Formulation of Optimum Beam Space Processing for Directional Transmission Phased Array Systems

Ruiting Yang, Douglas A. Gray and Waddah A. Al-Ashwal
University of Adelaide Radar Research Centre
EEE School, The University of Adelaide
Adelaide, Australia

***Abstract*—Standard beam space processing algorithms, such as minimum variance distortionless response, normally assume that the transmitter is omni-directional, and hence are mis-matched when the transmitter is directional. This paper derives a new formula for the case of directional transmission. An example shows that the new formula achieves significantly better performance than the standard beam space formula when the transmission is via directional beams. Also, the idea of adding virtual beams to reduce the response to out of sector signals is proposed. Finally, the applicability of the algorithm for non-stationary scatterers is examined.**

Keywords—beam space; directional transmission; optimum beamforming; virtual beams

I. INTRODUCTION

In phased array systems, applying optimum weights to each element output according to some optimisation criterion can achieve better performance than conventional beamforming. For example, the popular minimum variance distortionless response (MVDR)-Capon algorithm [1] minimises the output power subject to a look direction constraint. In practice, the receiver outputs may not be accessible and only beamformer output signals are available, as occurs in many phased array radar and sonar systems, and in such cases it is not always possible to modify the element (receiver) weights. However weights can be applied to the outputs of different beams to maximise the signal-to-noise ratio (SNR) [2]: such approaches are termed beam space (BS) methods. Popular direction finding algorithms such as MUSIC [4, 5], ESPRIT [6] can also be implemented in beam space. The BS approach may also be used to achieve lower dimensionality, less computation, reduced statistical variance [3] and higher robustness [7].

Current BS algorithms are formulated for either passive or active systems which assume that the transmitter is omni-directional. This implies that the return from any scatterer at the receive array is statistically the same for all beams. However in many practical cases, directional transmission where a single transmit beam and a single receive beam are synchronously scanned over a sector of interest is widely used. In these cases, the return from any particular scatterer at the receive array will vary from beam to beam as the amount of energy it receives from the transmitter will vary according to each scatterer's orientation relative to the maximum response axis of the transmit beam. In this case since the statistics of the receiver outputs for each transmit beam vary, different theoretical expressions for the element space (ES) and BS covariance matrices will result.

For ES approaches directional transmission does not pose a severe problem as for each transmitted beam an ES covariance matrix can be estimated and used to derive the optimum set of array weights provided the sampling rate is fast enough to allow a sufficient number of degrees of freedom to be used for its estimation. However the standard BS method assumes that the BS covariance matrix is a linear transformation of a fixed ES covariance matrix and so for directional transmission where the receiver ES covariance matrix varies with the transmit beam, the performance of the standard BS approach will be degraded. This paper derives a new BS MVDR beamforming formula for the directional transmission mode. It should be commented that this approach requires that the scatterers remain stationary, in a statistical sense (i.e., wide sense stationary), during the interval over which the beams scan the sector of interest.

In Section II the standard BS MVDR formula is described and the loss of performance when directional transmission is used is examined. A new BS MVDR formula for directional transmission problem is derived in Section III and the performance of the new formula is demonstrated by an example. When beam scanning is not over the full azimuthal and elevational ranges BS approaches show a high response to out of sector signals and an approach to alleviate this problem is described in section IV. The performance of the algorithm for both stationary and non-stationary scatterers is discussed in Section V. The paper is concluded in Section VI.

II. STANDARD BS FORMULATION AND PERFORMANCE FOR DIRECTIONAL TRANSMISSION

A. Formulation of standard BS MVDR processing

Consider a uniform linear array (ULA) of K receivers with separation d where the steering vector for a narrow band signal incident on the array from an angle θ can be defined as

$$\boldsymbol{v}(\theta) = [1 \quad e^{j2\pi d/\lambda \sin(\theta)} \quad \dots \quad e^{j2\pi(K-1)d/\lambda \sin(\theta)}]^T, \quad (1)$$

where $(\cdot)^T$ denotes transpose and λ is the wavelength. At a given range, there are L independent scatterers located at angles $\Theta = [\theta_{s1} \ \theta_{s2} \ \cdots \ \theta_{sL}]$ and their received complex amplitudes written as an $L\times 1$ vector $\boldsymbol{s} = [\sigma_{s1} \ \sigma_{s2} \ \cdots \ \sigma_{sL}]^T$

