A Study of a Gough-Stewart Platform-based Manipulator for Applications in Biomechanical Testing

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A thesis submitted in fulfillment of the requirements for the degree of Ph.D. in Mechanical Engineering on 15 October 2013. Qualified on 5 February 2014.
Abstract

This thesis investigates the development and application of a robotic system for *in-vitro* biomechanical testing to study the mechanisms leading to human joint injury and degeneration in an ethical and safe manner. Six degree of freedom (6-DOF) robotic-based systems, in particular Gough-Stewart platform-based systems, have been increasingly used in applications of biomechanical testing where 6-DOF mobility, large load capacity, and high stiffness and positioning accuracy are required from the testing machine. This study proposes a novel Gough-Stewart platform-based manipulator with ultra-high stiffness and accuracy for use in biomechanical testing and investigates its mechanism and control. Not only restricted to biomechanical testing, the proposed manipulator concept can also be applied to other robotic-based applications, particularly those requiring ultra-high accuracy positioning under large external loads (e.g. machining). Four main features of the proposed manipulator are individually studied in this thesis: namely, stiffness and control of a non-collocated actuator-sensor mechanism, active preload control using actuation redundancy for backlash elimination, adaptive velocity-based load control of human joints for unconstrained testing, and reproducing the *in-vivo* measured kinematics on human cadaveric joints.

**Stiffness and Control of the Non-collocated Actuator-Sensor Mechanism**

A novel Gough-Stewart platform-based mechanism is proposed with a fully decoupled actuator-sensor arrangement for passively compensating the structural compliance of the manipulator. The stiffness of the robot load frame and the sensing frame are respectively quantified using the robot kinematics error model combined
with finite element analysis (FEA) on the top and bottom assemblies. Numerical results demonstrate that the proposed mechanism improves the stiffness of the robotic testing system in excess of an order of magnitude on the translational axes and two orders of magnitude for rotational axes compared to a traditional actuator-sensor collocated design. Control disturbances arising from actuator-sensor non-collocation is addressed using decoupled control. Experimental results show that the proposed decoupled control algorithm improves the dynamic accuracy of the manipulator by approximately 25% on average.

**Active Preload Control Using Actuation Redundancy for Backlash Elimination**

This thesis investigates combining the benefits of both active and passive preload control methods, using actuation redundancy to prevent backlash on a general Gough-Stewart platform. Both the mechanical configuration and the dynamics model of the redundant manipulator are investigated for the ease of control. A novel online optimization algorithm combined with a feedback force control scheme is formulated to achieve a real-time method which is robust to both model inaccuracy and load disturbance. Simulation results demonstrate an effective preload efficacy by the redundant arrangement within the workspace of the robot. Simulation results also show that the proposed method can effectively achieve backlash-free positioning of the manipulator under large 6-DOF external loads. Experimental results further prove that the proposed method can eliminate backlash instabilities from control and consequently higher bandwidth control can be achieved by the robot with improved accuracy.

**Adaptive Velocity-based Load Control of Human Joint for Unconstrained Testing**

A novel adaptive velocity-based load control method is proposed in this thesis to more effectively achieve pure force or moments on human joints under unconstrained testing compared to existing methods. The force/moment control gains are designed to vary adaptively based on the tracking performance of the force/moment to make a compromise between load following and control stability, which makes the proposed method self-adaptive to unknown joint dynamics. Sheep functional spinal units are used to experimentally validate the method on the custom-built Gough-Stewart platform-based manipulator. Experimental results illustrate the efficiency of the
proposed method, which can be further improved when overcoming certain limitations of the system (e.g. load sensor noise, position inaccuracy arising from backlash, etc.)

Reproducing the *In-vivo* Measured Kinematics on Human Cadaveric Joints

This thesis develops a method to scientifically reproduce the general *in-vivo* kinematics measured from a living human on human cadaver joints using the custom-built Gough-Stewart platform-based manipulator. A human wrist is used as a typical example to elaborate the theory of the method and to assess the fidelity of the method. The proposed method uses a 3-D motion capture system to collect the *in-vivo* wrist kinematics from 12 patients undertaking hammering motion. In parallel, CT scans and static motion capture are undertaken on 8 cadaveric human wrist specimens in an effort to define the locations of the coordinate systems. Consequently the *in-vivo* measured wrist kinematics is transformed to the kinematics of the robotic testing system, which is used to reproduce the hammering motion. Experimental results show that the accuracy of the reproduced motion on the cadaveric samples is of similar magnitude to the measurement error of the motion capture system. Experimental results also show that the assumption of fixed wrist joint centre of rotation is valid for motion reproduction.
Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Boyin Ding and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Boyin Ding
Acknowledgement

I would like to acknowledges the efforts of all the people who have made a contribution towards this thesis. Many thanks to my parents, Guozheng Ding and Juzhen Li, who encouraged my English and engineering interests from an early age, and to their generous financial support while finishing this thesis.

