MATRIX ANALYTIC METHODS WITH MARKOV DECISION PROCESSES FOR HYDROLOGICAL APPLICATIONS

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بر ای الهلم

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Contents

Abstract

In general, a physical process is modelled in terms of how its state evolves over time. The main challenge of modelling is to describe this evolution without unnecessary computation or making unrealistic simplifying assumptions. Markov chains have found widespread applications in many fields of analytic study from engineering to biology to linguistics. One of their most notable applications in hydrological applications has been modelling the storage of reservoirs, as described in Moran's influential monograph (Moran, 1955). One of the fundamental properties of Markov chains is that the future evolution depends only on the present state, and not on any of the previous states. This property is simply stated as the "memory-less" property or the Markov property.

In a Markov chain model the states representing the physical process are discrete, but time can be modelled as either discrete or continuous. In this thesis, time is modelled in discrete units because this is consistent with the well-established theory of Markov decision processes. The discrete states need not be a practical limitation because of continuous state variables, as in this case storage in a reservoir, can be discretised as a reasonable approximation.

There have been many advances in Markov chain modelling techniques in other fields, most notably in telecommunications with the development of matrix analytic methods. Matrix analytic methods exploit the structure of certain types of Markov chains in order to more efficiently calculate properties of the models. This thesis examines how these methods can be applied to hydrological applications with the goal of providing a framework for which more precise modelling can be achieved without extending computational times. There are many unique challenges due to the seasonal nature of hydrology as well as the tendency for persistence of hydrological conditions. This thesis explores some of these problems in four papers.

The first paper looks at the issues surrounding hydrological persistence and its incorporation into Markov decision processes using the Southern Oscillation Index as proxy.

The second paper looks at modelling using matrix analytic methods of spate flows in the Cooper Creek, which is an ephemeral river located in the South Australia.

The third paper looks at a way of modelling hydrological persistence with underlying hidden states in contrast to assumed dependence on the Southern Oscillation Index.

The final paper looks at multi-objective optimisation using first-passage time distributions with an application to a two reservoir system in South East England. The Pareto front of Pareto optimal policies is shown.

Statement of Originality

I, Aiden James Fisher, certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide.

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Published Works

Fisher, A. J., Green, D. A., Metcalfe, A. V., and Webby, R. B. (2008). "Optimal Control of Multi-reservoir Systems with Time-dependent Markov Decision Processes." In "Proceeding from Water Down Under 2008," pages 2610–2619.

Fisher, A. J., Green, D. A., and Metcalfe, A. V. (2010). "Managing river flows in arid regions with matrix analytic methods." *Journal of Hydrology*, 382: 128–137.

Fisher, A. J., Green, D. A., and Metcalfe, A. V. (2011). "Modelling of hydrological persistence for hidden state Markov decision processes." *Annals of Operations Research*, 199: 215–224.

Fisher, A. J., Green, D. A., Metcalfe, A. V., and Akande, K. (2014). "First-passage time criteria for the operation of reservoirs." *Journal of Hydrology*, 519, Part B: 1836–1847.

Statement of Authorship

OPTIMAL CONTROL OF MULTI-RESERVOIR SYSTEMS WITH TIME-DEPENDENT MARKOV DECISION PROCESSES

In "Proceeding from Water Down Under 2008", 2008.

Fisher, A.J. (Candidate)

Constructed mathematical model, conducted analysis and interpreted results, wrote manuscript and acted as corresponding author.

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Webby, R.B.

Researched and developed application as decision process problem, researched and fitted rainfall - runoff model, assisted in developing model and contributed to the writing of the manuscript.

I hereby certify that the statement of contribution is accurate and I give permission for the inclusion of the paper in the thesis.

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Green, D.A.

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STATEMENT OF AUTHORSHIP

MANAGING RIVER FLOWS IN ARID REGIONS WITH MATRIX ANALYTIC METHODS

Journal of Hydrology, 2010.

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MODELLING OF HYDROLOGICAL PERSISTENCE FOR HIDDEN STATE MARKOV DECISION PROCESSES

Annals of Operations Research, 2011.

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FIRST-PASSAGE TIME CRITERIA FOR THE OPERATION OF RESERVOIRS

Journal of Hydrology, under review.

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Metcalfe, A.V.

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Date: 10th April 2014

Akande, K.

Provided description of, and data for, the case study.

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Signed:

Date: 3rd April, 2014

Preamble

This thesis has been submitted to the University of Adelaide for the degree of Doctor of Philosophy. According to the University's Specification for Thesis a Doctoral thesis may comprise,

> a combination of conventional written narrative presented as typescript and publications that have been published and/or submitted for publication and/or text in manuscripts,

and this thesis takes this form.

The thesis has been divided into seven chapters:

The first chapter is a very brief introduction to the history of water management, the problems currently facing the world's water supplies, and the future in a changing climate.

The second chapter gives a brief introduction to discrete-time Markov processes or Markov chains. This chapter is only intended as a cursory view of the topic. A more general and in depth discussion can be found in any textbook on the topic, for example Norris (1997). A reader familiar with this material could skip this chapter.

The third chapter introduces several more advanced Markov models, including; the phase-type distribution, Markov arrival processes, and hidden Markov models.

The fourth chapter discusses solution techniques for Markov decision processes.

The fifth chapter comprises of a literature review of mathematical modelling of reservoirs and methods for determining optimal policy using Markov decision processes.

The sixth chapter comprises four published papers that form the main component of the thesis, along with an outline of each paper. The published papers are presented in the format they were printed in. All the papers have been scaled to the largest size possible that will allow the pages to fit within the University's specifications for a thesis.

The final chapter outlines potential future research following on from this thesis.