are proportional to their radar cross section. The signal covariance matrix is given by

$$\boldsymbol{R_s} = \mathrm{E}\{\boldsymbol{ss^H}\}. \tag{2}$$

where $(\cdot)^{\mathrm{H}}$ denotes the Hermitian transpose. The output from K receivers is written in a vector as

$$\boldsymbol{x} = \boldsymbol{V_s}(\Theta)\boldsymbol{s} + \boldsymbol{n}, \tag{3}$$

where $$\boldsymbol{V_s}(\Theta) = [\boldsymbol{v}(\theta_{s1}) \quad \boldsymbol{v}(\theta_{s2}) \quad \cdots \quad \boldsymbol{v}(\theta_{sL})], \tag{4}$$

where the $K \times L$ matrix $\boldsymbol{V_s}(\Theta)$ contains steering vectors corresponding to the directions of arrival (DOA) of the signals and $\boldsymbol{n}$ is the $K{\times}1$ vector of uncorrelated receiver noise. The ES covariance matrix is given by

$$\boldsymbol{R_x} = \mathrm{E}\{\boldsymbol{xx}^H\} = \boldsymbol{V_s}(\Theta)\boldsymbol{R_s}\boldsymbol{V_s^H}(\Theta) + \sigma_n^2\boldsymbol{I}. \tag{5}$$

where σ_n^2 is the noise power and $\boldsymbol{I}$ is an identity matrix.

The ES MVDR algorithm is well known and its output power is given by [1]

$$p_{mvdr\_ES}(\theta) = (\boldsymbol{v}^H(\theta)\boldsymbol{R_x}^{-1}\boldsymbol{v}(\theta))^{-1}. \tag{6}$$

The output of a conventional beam steered in direction θ is given by

$$y(\theta) = \boldsymbol{v}^H(\theta)\boldsymbol{x}, \tag{7}$$

where the vector output of M conventional beams is

$$\boldsymbol{y} = [y(\theta_1) \quad y(\theta_2) \quad \cdots \quad y(\theta_2)]^T. \tag{8}$$

The BS covariance matrix is given by

$$\boldsymbol{R_y} = \mathrm{E}\{\boldsymbol{yy}^H\} = \boldsymbol{V^H}\boldsymbol{R_x}\boldsymbol{V}, \tag{9}$$

where $\boldsymbol{V}$ is the $K{\times}M$ matrix containing the steering vectors for each of the various beams and is given by

$$\boldsymbol{V} = [\boldsymbol{v}(\theta_1) \quad \boldsymbol{v}(\theta_2) \quad \cdots \quad \boldsymbol{v}(\theta_M)]. \tag{10}$$

The vector of beam outputs due to a single plane wave from direction θ is the constraint vector and is given by

$$\boldsymbol{h}(\theta) = \boldsymbol{V}^H\boldsymbol{v}(\theta). \tag{11}$$

The optimum BS beamformer is given as the solution of the problem (see [2] and [8], Chapter 6)

$$min\ \boldsymbol{w}^H\boldsymbol{R_y}\boldsymbol{w} \text{ subject to } \boldsymbol{w}^H\boldsymbol{h}(\theta) = 1, \tag{12}$$

and the solution is given by

$$\boldsymbol{w}(\theta) = \frac{\boldsymbol{R_y^{-1}}\boldsymbol{V}^H\boldsymbol{v}(\theta)}{\boldsymbol{v}^H(\theta)\boldsymbol{V}\boldsymbol{R_y}^{-1}\boldsymbol{V}^H\boldsymbol{v}(\theta)}. \tag{13}$$

The output power of the BS MVDR beamformer is

$$p_{mvdr\_BS}(\theta) = (\boldsymbol{v}^H(\theta)\boldsymbol{V}\boldsymbol{R_y^{-1}}\boldsymbol{V}^H\boldsymbol{v}(\theta))^{-1}. \tag{14}$$