I would also like to sincerely thank the following: Associate Professor Benjamin S. Cazzolato for his primary supervision, inputs and comments, encouragement and trust, and large amount of valuable time spent with me; Dr John J. Costi for his external supervision, expertise in biomechanics, and providing equipment and environment for experiment; Dr Steven Grainger for his co-supervision and comments; and Richard M. Stanley for his significant contribution in mechanical design, specimen preparation, and robot assembly. The author would also like to give thanks to the biomechanics research team: Dr Francois Fraysse, Dr Dominic Thewlis, Dr Duncan McGuire, and Dr Lucian B. Solomon for their expertise and cooperation in biomechanical testing. I am also indebted to the “electronics guys”, Craig Peacock and Damian Kleiss from the Flinders University and Norio Itsumi and Philip Schmidt from the University of Adelaide for all their help with the design, construction, and debugging of the electronics and experimental apparatus. In addition, I would like to acknowledge the technical assistance and previous work provided by Dr David Churchill, Mack G. Gardner-Morse and Professor Ian A.F. Stokes from the University of Vermont.

This work was supported by (in chronological order): Foundation Daw Park at the Repatriation General Hospital, the Health and Medical Research Fund Research Equipment program, South Australia Government, Flinders University, and the University of Adelaide which are gratefully acknowledged.
Finally, I would like to express my gratitude to the centre of my universe, Jiajing for here endless encouragement, support and patience during my Ph.D. candidature.
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Chapter 1

Introduction

1.1 Motivation of this Thesis

Joint diseases such as osteoarthritis, rheumatoid arthritis and more than 100 other forms of inflammatory conditions affect several hundred million people worldwide. This figure will sharply increase due to the predicted doubling in the number of people over 50 by the year 2020 (The bone and joint decade, 2010). Joint disease is the leading cause of disability in the United States and accounts for half of all chronic conditions in persons aged 65 and over. Although joint disease is more common in older people, younger people can develop it, usually as the result of injury, malformation, or a genetic defect in the cartilage (Buckwalter, 2004). Extensive human health research has been conducted into preventing joint disease, by weight reduction and activity modification, and by treatments such as physical therapy and joint replacement surgery (Valderrabano and Steiger, 2004, Burnett, 2005). There is no doubt that a good understanding of human joint behaviour is crucial to both prevention and treatment.

Musculoskeletal biomechanics involves studying the mechanical properties of human joints (e.g. knee, hip, spinal segment) in an effort to better understand the mechanisms leading to their injury and degeneration. The invasiveness required to study the in-vivo mechanics of human joints has led to in-vitro biomechanical testing of human cadaver specimens using machinery. Single-axis materials testing machines equipped with custom-built apparatus have been largely used for in-vitro testing of human joints (Adams et al., 1980, Latham et al., 1994, Bulter, et al., 1980, Fukubayashi et al.,
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1982, Hotchkiss, 1987). Although valuable information has been collected in this way, it remains difficult to simulate the six degree of freedom (6-DOF) motions and loads experienced by human joints during daily activities. Six-axis materials testing machine, e.g. Instron Bioplus and AMTI ADL-Force 5, is usually very expensive (greater than USD 0.7 million) and consequently various custom-built 6-DOF testing systems were developed specifically for biomechanical testing (Wilke et al., 1994, Hollis at al., 1991). These custom-built systems normally have highly complex mechanisms that result in high maintenance costs. In addition, their mechanical complexities largely increase the difficulties in modifying their functions for different testing purposes. Fujie et al. (1993, 1996) explored the use of robotics technology in biomechanical testing. A standard 6-DOF articulated robot arm was programmed to apply the major functions for studying human knees. Following on from their work, the use of 6-DOF articulated robots were employed in various biomechanics studies (Woo et al., 1999, Most, 2000, Frey et al., 2004, Gilbertson et al., 2000, Zantop et al., 2007). The main drawbacks of articulated robots are low stiffness and poor positioning accuracy due to their serial kinematic structures (Merlet, 2006). These limit the use of articulated robots in biomechanical testing, where large loads and high accuracy are required to test the stiff human joints.

In the last decade, the use of 6-DOF parallel robots—Gough-Stewart platform (or normally called a Stewart platform) has attracted the attention of many biomechanics researchers (Stokes et al., 2002, Howard et al., 2007, Matthew and Dickey, 2007, Liem et al., 2007, Goertzzen and Kawchuk, 2009, Arashidi, 2009). Compared with articulated serial robots that carry loads in bending, the Stewart platform, with its parallel elements carrying loads axially, is significantly stiffer, leading to improved accuracy under large payloads, as well as greater load carrying capacity, lower cost (for equivalent capacity), and a more compact size (Merlet, 2006). Due to these advantages, the Stewart platform is very suitable for use in biomechanical testing applications. Nevertheless, there remains a question as to whether the stiffness and accuracy of the Stewart platform-based testing system is sufficient for the purpose of testing stiff human joints. Both the compliance of the robot loading frame (Svinin et al., 2001) and the clearance/backlash in the robot joints (Nordin and Gutman, 2002) can significantly degrade the accuracy and precision of the system, and thus largely degrade the fidelity of the testing results. It remains a challenge to use current robotics
and control methodologies to effectively address these issues. In addition, it remains difficult to control the robotic manipulator to achieve specific biomechanical testing tasks. For example, the control of pure force or moment on human joints (Matthew and Dickey, 2007) and the reproduction of in-vivo kinematics on human joints (Howard et al., 2007) are both very challenging tasks.

