

# Modelling Power Market and Pricing Electricity Derivatives in a Regime Switching Framework

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# Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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*I dedicate this work to my parents,  
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# Abstract

The deregulation of power market has led to an increase in risk for both consumers and producers when trading the underlying. Random price variations require a proper risk hedging strategy; related securities like forwards, options and swaps are the main derivatives that investors resort to in order to reduce the risk. The electricity spot price however has a particular behaviour, a consequence of the physical nature of the underlying. The non elastic offer rate causes the market equilibrium price to jump to extreme high or low levels in addition to the mean reversion and seasonality effects.

After the Introduction to the thesis contents and the background given in Chapter I, Chapter II and III develop pricing using a stochastic discount factor with applications to power derivatives. Because of the multiple sources of randomness, the power market is incomplete and any risk neutral probability measure is not unique. Pricing derivatives under the historical measure using a stochastic discount factor is one way to overcome this issue. Chapter IV investigate a different type of power pricing model. We suggest a general form for the spot price model where the randomness is given by a compensated pure jump process. Chapter V considers a new model for electricity spot price driven by a unobserved Markov jump process and the jumps are modelled using an independent Markov chain driving the jump size. In the presence of an unobserved process, the calculation of the forward and option on forward prices is performed using filtering theory. A conclusion of the thesis is given in Chapter VI.