B. Model of the directional transmission mode

Consider an active system where the transmit energy is beamformed in a certain direction and at the receiver the array elements receive the scattered energy by forming a beam in the same direction. When a pulse from the m-th beam is directionally transmitted at an angle θ_m, and is reflected back by L scatterers located at angles Θ, the vector of receiver outputs can be expressed as

$$\boldsymbol{x}(\theta_m) = \boldsymbol{V_s}(\Theta)(\boldsymbol{V_s^T}(\Theta)\boldsymbol{v}^*(\theta_m)/K \circ \boldsymbol{s}) + \boldsymbol{n}. \tag{15}$$

where $(\cdot)^*$ denotes complex conjugate and $\circ$ denotes the Hadamard product. Thus the ES covariance matrix is transmit beam dependent and is given by

$$\boldsymbol{R_x}(\theta_m) = \boldsymbol{V_x}(\Theta,\ \theta_m)\boldsymbol{R_s}\boldsymbol{V_x^H}(\Theta,\ \theta_m). \tag{16}$$

where $$\boldsymbol{V_x}(\Theta, \theta_m) = [\boldsymbol{v_x}(\theta_{s1},\ \theta_m) \cdots \boldsymbol{v_x}(\theta_{sL},\ \theta_m)], \tag{17}$$

$$\boldsymbol{v_x}(\theta_{si}, \theta_m) = \boldsymbol{v}^H(\theta_m)\boldsymbol{v}(\theta_{si})\boldsymbol{v}(\theta_{si})/K,\ i = 1, 2 \cdots L. \tag{18}$$

The vector containing the beamformed receiver outputs corresponding to M different transmit beams is given by

$$\boldsymbol{y} = \begin{bmatrix} y(\theta_1) \\ y(\theta_2) \\ \vdots \\ y(\theta_M) \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}^H(\theta_1)\boldsymbol{x}(\theta_1) \\ \boldsymbol{v}^H(\theta_2)\boldsymbol{x}(\theta_2) \\ \vdots \\ \boldsymbol{v}^H(\theta_M)\boldsymbol{x}(\theta_M) \end{bmatrix}, \tag{19}$$

where the dependence of the outputs $\boldsymbol{y}$ on the chosen direction of the transmit beams is explicitly shown and for a single scatterer at angle θ_{si} reduces to

$$\boldsymbol{v_y}(\theta_{si}) = \begin{bmatrix} \boldsymbol{v}^H(\theta_1)\boldsymbol{v}(\theta_{si})\boldsymbol{v}^H(\theta_1)\boldsymbol{v}(\theta_{si})/K \\ \boldsymbol{v}^H(\theta_2)\boldsymbol{v}(\theta_{si})\boldsymbol{v}^H(\theta_2)\boldsymbol{v}(\theta_{si})/K \\ \vdots \\ \boldsymbol{v}^H(\theta_M)\boldsymbol{v}(\theta_{si})\boldsymbol{v}^H(\theta_M)\boldsymbol{v}(\theta_{si})/K \end{bmatrix}. \tag{20}$$

For the above case the BS covariance matrix is given by

$$\boldsymbol{R_y} = \boldsymbol{V_y}(\Theta)\boldsymbol{R_s}\boldsymbol{V_y^H}(\Theta) + K\sigma_n^2\boldsymbol{I}. \tag{21}$$

where $$\boldsymbol{V_y}(\Theta) = [\boldsymbol{v_y}(\theta_{s1}) \quad \boldsymbol{v_y}(\theta_{s2}) \quad \cdots \quad \boldsymbol{v_y}(\theta_{sL})]. \tag{22}$$

C. Application of the standard BS MVDR method to a directional transmission problem

It has been proved in [2] that the optimum BS gain equals the optimum ES gain when there are K independent beam spanning the full angular sector, because $\boldsymbol{V}$ becomes a $K \times K$ full rank invertible matrix so $\boldsymbol{x} = (\boldsymbol{V}^H)^{-1}\boldsymbol{y}$, and all the information in ES is retained in the BS formulation. In practice, signals of interest are normally located in a given subsector of the full angular region and so beams are only formed in that subsector and so the above result is not applicable.

In the following example a horizontal, uniform linear array of 64 elements spaced half a wavelength apart is considered. Two independent scatterers whose returns have SNRs of 10 and 6 dB at a single receiver are located at azimuth angles 8° and 15° respectively. Only 21 beams were formed spanning the azimuthal region of -2° to 18° with 1° separation. The theoretical output power values of the standard BS MVDR, the ES conventional and MVDR beamformer are plotted versus azimuth angle in Fig. 1.