The primary goal of the research presented in this thesis is to develop a novel Stewart platform-based manipulator for applications in biomechanical testing and to study the mechanism and control aspects of the manipulator in an effort to improve its stiffness and accuracy. Furthermore, this research also investigates the use of the Stewart platform-based manipulator to apply the pure forces or moments and to reproduce the in-vivo kinematics on human joints respectively. A full description of the research presented in this thesis to address the general research aim is given in Section 1.3.

1.2 Literature Review

This section provides a review of the literature specific to the content of this thesis. In Section 1.2.1 and 1.2.2, a discussion of the scope and implementation requirements of biomechanical testing is presented followed by a review of 6-DOF biomechanical testing systems. In Section 1.2.3 and 1.2.4, literature investigating compliance and backlash compensation methods is discussed. Section 1.2.5 presents a review of the literature exploring the pure force or moment control strategies for unconstrained testing. Finally, a discussion of the research on reproducing the in-vivo human joint kinematics is presented in Section 1.2.6.

1.2.1 Biomechanical Testing

As a relatively novel subject, biomechanical testing still remains unfamiliar to the public, however, within the scientific community it is developing rapidly towards building a database for human health research. This subsection briefly introduces the scope of biomechanical testing and discusses the general requirements for its implementation.

1.2.1.1 Scope

Biomechanical testing involves experimentally evaluating the mechanical properties (for example strength, fatigue and stability) of the biological and synthetic systems in
the human body (as well as other living species) for the purpose of solving specific clinical problems. Human joints such as the hip, knee and spine are the main focus in biomechanical testing as they are subject to large loads during daily activities of up to 15 times body weight and consequently have a high risk of injury and degeneration (Mow and Huiskes, 2005). Consisting of bones, ligaments, tendons, cartilages and muscles, the mechanical behaviour of human joints are extremely complex. By better understanding the mechanisms leading to injury and degeneration through biomechanical testing, better treatments can be developed for patients and better prevention advice can be given to healthy people. As an example, replacing the degenerated knee joint with an artificial joint through surgery is an effective treatment to relieve pain. However, current joint replacement technology can only guarantee the implanted joint will last for up to 20 years and the instability of the implanted joint is a major issue (Williams, et al., 2010). In order to achieve life-long implanted joints with better stability, more research and testing are required to be conducted on both the intact knee and treated knee. For another example, affecting at least 80% of the population, low back pain accounts for one of humankind’s most common complaints. Although the causes of low back pain in most cases are unclear, disc degeneration is thought to be of primary importance and may even lead to the majority of the physical disorders of the lumbar spine (Costi, 2004). This has become a very popular research topic in biomechanical testing. Besides joint injury and degeneration, bone fracture and muscle tears also belong to the scope of biomechanical testing.

1.2.1.2 Requirements to Implement Biomechanical Testing

The implementation of biomechanical testing is not trivial. The \textit{in-vivo} testing on a living human is restricted by ethics and safety issues, particularly when measuring the loading which is highly invasive. Consequently, the use of testing machines to conduct \textit{in-vitro} testing on human cadaver joints has become a trend (Matthew and Dickey, 2007). To achieve the fundamental goals of biomechanical testing, the testing machine should have the following specifications. Firstly, 6-DOF movement is required from the machine in order to study the three-dimensional (3D) behaviour of human joints. During daily activities, our joints (e.g. a functional spinal unit shown in
1.2. Literature Review

Figure 1.1. Degrees of freedom of a functional spinal unit (FSU). The human joint is able to rotate and translate in all three planes and therefore has six degrees of freedom. These degrees of freedom can be broken down into three rotations—left and right axial rotation, flexion and extension, and left and right lateral bending—and three translations—anterior and posterior shear, compression and decompression, and left and right lateral shear. (Wilke et al., 1998)

