Towards Unsupervised Online Band Selection in Hyperspectral Imaging

by

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To my Beautiful Wife, Loving Parents and Omniscient Teachers
Contents

Chapter 1. Introduction
1.1 Overview ....................................................... 2
1.2 Hyperspectral Imaging (HSI) .............................. 4
1.3 Hyperspectral Band Selection ........................... 8
1.4 Problems Addressed .......................................... 9
  1.4.1 Unsupervised Band Selection to improve Model Estimation Accuracy of a Scene .......... 9
  1.4.2 Inferring Appropriate Bands to find Anomalies in the Scene . . . 10
  1.4.3 Pixel-by-Pixel Online Band Selection for Band Cueing using Sub-Pixel Mixing Criteria ......... 11
1.5 Contributions and Publications .......................... 12
1.6 Thesis Structure .............................................. 13

Chapter 2. Preliminaries ........................................ 15
  2.1 Introduction ................................................. 16
Chapter 3. Unsupervised Band Selection using Gaussian Mixtures and Maximum Likelihood Criteria

3.1 Introduction .................................................. 30
3.2 Background ................................................... 31
  3.2.1 Gaussian Mixtures for Hyperspectral Data .............. 31
  3.2.2 Nonlinear vs Linear Band Scoring ...................... 31
3.3 Existing Work ............................................... 32
3.4 Maximum Likelihood Criteria for Band Scoring ............ 34
  3.4.1 Motivation ............................................... 34
  3.4.2 Proposed Model ........................................ 35
  3.4.3 EM Algorithm ......................................... 37
  3.4.4 Non-linear Band Scoring using Convex Optimisation .. 40
  3.4.5 EM CVX Algorithm ................................... 41
  3.4.6 Proof of Concavity for the Band Selection Objective .. 42
3.5 Experiment A ............................................... 43
3.6 Conclusions and Limitations ................................ 44
Chapter 4. Inferring Appropriate Bands To Find True Anomalies

4.1 Introduction ..................................................... 52
4.2 Existing Work .................................................... 53
  4.2.1 Band Selection Criteria ..................................... 53
  4.2.2 Band Selection or Reduction Process ..................... 55
  4.2.3 Summary of Work and Contributions ..................... 56
4.3 Methodology .................................................... 58
  4.3.1 Problem Formulation ...................................... 58
  4.3.2 Labelling Outliers and Partial Backgrounds using Convex Relaxation ............................................. 61
  4.3.3 Maximum Likelihood Estimation of Gaussian Mixture Band-Subsets .................................................. 64
  4.3.4 A Kullback-Leibler Divergence for Maximising Partially Labelled Gaussian Mixtures ............................................. 67
  4.3.5 Anomaly Detection and Band Ranking Using Convex Relaxation .................................................. 69
4.4 Experiments ..................................................... 72
  4.4.1 Experiment A: Simulated Gaussian Mixture data ............ 72
  4.4.2 Experiment B: Real Hyperspectral Data .................... 73
4.5 Discussion ..................................................... 76
4.6 Conclusion ..................................................... 77

Chapter 5. Band Sparsity for Compositional Models in Hyperspectral Imaging

5.1 Introduction ..................................................... 84
  5.1.1 Motivation and Significance ................................ 86
  5.1.2 Summary of Work and Contributions ..................... 87
5.2 Existing Work .................................................... 89
5.3 Problem Formulation ......................................... 91
5.4 Background ..................................................... 94
  5.4.1 Representing End-members using Gaussian Processes ........ 94
# Contents