As the covariance matrix varied for each different transmit/receive beam, the ES results were obtained by processing the 21 sectors separately and the covariance matrix corresponding to the nearest transmit beam was used to derive the output power. Note in this case the standard BS MVDR algorithm assumes omni-directional transmission whilst the data comes from directional transmission, thus resulting in model mismatch. Whilst the direction of arrival of each signal can be estimated by the standard BS MVDR method, the mismatch has resulted in very low SNRs for both signals. Obviously, the standard BS MVDR formula has very poor performance when a directional transmitter is used.

III. Formulation of Directional Transmission BS MVDR Processing and Performance Evaluation

According to (9), the BS covariance matrix is a linear transformation of the ES covariance matrix and the formula (14) is based on a condition that ES covariance matrix is constant. For the directional transmission problem, the ES covariance matrix varies as the transmit beam changes so the formula is no longer applicable. In this section, the derivation of a modified BS algorithm takes into account the scanning beampattern of the transmit array.

A. Formulation of directional transmission BS MVDR processing

To modify the BS approach, we consider stacking all the receiver outputs for each of the M different transmit beams to form a new stacked $KM\times 1$ vector

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}(\theta_1) \\ \boldsymbol{x}(\theta_2) \\ \vdots \\ \boldsymbol{x}(\theta_M) \end{bmatrix}. \tag{23}$$

The corresponding $KM\times KM$ covariance matrix is given by

$$\boldsymbol{R}_x = E\{\boldsymbol{x}\boldsymbol{x}^H\}$$
$$= E\left\{\begin{bmatrix} \boldsymbol{x}(\theta_1)\boldsymbol{x}^H(\theta_1) & \boldsymbol{x}(\theta_1)\boldsymbol{x}^H(\theta_2) & \cdots & \boldsymbol{x}(\theta_1)\boldsymbol{x}^H(\theta_M) \\ \boldsymbol{x}(\theta_2)\boldsymbol{x}^H(\theta_1) & \boldsymbol{x}(\theta_2)\boldsymbol{x}^H(\theta_2) & \cdots & \boldsymbol{x}(\theta_2)\boldsymbol{x}^H(\theta_M) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{x}(\theta_M)\boldsymbol{x}^H(\theta_1) & \boldsymbol{x}(\theta_M)\boldsymbol{x}^H(\theta_2) & \cdots & \boldsymbol{x}(\theta_M)\boldsymbol{x}^H(\theta_M) \end{bmatrix}\right\}. \tag{24}$$

The stacked receiver beam outputs are given by

$$\boldsymbol{y} = \begin{bmatrix} y(\theta_1) \\ y(\theta_2) \\ \vdots \\ y(\theta_M) \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}^H(\theta_1)\boldsymbol{x}(\theta_1) \\ \boldsymbol{v}^H(\theta_2)\boldsymbol{x}(\theta_2) \\ \vdots \\ \boldsymbol{v}^H(\theta_M)\boldsymbol{x}(\theta_M) \end{bmatrix} = \boldsymbol{U}^H\boldsymbol{x}, \tag{25}$$

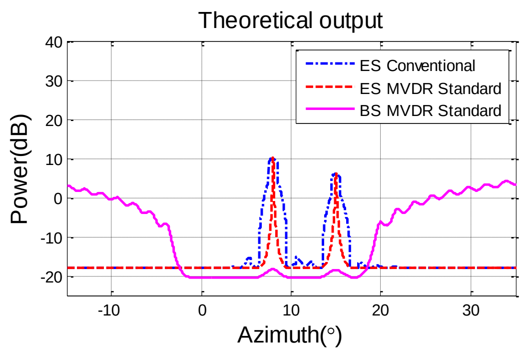

Fig. 1. An example of applying the BS MVDR standard formula to directional transmission mode and comparing with the ES conventional and MVDR beamforming. The conventional beamforming outputs are normalised.

where
$$\mathbf{U} = \begin{bmatrix} \boldsymbol{v}(\theta_1) & 0 & \cdots & 0 \\ 0 & \boldsymbol{v}(\theta_2) & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{v}(\theta_M) \end{bmatrix}. \tag{26}$$