Fig. 1.1) experience 3D motions and therefore have 6-DOF which can be expressed as translations and the rotations about three orthogonal axes. In joint spinal biomechanics, the three translations are normally termed anterior and posterior shear, compression and decompression, and left and right lateral shear while the three rotations are named left and right axial rotation, flexion and extension, and left and right lateral bending (Wilke et al., 1998). A 6-DOF machine allows simulation of not only the motion in each of the six axes, but also the combined motion in more than two axes. Secondly, the machine is required to have a large load capacity to fail the joints. Human joints are designed to sustain very high loads during movement. For example, the knee joint can experience more than 600% body weight (BW) impact when jumping and landing (Cleather et al., 2013) and a spinal disc can experience 420% BW compressive load when lifting a 20 kg object (Wilke et al., 2001). The peak moments experienced by the joints during daily activities are also not trivial, e.g. 6% body weight times height (BW*Ht) for the knee joint (Thambyah et al., 2004). This
means the machine should be capable of generating more than 10,000N forces and 100Nm moments for failure testing. Last but not least, high accuracy and precision is required from the machine to obtain a reliable testing result. Human joints are normally very stiff mechanisms. For example, the compressive stiffness of a functional spine unit (FSU) is about 5,000N/mm (Costi, 2008). Consequently, when testing a FSU in compression, a translational error as small as 0.1mm can result in a force error as high as 500N. The accuracy of the machine is highly dependent upon its stiffness, particularly under large payloads. To ensure that the machine inaccuracy is negligible compared with the joint deformation, the stiffness of the machine needs to be much higher than the stiffness of the joint under testing. For example, when testing a FSU, a machine with 50,000N/mm compressive stiffness can cause 10% testing error while a machine with 500,000N/mm compressive stiffness can lead to only 1% testing error. Besides the machine stiffness/compliance issue, the backlash in the machine mechanical system is the other main factor impacting on the accuracy as well as the precision of the machine. Since backlash is necessary for clearance to accommodate manufacturing errors, provide space for lubrication and allow for thermal expansion of components (Wittenstein Inc., 2006), it is impossible to eliminate backlash in mechanical design. Consequently, a backlash compensation algorithm is required when controlling mechanical systems with high accuracy and precision. A review and comparison of the 6-DOF machines developed for biomechanical testing thus far is presented in Section 1.2.2.

Besides the fundamental requirements discussed above, there are two specific requirements in evaluating human joints—unconstrained testing and in-vivo kinematics reproduction. Unconstrained testing (or flexibility testing) was proposed and described by Panjabi (1988), where a pure force or moment is applied about one axis and the five remaining degrees of freedom are left unconstrained. Compared with constrained testing which constrains the joint movements in all 6-DOF, unconstrained testing allows complete freedom of movement of human joints, thereby allowing natural joint behaviour to take place. Furthermore, unconstrained testing effectively decouples the fully coupled 6-DOF properties of human joints, thereby largely simplifying their mathematical models (Most, 2000). As a result, a large number of special loading apparatuses, 6-DOF systems and robotic-based methods have been developed to enable unconstrained testing on human joints, which are described in
Section 1.2.5. The other specific requirement is to reproduce the *in-vivo* kinematics on cadaver joints and measure the resulting loads. This simulates and studies the joint dynamics in a more physiological condition where important data can be obtained to improve diagnostic analyses, reconstruction techniques, and rehabilitation protocols (Moore, 2006). A few attempts to reproduce the *in-vivo* joint kinematics with robotics-based systems are reviewed in Section 1.2.6.

### 1.2.2 6-DOF Biomechanical Testing Systems

The study of joint biomechanics has grown rapidly in the last three decades. Consequently, many testing machines were developed. As 6-DOF motion is fundamental for studying the 3D movement of human joints, a comprehensive review on the 6-DOF systems developed for biomechanical testing has been conducted.

#### 1.2.2.1 Commercial Material Testing Systems

Although there are hundreds of commercial materials testing systems on the market (e.g. Instron, AMTI, Zwick and Tinius Olsen), most of them are uni-axial machines. Even in the category of multi-axial machines, there are few that are capable of 6-DOF movement. Instron Biopuls-Spine (Fig. 1.2) and AMTI Force-5 (Fig. 1.3) are two typical 6-DOF machines designed specifically for biomechanical testing. Biopuls-Spine was proposed as a spine testing machine. It consists of a shear table at the bottom of the device which enables the movements along anterior-posterior and lateral shear, and a servo actuator box at the top, which is capable of moving the specimen along the remaining four degrees of freedom. AMTI Force-5 was proposed for testing hip, knee, and spine joints. The machine itself only has 4-DOF—axial rotation is achieved by the bottom torsion table, while anterior-posterior shear, compression-decompression, and flexion-extension are realized by the servo actuator box at the side. With the help of additional kits (e.g. hip/spine kit and knee kit), lateral bending and shear can be achieved, although only uncontrolled passive motion is allowed along lateral shear. In both Biopuls-Spine and Force-5, a 6-DOF load-cell is mounted beneath the specimen for measuring the loads.

Table 1.1 shows the main specifications of Biopuls-Spine and Force-5 in each degree of freedom. Both the machines have rather small load capacity, particularly on the rotational axis as each axis is controlled by only one actuator. The strokes of Biopuls-
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Figure 1.2. Instron Biopuls-Spine showing testing of a FSU. A shear table at the bottom is able to achieve anterior-posterior and lateral shear, while a servo actuator box is able to produce compression-decompression, axial rotation, and flexion-extension, and lateral bending. A 6-DOF load-cell is mounted beneath the specimen for measuring the loads at the specimen. (Jones, 2012)

Figure 1.3. AMTI Force-5 showing testing of an artificial hip joint. A torsion table at the bottom is able to achieve axial rotation while a servo actuator box at the side is able to reproduce anterior-posterior shear, compression-decompression, and flexion-extension. Lateral bending can be achieved by assembling a hip/spine kit on the machine. A 6-DOF load-cell is mounted beneath the specimen for measuring the loads at the specimen. (AMTI Inc., 2011)
Table 1.1. Main specifications of Instron Biopuls-Spine and AMTI Force-5. *Accuracy here represents the tracking performance of the machine in each axis during a dynamic condition. These values are typical for testing at a 1Hz repetition rate while running the ISO waveforms.

