engendered by personal example, and the staff and students of the Centre for the Subatomic Structure of Matter (CSSM) at the University of Adelaide, and its director, Professor Tony Thomas, for providing a hot-house atmosphere in which to research physics. Thanks are also due to Ramona Adorjan, eResearch SA for the use of their supercomputer facilities, Pat at SYC and Dr. Fred Hawes. I also wish to express my appreciation to the late Professors Bert Green and Angus Hurst from the old Department of Mathematical Physics at the University of Adelaide; and finally, for everyone whose patience I have tried over the years.

For my mother and father.