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UNIVERSITY OF ADELAIDE

DOCTORAL THESIS

Propagation Effects on HF Skywave MIMO Radar

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Declaration of Authorship

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“Those who are in love with practice without knowledge are like the sailor who gets into a ship without rudder or compass, and who never can be certain where he is going.”

Leonardo Da Vinci

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Abstract

Doctor of Philosophy

Propagation Effects on HF Skywave MIMO Radar

by Sonia TOMEI

MIMO technology has been suggested as an effective tool to overcome some of the issues typical of conventional OTH skywave radars. The advantages of the application of MIMO technology to HF Skywave radars is based on the transmission of multiple linearly independent waveforms and their separation at the receiver. Notwithstanding, the high instability of the ionosphere is responsible for severe signal fading and degradation that can prevent the separation with consequences on the radar performance. The present thesis is concerned with the problem of the effects of ionospheric propagation, which are analyzed from a theoretical point of view at first, through the description of the ionosphere morphology and the disturbances that affect the ionospheric electron density structure. The relation between structural variations in the ionosphere and the transmitted signal parameters has been then derived. A radar signal simulator has been realized accordingly to the signal model proposed in the thesis. The results of the thesis concern three different aspects of propagation in HF MIMO radars. The orthogonality of the transmitted waveforms after ionospheric propagation is analyzed first, while the effects of ionospheric propagation on the results of conventional beamforming is studied secondly. The performance of the radar receiver are evaluated in terms of ROCs in case of multipath propagation and compared to the single path case.

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Abbreviations

AWG	A tmospheric G ravity W ave
BS	B eam S pace
CIT	C oherent I ntegration I nterval
CNR	C lutter-to- N oise R atio
DSTO	D efence S cience T echnology O rganization
e.m.	electromagnetic
ES	E lement S pace
FM-CW	F requency- M odulated C ontinuous W ave
FMS	F requency M anagement S ystem
FR	F araday R otation
HF	H igh F requency
HILOW	H igh frequency L -shaped O rthogonal W aveform
i.i.d.	independent and i dentically d istributed
IRI	I nternational R eference I onosphere
JORN	J indalee O perational R adar N etwork
LPF	L ow P ass F ilter
NOSTRADAMUS	N Ouveau S ysteme T RAnshorizon D ecametrique Appliquant les M ethodes U tilisees en S tudio
LFM-CW	L inear F requency- M odulated C ontinuous W ave
LoS	L ine of S ight
LPF	L ow P ass F ilter
LSTID	L arge S cale T ravelling I onospheric D isturbance
MIMO	M ultiple I nput M ultiple O utput
MISO	M ultiple I nput S ingle O utput

MF	M atched F ilter
MSTID	M edium S cale T ravelling I onospheric D isturbance
NRL	N aval R esearch L aboratory
O	O rdinary
ONERA	O ffice N ational d'Etudes et de Recherches A érospatiales
OTH	O ver T he H orizon
PCA	P olar C ap A bsorption
pdf	probability density function
P.P.	P erturbed P ath
PRF	P ulse R epetition F requency
RADAR	R adio D etection A nd R anging
RCS	R adar C ross S ection
RIAS	R adar a I mpulsion S ynthetique
RVP	R esidual V ideo P hase
rx	receiver
SAR	S ynthetic A perture R adar
SIAR	S ynthetic I mpulse A perture R adar
SID	S udden I onospheric D isturbance
SISO	S ingle I nput S ingle O utput
SSTID	S mall S cale T ravelling I onospheric D isturbance
TID	T ravelling I onospheric D isturbance
tx	transmitter
UHF	U ltra H igh F requency
U.P.	U nperturbed P ath
VHF	V ery H igh F requency
X	e Xtraordinary

Physical Constants

Speed of Light $c = 2.997\,924\,58 \times 10^8 \text{ ms}^{-\text{s}}$ (exact)