<table>
<thead>
<tr>
<th>Platform</th>
<th>Load Capacity</th>
<th>Stroke</th>
<th>Displacement Resolution/Accuracy*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Biopuls</td>
<td>Biopuls</td>
<td>Biopuls</td>
</tr>
<tr>
<td>An/Posterior Shear</td>
<td>500N</td>
<td>±50mm</td>
<td>±25mm 30µm/0.75mm 0.1mm/0.5mm</td>
</tr>
<tr>
<td>Lateral Shear</td>
<td>500N</td>
<td>±50mm</td>
<td>±6mm 30µm/0.75mm Na</td>
</tr>
<tr>
<td>Axial Load</td>
<td>4400N</td>
<td>±50mm</td>
<td>±25mm 0.1µm/0.25mm 0.1mm/0.5mm</td>
</tr>
<tr>
<td>Lateral Bending</td>
<td>30Nm</td>
<td>±20°</td>
<td>±9° 0.01°/0.2° 0.1°/0.09°</td>
</tr>
<tr>
<td>Flex/Extension</td>
<td>25Nm</td>
<td>±30°</td>
<td>±100° 0.01°/0.2° 0.1°/1°</td>
</tr>
<tr>
<td>Axial Rotation</td>
<td>30Nm</td>
<td>±20°</td>
<td>±100° 0.01°/0.2° 0.1°/1°</td>
</tr>
</tbody>
</table>

Spine are very reasonable for spinal testing, however they have limitations when testing knee and hip joints whose full range of movement in flexion and extension is normally very large (e.g. 140° for knee as stated by Laubenthal in 1972). On the other hand, the Force-5 has very large strokes in flexion and extension and axial rotation but has very limited strokes in lateral bending and shear. In terms of system accuracy, both machines have appropriate dynamic tracking accuracy on each axis relative to the stroke of that axis. However, the absolute accuracy of the machines under static condition is expected to be relatively poor as relatively low system stiffness and accumulated backlash must arise from their mechanisms where six independent stages are arranged in series to collectively achieve 6-DOF motion. Another main limitation of these machines is that they can only rotate about a fixed spatial point. This greatly increases difficulties in cadaver preparation. If there is an error in aligning the joint centre of rotation (COR) with the machine COR, the testing is no longer valid. In addition, these machines are prohibitively expensive for the majority of research groups (> US$700,000).

1.2.2.2 Custom-built systems

Compared with commercial 6-DOF testing machines, custom designed and built 6-DOF testing systems are more common and cost effective. Figure 1.4 shows a representative custom-built system developed by Wilke et al. (1994). The core part of the system is a specially designed gimbal that allows rotation around all three axes.
Figure 1.4. Custom-built spine testing system developed by Wilke et al. (1994). Stepper motors were used to produce translations and rotations about all three axes, and a pneumatic system to maintain a preload on the specimen and to also control muscle forces. A 6-DOF load-cell was used to measure the loads at the specimen.

and is able to move up and down in a vertical direction. A stepper motor with a torque of 55Ncm and 1.8° per step is connected to each of the three axes of the gimbal. Harmonic drive gears with a ratio of 1:160 increase the moment up to 50Nm. The motors then flex, bend, or torque the specimens with about one-ninetieth of one angular degree. The gimbal allows a lateral bending up to ±90°, and an axial rotation and a flexion/extension angle up to ±45°. Two parallel pneumatic cylinders are used to compress or decompress the specimens in the vertical direction. The apparatus can apply an axial preload up to 1000N. Shears in anterior-posterior or lateral directions are applied with additional stepper motors. Cables can be attached at the top slide or directly to the specimen, then guided with pulleys, and rolled up or off from a special spindle. A 6-DOF load-cell measures the loads at the specimen.

Although very sophisticated, the custom-built system is extremely complex and sufficient expertise must exist “in-house” to create, operate and maintain such a system. Moreover, its mechanical complexity results in very low system stiffness and significant accumulated backlash in all 6-DOF. Like the commercial testing machines, the custom-built system has a fixed centre of rotation (COR) and thus is not able to adjust rotations when the joint COR changes during movement.
1.2.2.3 Serial Robot-based systems

Fujie et al. (1993) are the pioneers who explored the use of robotics technology in biomechanical testing. A commercial 6-DOF articulated robot arm was programmed to realize the major functions for studying human knees. Following on from their work, the use of commercial 6-DOF articulated robots was widely adopted in various biomechanics studies (Woo et al., 1999, Most, 2000, Frey et al., 2004, Gilbertson et al., 2000, Zantop et al., 2007). Figure 1.5 shows a typical example. The robot (KR 125, KUKA Robots) is a six-joint, serially articulated manipulator, which allows 6-DOF movement of the knee. The universal force moment sensor (UFS) allows the measurement of three forces and three moments along a Cartesian axis system. The knee is mounted to the system with the tibia attached to the end-effector via the UFS while the femur is mounted to the base. Compared with materials and custom-built testing machines, the use of robots in biomechanical testing is more flexible. By establishing coordinate systems, the joint COR can be virtually defined in software and therefore the robot COR is changeable.