The resulting BS covariance matrix is given by

$$\boldsymbol{R}_y = E\{\boldsymbol{y}\boldsymbol{y}^H\} = \boldsymbol{U}^H\boldsymbol{R}_x\boldsymbol{U}. \tag{27}$$

The stacked vector for constraining to unity the output for a plane wave incident on the receive array from direction at θ is given by

$$\boldsymbol{v}_p(\theta) = \begin{bmatrix} \boldsymbol{v}^H(\theta_1)\boldsymbol{v}(\theta)\boldsymbol{v}(\theta)/K \\ \boldsymbol{v}^H(\theta_2)\boldsymbol{v}(\theta)\boldsymbol{v}(\theta)/K \\ \vdots \\ \boldsymbol{v}^H(\theta_M)\boldsymbol{v}(\theta)\boldsymbol{v}(\theta)/K \end{bmatrix} = (\boldsymbol{V}^H\boldsymbol{v}(\theta)\otimes\boldsymbol{v}(\theta)/K. \tag{28}$$

where $\otimes$ denotes the Kronecker product. It can be shown that with this formulation the output power of the optimum BS beamformer at θ is given by

$$p_{opt\_BS}(\theta) = (\boldsymbol{v}_p^H(\theta)\boldsymbol{U}\boldsymbol{R}_y^{-1}\boldsymbol{U}^H\boldsymbol{v}_p(\theta))^{-1}. \tag{29}$$

The optimum BS weight for the directional transmission at angle θ is given by

$$\boldsymbol{w}_{opt\_BS}(\theta) = \frac{\boldsymbol{R}_y^{-1}\boldsymbol{U}^H\boldsymbol{v}_p(\theta)}{\boldsymbol{v}_p^H(\theta)\boldsymbol{U}\boldsymbol{R}_y^{-1}\boldsymbol{U}^H\boldsymbol{v}_p(\theta)} = \frac{\boldsymbol{R}_y^{-1}\boldsymbol{v}_y(\theta)}{\boldsymbol{v}_y^H(\theta)\boldsymbol{R}_y^{-1}\boldsymbol{v}_y(\theta)}. \tag{30}$$

B. Performance evaluation of new BS MVDR formula

The new BS approach was applied to the same signal model as in Section II. The theoretical output power values of the directional transmission BS MVDR formula, ES conventional and MVDR are plotted versus azimuth in Fig. 2.

The performance of the directional transmission formula is significantly better than the standard BS MVDR formula. The power of signal of interest is estimated accurately and the resolution is much higher than the conventional ES beamforming, even slightly better than the standard ES MVDR.

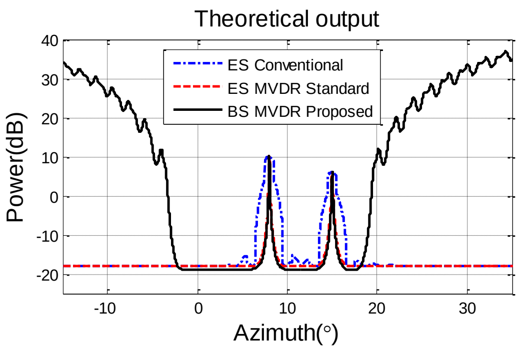

Fig. 2. An example of applying the proposed BS MVDR formula to directional transmission mode and comparing with ES conventional and MVDR beamformer. The conventional beamforming outputs are normalised.

IV. Out of Sector Interference

In Fig. 2, it is apparent that the BS MVDR output power in the region not contained within the sector of interest is very strong. Whilst this may not be an issue as there has been no energy transmitted into this region, the high response in this area may cause a problem with receiver noise and sensitivity. Notice this effect occurs for both conventional and directional BS approaches. The reason is that no eigenvectors in the beam space covariance matrix $\boldsymbol{R_y}$ correspond to out of sector steering vectors , so a large value is obtained by (14) or (29) and no suppression is made by the MVDR algorithm.