5.4.2 Using Gamma and Dirichelet Distribution to represent Abundance 94

5.5 Posterior Probability Estimates for a Naive Gibbs Sampler 96

5.5.1 Estimating the Endmember Posterior 96

5.5.2 Estimating the Abundance Posterior 97

5.5.3 Abundance Sampling - Technique A: Gamma Dirichelet Relation 97

5.5.4 Abundance Sampling - Technique B: Non-Linear Transformation 99

5.5.5 Gibbs Sampler using Abundance Sampling Technique A 100

5.5.6 Gibbs Sampler using Abundance Sampling Technique B 102

5.6 Recursive Band Selection using Beta Processes 102

5.6.1 Estimating Base Measure using Convex Relaxation 103

5.6.2 Beta and Bernoulli Processes 106

5.6.3 RSBS Algorithm Summary 108

5.7 Experiments 108

5.7.1 Experiment A 109

5.7.2 Experiment B 110

5.8 Discussion 112

5.9 Conclusion 119

5.10 Appendix 120

## Chapter 6. Concluding Remarks 123

6.1 Conclusion 124

6.2 Recommendations on Future Work 125

References 127
Abstract

This thesis explores the problem of unsupervised selection of a set of spectral wavebands in a hyperspectral sensor for a surveillance task. Selecting a subset of wavebands for surveillance has the advantage of reducing data throughput and hence network bandwidth requirements, computational complexity for processing the data and storage requirements in a ground-station. For the sensor designer, Signal-To-Noise Ratio and other sensor-band improvements can be made on those bands deemed critical for the surveillance task. In chapters 3 and 4, we propose the use of locally correlated high-dimensional Gaussian Mixture models to account for band overlap where maximum likelihood estimates of the parameters of such a model are provided using the SAGE-EM (Space Alternating Generalised Expectation Maximisation) algorithm. In both these chapters convex-relaxation strategies are proposed to handle the combinatorial complexity of selecting a subset-of bands that are locally correlated and contain non-Gaussian measurements. However, in chapter 4, we select bands according to anomaly detection criteria as opposed to modelling estimation accuracy (likelihood) as done in chapter 3. We breakdown the problem such that any pixel contains band measurements that belong to either an outlier or partial background distribution, where the distributions diverge across band-subsets in a Kullback-Leibler (KL) divergence sense. A pixel is deemed as an anomaly if it contains a certain number of outliers. We identify the bands that contain the most number of contiguous outlier measurements and also subsequently reveal the presence of anomalies. Finally, in the last chapter we solve the problem of online band selection for sub-pixel compositional hyperspectral models using a Bayesian approach. Online band-selection enables spectral-band cueing and automation for adaptive focal plane arrays where not all bands are used to measure each pixel. We apply beta process models to provide a recursive strategy to select bands based on prior knowledge of their utility as well as bands used in neighbouring pixels. Band utility is measured through convex-relaxation as the subset of bands that provides the best abundance estimation accuracy of training data. The
Abstract

combination of a Gaussian process prior for possible end-members (pure materials) as well as a Gamma distributions for the abundance, enables efficient posterior sampling from a joint Normal-Gamma distribution. Furthermore, natural spectral band variations are retained making the model suitable for band selection, where approximate sum-to-one constraints are enforced through an intelligent update of the Gamma hyperparameters, based on the Dirichelet-Gamma relation. Experiments are conducted on synthetic Gaussian Mixture data with additive noise (Chapters 3, 4), Rochester Institute of Technology (RIT) Target Detection Test using the HyMAP sensor, (Chapter 4), synthetic sub-pixel data created using USGS spectral database [1] (Chapter 4) and AVIRIS-Cuprite dataset used by Mittelman et.al. in [2] (Chapter 4).
Statement of Originality

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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List of Figures

1.1 a) A slab of measurements are recorded by multiple frequency band snapshots as the sensor platform moves along track. The X and Y axes correspond to the 2D detector array. The multi-dimensional radiance signal recorded across multiple bands, for a single pixel is also shown. The acronyms VIS refers to Visible, NIR refers to the near-infrared region of the electro-magnetic spectrum and SWIR refers to the short-wave-infrared region. .......................................................... 6

3.1 Exp. A1: Combined Band MSE; EM-CVX Algorithm; Selected 5 out of 10 frequency bands, 0 dB SNR across all frequency bands ................ 45

3.2 Exp. A2: Combined Band MSE; EM-CVX; Select 20 out of 100 frequency bands, 0 dB SNR across all channels .......................... 46

3.3 Exp. A3: Combined Band MSE; EM-CVX; Select 25 out of 200 frequency bands, 20 dB SNR across all frequency bands .................. 47