Electron Charge $e = -1,602 \times 10^{-19} \text{ C}$

Electron Mass $m_e = 9,109 \times 10^{-31} \text{ Kg}$

Vacuum Permittivity $\epsilon_0 = 8,854 \times 10^{-12} \text{ F}^{-\text{m}}$

Symbols

A

A_{FR}	signal attenuation due to FR
$A^{(r)}$	FR attenuation along the r^{th} path in the MISO case
$A^{(r,q)}$	FR attenuation in the MISO case
$A_m^{(r,q)}$	FR attenuation for the m^{th} transmitted signal in the SISO case
$A_{mn}^{(r,q)}$	FR attenuation for the m^{th} tx signal at the n^{th} rx in the MIMO case

B

B	Earth's magnetic induction field
B	signal bandwidth
B_{av}	available bandwidth
B_g	guard bandwidth
B_{LPF}	low pass filter bandwidth
B_m	bandwidth of the m^{th} transmitted signal

C

C_{att}	constant accounting for signal attenuations and losses
-----------	--

D

D_{array}	maximum dimension of the array
ds	unit vector of length along the wave path

E

F

f_0	wave frequency
$f_{beat,max}$	maximum beat frequency
f_{off}	frequency offset

f_{opt}	optimum carrier frequency at the output of the FMS
\hat{f}_{opt}	estimated optimum carrier frequency at the output of the FMS
f_{pi}	plasma frequency of the i^{th} ionospheric layer
f_{pi0}	plasma frequency of the unperturbed ionosphere
f_{pi}	plasma frequency of the perturbed ionosphere
f_{pmax}	maximum plasma frequency value
f_{out}	demodulated signal frequency after deramping
$f_{\mathbf{u} H_0}$	pdf of the measured signal under H_0
$f_{\mathbf{u} H_1}$	pdf of the measured signal under H_1
$f_{\mathbf{w}}$	pdf of noise
f_{σ}	pdf of target's RCS
	<u>G</u>
	<u>H</u>
H	global propagation channel matrix
H_0	hypothesis of no target in the range cell under test
H_1	hypothesis of target present in the range cell under test
\mathbf{h}_{kn}	propagation channel factors vector
\mathbf{H}_n	matrix of the propagation channel factors at the n^{th} rx
	<u>I</u>
	<u>J</u>
	<u>K</u>
k_0	wavenumber
	<u>L</u>
$L^{(r)}$	signal losses along the r^{th} path in the MISO case
$L^{(r,q)}$	signal losses in the MISO case
$L_m^{(r,q)}$	signal losses for the m^{th} tx signal in the SISO case
$L_{mn}^{(r,q)}$	signal losses for the m^{th} tx signal at the n^{th} rx in the MIMO case
	<u>M</u>
$m_m(t)$	baseband signal emitted by the m^{th} tx
$M_m(f)$	baseband signal emitted by the m^{th} tx in the frequency domain
\mathbf{M}_p	matrix for coordinates transformation

N

N_B	number of beams in the BS signalling technique
n_e	electron density
nmi	nautical miles
N_{rx}	number of rx elements
N_{tx}	number of tx elements
N_w	number of orthogonal waveforms

O

O	centre of the local system of reference $T_p(p, q, r)$
-----	--

P

\mathbf{p}	vector of coordinates in the local system of reference
$P(\theta)$	power associated with the tx signals at location given by θ
\mathbf{p}_{Tm}	m^{th} tx coordinates in the local system of reference
\mathbf{p}_T	target's coordinate in the local system of reference
\mathbf{p}_{Rn}	n^{th} rx coordinates in the local system of reference
$\mathbf{p}_{Tx,m}$	m^{th} tx coordinates in the local system of reference
P_{total}	total phase path length

Q

Q	number of paths between the target and the rx in the MISO case
-----	--

R

\mathbf{R}	correlation matrix of the transmitted signals
R_g	range distance
R_{max}	maximum range
R	number of paths between the tx and the target in the MISO case
R_m	number of paths between the m^{th} tx and the target in the SISO case
\mathbf{R}_w	noise covariance matrix
\mathbf{R}_σ	target's RCS covariance matrix