To remove these high outputs, N extra virtual beams at selected angles in the "out of sector" region $\Theta_{extra} = [\theta_{M+1} \quad \theta_{M+2} \quad \cdots \quad \theta_{M+N}]$ are added and are assumed to contain uncorrelated noise only. The modified covariance matrix is given by

$$\boldsymbol{R_{y\_All}} = \begin{bmatrix} \boldsymbol{R_y} & \boldsymbol{0} \\ \boldsymbol{0} & K\sigma_n^2 I \end{bmatrix}. \tag{31}$$

Inspection of the above augmented BS covariance matrix indicates that the effect is to add additional eigenvalues at the noise level. For the augmented system, the stacked steering vector in (28) becomes

$$\boldsymbol{v_p}(\theta) = \begin{bmatrix} \boldsymbol{v}^H(\theta)\boldsymbol{v}(\theta_1)\boldsymbol{v}(\theta)/K \\ \boldsymbol{v}^H(\theta)\boldsymbol{v}(\theta_2)\boldsymbol{v}(\theta)/K \\ \vdots \\ \boldsymbol{v}^H(\theta)\boldsymbol{v}(\theta_{M+N})\boldsymbol{v}(\theta)/K \end{bmatrix}. \tag{32}$$

An example is shown here, where the scenario was the same as that previously used but an extra interference at 20° with 40dB interference-to-noise ratio (INR) has been added. Virtual beams from -90° to -3° and 19° to 90° with 1° separation were added and the virtual beams contained only noise. As discussed above, the output power values of directional transmission BS MVDR processing with and without virtual beams are shown in Fig. 3. It shows that the high out of sector response has been effectively removed, and the out of sector signal (interference) has been attenuated, as we assumed the beam output at the direction of interference is only noise; additionally, the output power at the angle of the source signal is attenuated very slightly.

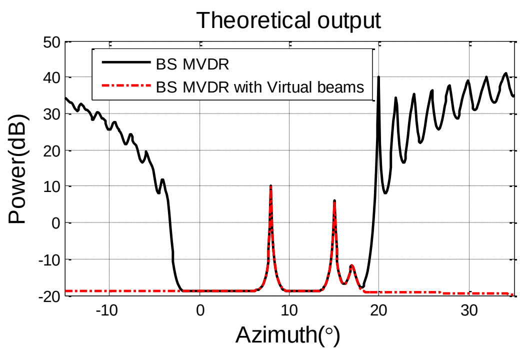

Fig. 3. The output power values of directional transmission BS MVDR processing with and without virtual beams

V. Performance of the Algorithm for Both Stationary and Non-Stationary Scatterers

As discussed, the modified beam space approach requires that scatterers remain stationary, but in practice non-stationary scatterers are quite common. To check how the mobility of a scatterer affects the performance of the algorithm, the proposed beam space algorithm has been applied to a scenario containing both stationary and non-stationary scatterers. In this example a target moving at a constant speed and at a constant range is considered, thus eliminating the need to consider range and Doppler effects.

The beam scanning strategy will have an impact on the algorithm's performance against a moving target. The example considered here is where a burst of P pulses is transmitted at each steering angle of the phased array. The initial azimuth angle of a non-stationary scatterer is θ_{si} and between adjacent pulses the target moves $\Delta\theta$. The BS covariance matrix is estimated by

$$\boldsymbol{R_y} = E\{\boldsymbol{y}\boldsymbol{y}^H\} = \frac{1}{P}\sum_{p=1}^{P} \boldsymbol{y}(p)\boldsymbol{y}^H(p), \tag{33}$$

where $\boldsymbol{y}(p)$ denotes all the beam outputs at the p-th pulse in each beam. The covariance matrix is a composite of stationary scatterers, non-stationary scatterers and noise components, given by

$$\boldsymbol{R_y} = \boldsymbol{R_{y\_1}} + \boldsymbol{R_{y\_2}} + K\sigma_n^2 \boldsymbol{I}, \tag{34}$$

where $\boldsymbol{R_{y\_1}}$ and $\boldsymbol{R_{y\_2}}$ are the stationary and non-stationary scatterers components respectively. For a single non-stationary scatterer whose complex amplitude is σ_{si}, the i, j entry of matrix $\boldsymbol{R_{y\_2}}$ is given by

$$\left(\boldsymbol{R_y}\right)_{ij} = \frac{\sigma_{si}^2}{P}\sum_{p=1}^{P} \boldsymbol{v_y}\big(\theta_{si} + \big((i-1)P + p - 1\big)\Delta\theta\big) \\ \boldsymbol{v_y^H}\big(\theta_{si} + \big((j-1)P + p - 1\big)\Delta\theta\big). \tag{35}$$