3.4 Combined Band MSE; EM-CVX; Select 10 frequency bands, no removal of frequency bands .......................... 48

3.5 Exp. A5: Combined Band MSE; EM-CVX; Select 10 out of 50 frequency bands, 20, 10, 5 dB SNR evenly distributed across all frequency bands for two arbitrary initialisation points .......................... 48
4.1 a) Proposed algorithm simultaneously identifies critical band-subsets which reveal the presence of anomalies. Outlier (O) and partial backgrounds (PB) measurements are first identified across $R$ band-subsets. The diagram on the right shows $P < N$ anomaly pixels (A) that produce the greatest KL divergence between PB and O distributions using a subset of bands from a critical band rank. b) The graphical model describes the generative process for each $r$-th band-subset which contains $Q(r)$ bands. Indicator variable $T(r)$ indicates the membership of a spectral sample from the $r$-th band-subset to outlier, $Z(r)$ and partial background $Z(r)^c$ subsets. The full circles are random variables whilst the square plates around the circles indicate number of measurements, bands or components of the variable in the circle whilst dotted circles represent parameters that are non-random variables. A measurement window contain a maximum of $P$ anomalies where anomalous pixels exceed $P + 1$ thresholds. Both anomalies and critical bands are derived from convex matrix $\Phi$ which is restricted by inequality constraints $\lambda_1, \lambda_2$. $Z_n^A, Z_n^B$ are binary matrices that indicate membership of the $n$th pixel to anomaly and background groups.

4.2 Experiment A: a) Subset of backgrounds (blue) and all true anomalies (red) detected for the least difficult scenario considered where there are 7/10 band-subsets that contain outliers. b) Arbitrary backgrounds (blue) and all true anomalies (red) detected for the tougher case where there are 4/10 band-subsets that contain outliers.
4.3 Experiment A: a) $P_D$ vs. FAR for $M = 7, 6, 5, 4, 3$ anomalous band-subsets out of $R = 10$ total band-subsets. They are represented by red, blue, purple, black and green curves respectively. Each band-subset consists of $Q(r) = 3$ bands hence making the total number of bands equal to 30. Error bars (referred to as SD in the legend) indicate the accuracy range for $E = 1000$ simulations. b) Band Ranking performance is measured according to how many times critical band-subsets that contain unique outliers were actually chosen. It is represented as a function of the cumulative Cauchy-Schwarz distances between partial background and outlier distributions and as a function of the anomalous band-subsets is indicated in the legend.

4.4 Spectral measurements of anomaly vs background in local window consisting grass, tree and soil. Green asterix indicates critical bands inferred for each material. Note how the locations vary. For anomaly, $F_4$, which produced the worst result in terms of $P_D$, 34 bands are required to obtain the result. Critical bands identified for $F_1$ can be validated visually, whereas the inferred critical bands are not so obvious for the others.

4.5 Experiment B: $P_d$ vs FAR for finding 54 anomalies out of 10000 pixels: $F_1, F_2, F_3, F_4$. 
5.1 This directed graphical model represents the generative model used to capture linear sub-pixel mixing phenomena described in equation (5.1). Random variables (circles) and hyperparameters (smooth boxes) are unknown and inferred using a Gibbs Sampler. The lower-case symbols used inside the circles represents samples of those random variables. Arrows indicate the dependencies between random variables. The exception to this rule is the pixel $y_n$ which is an observation. In this model the $kth$ abundance of the $nth$ pixel is represented by $x_{k,n}$ and $g_{k,n}$ is the $kth$ endmember that is present in the $nth$ pixel and sampled from posterior probabilities of random variables $X_k, G_k$. The endmember and abundance are conditionally dependant given the measurement at the $nth$ pixel $y_n$. Hyperparameters $\alpha_{k,n}$ (shape) and $\beta_{k,n}$ (scale) vary for each $nth$ pixel. In this model, the measurement at each $nth$ pixel is assumed to be independent of remaining $N-1$ pixels. Indicator matrix $T_{d,n}$ is not a random variable and is iteratively inferred to determine whether $M_n$ bands are sufficient to describe the pixel. $y_n$. The band selection aspect of the model is specific to training data and is used to estimate the base distribution of the Beta process $B_0$.