S

$s_m(t)$	m^{th} waveform of a set of orthogonal waveforms
----------	--

T

$T_p(p, q, r)$	local Cartesian system of reference T_{rep}
----------------	---

T_{sw}	ramp duration
$T_x(x, y, z)$	geocentric system of reference
	<u>U</u>
\mathbf{u}	vector notation of the received signal
u_{kn}	vector notation of the output of the k^{th} MF at the n^{th} rx
\mathbf{u}_n	vector of the outputs of all the MFs at the n^{th} rx
	<u>V</u>
	<u>W</u>
\mathbf{w}	total noise contribution at the receiver at the output of the MFs
w_{kn}	noise at the output of the k^{th} MF at the n^{th} rx
\mathbf{w}_n	output noise of all the MFs at the n^{th} rx
	<u>X</u>
\mathbf{x}	vector of coordinates in the local system of reference
x	horizontal direction of the plasma frequency modulation
$x_m(t)$	signal emitted by the m^{th} tx
	<u>Y</u>
$y_T(t)$	signal at the target location in the MISO case
	<u>Z</u>
$z(t)$	received signal in the MIMO case
$z_m(t)$	signal emitted by the m^{th} tx at the rx site
$z_{miso}(t)$	received signal in the mISO case
$z_{T,nk}$	output of the k^{th} matched filter at the n^{th} rx
$z_{siso}(t)$	received signal in the SISO case
$z_{siso,m}(t)$	received signal from the m^{th} tx in the MISO case

Greek Symbols

	<u>α</u>
α_m	amplitude of the m^{th} transmitted signal
α_{mn}	amplitude of the m^{th} transmitted signal at the n^{th} rx
	<u>β</u>
β_i	direction of the i^{th} beam in the BS signalling technique

	<u>γ</u>	
γ		chirprate
γ_0		pdf multiplying factor under H_0
γ_1		pdf multiplying factor under H_1
	<u>δ</u>	
$\delta\mu$		refractive index contribution due to ionospheric perturbation
δ_R		range resolution
δ_{CR}		cross range resolution cell dimension
	<u>ϵ</u>	
ϵ		amplitude modulation of the plasma frequency
	<u>θ</u>	
$\overline{\delta_R}$		fixed range resolution
ϑ_0		initial phase of the plasma frequency modulation
	<u>λ</u>	
λ		wavelength
	<u>μ</u>	
μ_i		refractive index of the i^{th} ionospheric layer
μ_p		refractive index for a perturbed ionosphere
	<u>ρ</u>	
$\phi_{inc,i}$		angle with an e.m. wave strikes into the i^{th} ionospheric layer
	<u>σ</u>	
σ		vector of the target's RCS
$\sigma(\xi)$		scaling factor accounting for the target RCS
σ_{kn}		vector of target's reflectivity
σ_n		vector containing the target RCS at the n^{th} rx
$\sigma^{(r,q)}$		target's RCS for the signal propagating in the MISO case
$\sigma_m^{(r,q)}$		target's RCS for the m^{th} transmitted signal in the SISO case
	<u>τ</u>	
τ		time delay
τ_{max}		maximum time delay
τ_{off}		time offset of the transmitted signals

$\tau_{p,mn}$	propagation delay between the m^{th} tx to the n^{th} rx via target
$\tau_{G,T}^{(r)}$	group delay at the target location
$\tau_G^{(r,q)}$	group delay of the signal propagating in the MISO case
$\tau_{Gm}^{(r,q)}$	group delay of m^{th} transmitted signal in the SISO case
$\tau_{P,T}^{(r)}$	phase delay at the target location
$\tau_P^{(r,q)}$	phase delay of the signal propagating in the MISO case
$\tau_{Pm}^{(r,q)}$	phase delay of the m^{th} transmitted signal in the SISO case
$\tau_T^{(r)}$	phase delay between the tx and the target along the r^{th} path
	<u>ϕ</u>
ϕ_{el}	elevation angle of an e.m. wave
Φ	amount of FR
$\Delta\varphi$	phase shift
	<u>χ</u>
χ	LRT threshold
χ_1	LRT modified threshold
	<u>ω</u>
ω_0	angular carrier frequency

Math Operators

$E\{\}$	expectation
H	Hermitian operator
$\delta_{i,j}$	Kronecker delta function
T	transpose operator
$*$	conjugate operator