Figure 1.5. 6-DOF articulated robot (KR 125, KUKA Robots) testing a cadaver knee joint (Zantop et al., 2006). (a) The robotic manipulator can move the knee in 6-DOF, while the universal force moment sensor (UFS) can measure three orthogonal forces and moments. (b) The knee is mounted to the system with the tibia attached to the end-effector of the robot via the UFS while the femur is mounted to the base.
Table 1.2. Specifications of KUKA Robots KR 125 and KR 1000 TITAN. (KUKA Robots, 2013)

<table>
<thead>
<tr>
<th></th>
<th>Payload</th>
<th>Repeatability</th>
<th>Weight</th>
<th>Work Envelope</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>KR 125</strong></td>
<td>1,250N</td>
<td>±0.2mm/±0.02°</td>
<td>975kg</td>
<td>39m³</td>
<td>USD$100,000</td>
</tr>
<tr>
<td><strong>KR 1000 TITAN</strong></td>
<td>10,000N</td>
<td>±0.1mm/±0.01°</td>
<td>4950kg</td>
<td>69.5m³</td>
<td>USD$200,000</td>
</tr>
</tbody>
</table>

The specifications of the robot (KR 125) are shown in Table 1.2. The repeatability of the system is 0.2mm and 0.02° for position and orientation of the end-effector. Although the system is only capable of generating 1,250N rated payload, the weight of the system is as high as 975kg. Like most articulated robots, the KR 125 has a very large work envelope (39m³). The cost of this robot is approximately USD$100,000 not including installation, design, and manufacture of specialized fixation devices for testing joints. An additional quote was sought for the strongest and largest 6-axis articulated robot on the market (KR 1000 TITAN, KUKA Robots) with a rated payload of 10,000N and a weight of almost 4950kg. The cost for this large, heavy robot is approximately USD$200,000. There is no doubt the load-to-weight ratio of serial articulated robots is very low. This leads to relatively high cost and low system dexterity of the robot to achieve the required load capacity for biomechanical testing. The other main drawback of an articulated robot is its very low system stiffness and significant accumulated backlash arising from its open chain kinematics formed by links connected in series.

### 1.2.2.4 Parallel Robot-based systems

The Stewart platform—a classic 6-DOF parallel robot design— was proposed in 1965 as a flight simulator (Dasgupta and Mruthyunjaya, 1998). Since then, a wide range of applications including aerospace, automotive, nautical, and machine tool technology have benefited from the design. As shown in Figure 1.6, the Stewart platform consists of a moving platform at the top and a rigid base at the bottom which are connected via six linearly actuated legs with passive spherical joints attached at both ends. The coordinated motions of the legs allow the top platform to move in 6-DOF. Compared with articulated serial robots which by their very design carry loads in bending, the Stewart platform with its parallel elements carrying loads axially is significantly stiffer, leading to much better accuracy under large payloads, has a greater load carrying capacity, costs less (for equivalent capacity), and is more compact in size.
1.2. Literature Review

(Merlet, 2006). Furthermore, inaccuracies resulting from the robot joints (e.g. backlash) are averaged instead of accumulated due to its parallel kinematics structure. In the last decade, there were a few attempts to use parallel robots based on the concept of the Stewart platform in biomechanical testing.

Figure 1.7 shows a commercial 6-DOF parallel robot (R-2000 Rotopod, Parallel Robotic Systems Corporation) commonly used in biomechanical testing. The robot design is based on a Stewart platform, whereby instead of the leg lengths being actuated, six fixed length legs are actuated about a circular track. The change in
Table 1.3. Specifications of R-2000 Rotopod. (Parallel Robotic Systems Corporation, 2013)

<table>
<thead>
<tr>
<th></th>
<th>Payload</th>
<th>Repeatability/Accuracy</th>
<th>Weight</th>
<th>Work Envelope</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>R 2000</strong></td>
<td>2,270N</td>
<td>0.025mm/0.05mm</td>
<td>545kg</td>
<td>Tx, Ty: 200mm</td>
<td>USD$100,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Tz: 200mm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rx, Ry: ±15°</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Rz: ±720°</td>
<td></td>
</tr>
</tbody>
</table>

The relation between the bases of the legs allows the robot top platform to move in 6-DOF. The testing sample (e.g. a spine model in the picture) is mounted between a cross beam attached to the frame surrounding the robot and a six-axis load cell which is attached to the robot top platform. The specifications of this commercial robot are shown in Table 1.3. Compared with commercial articulated robots, this parallel robot has a much higher repeatability and accuracy, a higher load to weight ratio, but a smaller working envelope. The unique design of its bottom circular track significantly increases the motion range about the vertical axis of the robot. As this commercial robot was not specifically developed for biomechanical testing, it has limitations in both load capacity and system stiffness.