5.2 The following graphical model applies to test data where prior band utility is described by a base measure $B_0$ obtained from training data. Each $nth$ pixel is represented by no more than $Q_n \subset D$ total number of bands in the sensor array, where value $Q_n$ is drawn from a Poisson random variable, $\pi_n(d)$ is the posterior band utility which forms the posterior Beta process that depends on base distribution $B_0$ derived from training data and prior band labels. Hyperparameters for the Beta process are indicated by $\rho_1,c$. Both sets of weights are combined in estimating binary random matrix $Z = z_1(d), \ldots, z_N(d)$ which is a result of successive draws from a Bernoulli random variable and forms the posterior Bernoulli process.
5.3 The figure displays the base probability measure used for the Beta process, $B_0$ in experiment A and experiment B, where the prior probabilities indicate band utility. For both experiments, bands between $170 - 200$ are more useful than the remainder. .................................................. 112

5.4 Experiment A a), c): Posterior Beta process measured after 20, 100 pixels, where the concentration parameter $c = 1$ and $\gamma = 15$. Experiment B b), d): Posterior Beta process measured after 20, 100 pixels, with the same hyper-parameters. Prior band utility is captured from the discrete base measures $B_0$ as is evident from the number of bands chosen after band 100. The small size of $c$ ensures that bands used to describe previous pixels is captured. The size of the $\gamma$ value ensures that a sufficient number of new bands are sampled as evident from bands with a small number of counts. .................................................. 114

5.5 Experiment A a), c): Posterior Beta process measured after 1000, 10000 pixels, where the concentration parameter $c = 1$ and $\gamma = 15$. Experiment B b), d): Posterior Beta process measured after 1000, 6400 pixels, with the same hyper-parameters. Prior band utility is captured from the discrete base measures $B_0$ as is evident from the number of bands chosen after band 100. The small size of $c$ ensures that bands used to describe previous pixels is captured. The size of the $\gamma$ value ensures that a sufficient number of new bands are sampled as evident from bands with a small number of counts. .................................................. 115

5.6 a) Experiment A: shows 20, 22, 23, 27 bands used at pixel locations 20, 100 and 1000, 10000. Bands greater than 140 or $1500 \text{nm}$ are used consistently across all 4 locations. This is agreeable with the base measure which contains larger probabilities at these locations. b) Experiment B: Only 10 – 14 bands are used at the four locations 20, 100, 1000, 6400 pixels. Bands greater than 1700 nm are used consistently across these locations. 117

5.7 Experiment B: Abundance Map of AVIRIS-Cuprite image subset. a) Original Image (courtesy Mittleman et. al. [2]). b) Kaolinite 1 c) Kaolinite 2 d) Alunite e) Montmorollinite f) Sphene .................................................. 118
5.8 Experiment B: a) Posterior endmember estimates at the first pixel of Kaolinite 1 (blue), Kaolinite 2 (green), Alunite (red), Montmorillonite (cyan) and Sphene (magenta). The signatures marked with crosses represent the estimate whilst those without any markers represent the true value.
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Band Selection Experiment Summary</td>
<td>44</td>
</tr>
<tr>
<td>4.1</td>
<td>Experiment B: Anomaly Details</td>
<td>74</td>
</tr>
<tr>
<td>4.2</td>
<td>Experiments A,B: Parameter Summary (* Parameter setting for Atmospheric Bands)</td>
<td>75</td>
</tr>
<tr>
<td>4.3</td>
<td>Critical Bands, True Anomaly Detection Summary</td>
<td>75</td>
</tr>
<tr>
<td>5.2</td>
<td>Experiment B: Endmember SSE against USGS ground-truth. Mean and Standard Deviation with added Gaussian Noise at 10dB SNR: RSBS vs SCU, VCA, BLU</td>
<td>113</td>
</tr>
<tr>
<td>5.3</td>
<td>Experiment A: Bands Used at different Pixel Locations</td>
<td>116</td>
</tr>
</tbody>
</table>