To demonstrate the performance of the modified beam space algorithm with a non-stationary scatterer, the same scenario as that for Fig. 2 was used with the first scatterer (whose SNR was 10dB and initial azimuth angle was 8°) was moving at a fixed speed in azimuth. The second scatterer was stationary and located at 15°. The ratio of the target's location change in azimuth between beams to beam width, BW, is defined as

$$\rho = P\Delta\theta/BW, \tag{36}$$

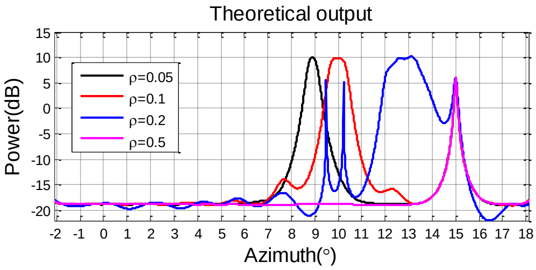

Fig. 4. The output power values of directional transmission BS MVDR processing for a stationary and a non-stationary scatterer with different moving speeds

and it indicates the fraction (or multiple) of a beamwidth that the target moves when the transmitted/received beam changes from one steering direction to the next. Values of ρ equal to 0.05, 0.1, 0.2 and 0.5 correspond to angular changes of the scatterer's azimuthal location over the total assumed "coherent processing interval" of 1.67°, 3.33°, 6.66° and 16.65° respectively. The processing results with different ρ are shown in Fig. 4 which indicates that the received signal reflected by a slow moving scatterer, i.e., $\rho = 0.05$, can still be effectively beamformed and hence detected; with increasing speed, i.e., $\rho = 0.1$ the non-stationary scatterer can be detected when it is in the mainlobe of a beam but the sharp peak is widened by motion of the scatterer. Increasing the speed to $\rho = 0.2$ introduces wrong peaks. When the speed is increased to $\rho = 0.5$, the non-stationary scatterer is moving too fast to be detected, as it is not in the mainlobe of any of the beams. The processing result of the stationary scatterer is not affected unless the non-stationary and stationary scatterers fall into the same beam. To improve the performance for non-stationary scatterers, a quicker beam scanning strategy and a new algorithm which would detect and compensate for scatterer motion is required.

VI. Conclusion

A formula for directional transmission beam space MVDR beamforming has been derived. As demonstrated by an example, the derived formula works effectively for the case of scanning directional transmission systems, while the standard beam space formula fails. The use of virtual beams to add extra eigenvalues in beam space covariance matrix to reduce high level out of sector power has also been demonstrated. The algorithm can be used for slowly moving non-stationary scatterers but performance is degraded by increasing mobility.

References

[1] J. Capon, "High resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57, no.8 pp. 1408-1418, Aug. 1969.

[2] D. A. Gray, "Formulation of the maximum signal to noise ratio array processor in beam space," *The Journal of the Acoustical Society of America*, vol. 72, no.4, pp. 1195-1201, Oct. 1982.

[3] D. A. Gray, "Output statistics of the sample matrix inverse beamformer implemented in beam space," *The Journal of the Acoustical Society of America*, vol. 80, no. 6, pp. 1737-1739, Dec. 1986.

[4] H. B. Lee and M. S. Wengrovitz, "Resolution threshold of beamspace MUSIC for two closely spaced emitters," *IEEE Trans. Acoustics, Speech and Signal Processing*, vol. 38, no. 9, pp. 1545-1559, Sep. 1990.

[5] P. Stoica and A. Nehorai, "Comparative performance study of element-space and beam-space MUSIC estimators," *Circuits, Systems and Signal Processing*, vol. 10, no. 3, pp. 285-292, 1991.

[6] G. Xu, S. D. Silverstein, R. H. Roy, and T. Kailath, "Beamspace ESPRIT," *IEEE Trans. Signal Processing*, vol. 42, no. 2, pp. 349-356, Feb. 1994.

[7] A. Hassanien and S. A. Vorobyov, "A Robust Adaptive Dimension Reduction Technique With Application to Array Processing," *IEEE Signal Processing Letters*, vol. 16, no. 1, pp. 22-25, Jan. 2009.

[8] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Part IV, Optimum Array Processing*. New York: Wiley, 2002.