Figure 1.8 shows a custom-built parallel robot called HexaSpine developed by Liem et al. (2007) for spinal testing. The main innovation of this design is the use of fluidic (pneumatic) muscles as the actuators to avoid slip-stick effects. For control and testing purposes, each actuator is equipped with sensors measuring force, pressure and stroke. This results in complexity in its mechanical design which can largely degrade
Table 1.4. Load capacity of HexaSpine parallel platform (Forces in N and Torques in Nm)

<table>
<thead>
<tr>
<th></th>
<th>Fx</th>
<th>Fy</th>
<th>Fz</th>
<th>Mx</th>
<th>My</th>
<th>Mz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative direction</td>
<td>-622.8</td>
<td>-709.4</td>
<td>-1117.3</td>
<td>-51.5</td>
<td>-57.9</td>
<td>-61.6</td>
</tr>
<tr>
<td>Positive direction</td>
<td>582.6</td>
<td>704.5</td>
<td>2734.7</td>
<td>52.3</td>
<td>57.9</td>
<td>61.6</td>
</tr>
</tbody>
</table>

...the stiffness of the robot. By measuring the forces on each leg and transforming these forces according to the geometry of the robot, the 6-DOF forces and moments acting at the specimen can be obtained. As shown in Table 1.4, the load capacity of the HexaSpine parallel platform is suitable for simulating daily loading conditions but is not high enough for failure testing.

Figure 1.9 shows another custom-built parallel robot-based testing system developed by Walker and Dickey (2007) at the University of Guelph. Due to budget constraints, the structure of this custom-built system exhibits large flexibility that directly results in very poor system accuracy during testing. Although there is large deficiency in its design, the concept to use the inner space of the robot to mount specimens is very clever. This not only makes the system more compact but also avoids introducing compliance from additional framework.

Figure 1.9. Parallel robotic-based testing system developed by Walker and Dickey (2007). A porcine lumbar spine specimen is mounted within the manipulator and is faced anteriorly. Captions describe the base and top plates, linear actuator, leg potentiometer, UFS load cell, spine mounting cup, aluminium plate fixators, and the intervertebral joint.
Walker and Dickey (2007) are not the only ones to use the inner space of the robot for testing. Stokes et al. (2001) proposed a sophisticated parallel robot-based testing system at the University of Vermont (Figure 1.10). The moving platform of the robot is driven by stepper motors coupled to precision lead screw actuators. A 6-DOF load-cell is mounted between the moving platform and a specimen mounting plate. Six linear encoders are used to measure and control the displacements of the robot. The encoders are mounted nominally parallel to the actuators, but are not physically linked to them. Instead, the encoders span between the base, and the specimen mounting plate. As a result, they provide a measure of the position of the specimen mounting plate independent of load-cell compliance. The specifications of this parallel robot-based system are shown in Table 1.5. The system has an appropriate load capacity and a good screw rate. The stroke of the robot is relatively small as it was proposed for spinal testing. The system accuracy is fine but is no doubt degraded by the compliance of the robot frame and the backlash in the system.

![Parallel robotic-based testing system developed by Stokes et al. (2001).](image)

**Table 1.5. Specifications of the parallel robotic-based testing system developed by Stoke et al. (2001)**

<table>
<thead>
<tr>
<th>Load Category</th>
<th>Load Capacity</th>
<th>Stroke</th>
<th>Resolution</th>
<th>Accuracy</th>
<th>Maximum Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>An/Posterior Shear</td>
<td>2000N</td>
<td>±80mm</td>
<td>0.005mm</td>
<td>0.05mm</td>
<td>100mm</td>
</tr>
<tr>
<td>Lateral Shear</td>
<td>2000N</td>
<td>±85mm</td>
<td>0.005mm</td>
<td>0.05mm</td>
<td>100mm</td>
</tr>
<tr>
<td>Axial Load</td>
<td>4500N</td>
<td>±50mm</td>
<td>0.005mm</td>
<td>0.05mm</td>
<td>100mm</td>
</tr>
<tr>
<td>Lateral Bending</td>
<td>100Nm</td>
<td>±15°</td>
<td>0.002°</td>
<td>0.05°</td>
<td>25°</td>
</tr>
<tr>
<td>Flex/Extension</td>
<td>100Nm</td>
<td>±15°</td>
<td>0.002°</td>
<td>0.05°</td>
<td>25°</td>
</tr>
<tr>
<td>Axial Rotation</td>
<td>100Nm</td>
<td>±15°</td>
<td>0.002°</td>
<td>0.05°</td>
<td>25°</td>
</tr>
</tbody>
</table>
1.2.2.5 Conclusion

This subsection reviewed four types of 6-DOF systems developed for biomechanical testing. A comprehensive comparison between the specifications of these systems is concluded in Table 1.6 as well as the specifications required from an ideal system. Obviously, among the four types of systems, the parallel robot-based system demonstrates superior performance in almost all the aspects, particularly in load capacity, stiffness, backlash level, size/weight and dexterity which are critical for the applications of biomechanical testing. The only drawback of the parallel robot-based system is its relatively small range of motion in its rotational axis (normally within ±35° for a normal Stewart platform design). Even so, it is acceptable for most biomechanical testing and an offset can be set in the specimen fixation device to locate the specimen into the desired testing range. Therefore, the Stewart platform is regarded as mostly approaching the ideal system. The capacities required from the ideal system are used as the objectives to develop the Stewart platform testing system. Some of the specifications can be achieved by better design (e.g. load capacity), whilst some requires further research work (e.g. rigidity and accuracy). Although the Stewart platform is well known for its high stiffness, there still remains a question

<table>
<thead>
<tr>
<th></th>
<th>Commercial System</th>
<th>Custom-built System</th>
<th>Serial Robot-based System</th>
<th>Parallel Robot-based System</th>
<th>Ideal System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load Capacity</td>
<td>Medium</td>
<td>Low</td>
<td>Low</td>
<td>High</td>
<td>&gt;10kN, &gt;100Nm</td>
</tr>
<tr>
<td>Stiffness</td>
<td>Medium</td>
<td>Low</td>
<td>Low</td>
<td>High</td>
<td>&gt;500,000N/mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>&gt;1000Nm/°</td>
</tr>
<tr>
<td>Backlash</td>
<td>Accumulated</td>
<td>Accumulated</td>
<td>Accumulated</td>
<td>Averaged</td>
<td>&lt;0.01mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>&lt;0.01°</td>
</tr>
<tr>
<td>Range of Motion</td>
<td>Medium</td>
<td>Large</td>
<td>Large</td>
<td>Medium</td>
<td>&gt;60°</td>
</tr>
<tr>
<td>COR</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Changeable</td>
<td>Changeable</td>
<td>Changeable</td>
</tr>
<tr>
<td>Size/Weight</td>
<td>Large</td>
<td>Large</td>
<td>Large</td>
<td>Compact</td>
<td>Compact</td>
</tr>
<tr>
<td>Dexterity</td>
<td>Medium</td>
<td>Medium</td>
<td>Low</td>
<td>High</td>
<td>&gt;50°/s</td>
</tr>
<tr>
<td>Ease of Use</td>
<td>Medium</td>
<td>Low</td>
<td>Medium</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>Maintenance Fee</td>
<td>High</td>
<td>High</td>
<td>Medium</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>Price</td>
<td>High</td>
<td>Medium</td>
<td>Medium</td>
<td>Medium</td>
<td>&lt;USD200,000</td>
</tr>
</tbody>
</table>
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if its stiffness is sufficiently high for the purpose of biomechanical testing. To reach the desired system stiffness while maintaining the dexterity and cost of the system, additional methods may be required to compensate the compliance in the Stewart platform-based testing system. Therefore, a review on the existing compliance compensation methods is presented in Section 1.2.3. Besides compliance, backlash is the other main reason causing inaccuracy (normally higher than 0.05mm) in the Stewart platform, although the backlash is averaged due to its parallel kinematics structure. The methods attempting to compensate inaccuracy arising from backlash were generally reviewed and are described in Section 1.2.4.

1.2.3 Compliance Error Compensation

In a general robotic system, the compliance error of the robot structure cannot be observed by its internal sensors, and therefore cannot be directly compensated using robot control. This can largely decrease the accuracy of the robot in applications where it is subjected to very high external forces/torques, e.g. biomechanical testing. The Stewart platform is well known to provide higher stiffness compared to a similar sized serial robot, and therefore has been used in biomechanical testing by several biomechanics research groups. However, none of them performed an analytical study on their system stiffness, which can be largely degraded by sources of compliance from the robot frame and specimen fixation devices. This subsection reviews the existing methods developed to compensate for the compliance error in robotic systems.

1.2.3.1 Active Compensation Based on Control Software Modification

A number of studies attempted to actively compensate the compliance error in robotic systems based on control software modification. Usually, the problem can be solved in two ways that differ in degree of modification of the robot control software: a) manipulator model modification b) prescribed trajectory modification. As shown in Figure 1.11(a), the first approach integrates the stiffness model into the whole robot model and consequently takes the deformation of the manipulator into consideration during control. This approach suits real-time manipulator compensation implemented online although it displays the following drawbacks. Firstly, the stiffness model of the robot is essentially more complicated than the geometrical model and requires rather intensive computations (Su and McCartney, 2006). Secondly, the compensation results
Figure 1.11. Robot error compensation methods: a) modification of the manipulator model b) modification of the target trajectory. (Klimchik et al., 2013)

are largely subjected to the accuracy of the stiffness model, which is very sensitive in practice (Pashkevich et al., 2011). In addition, it is difficult to include the stiffness model into the controller of an integrated commercial robotic system. As shown in Figure 1.11(b), the second approach uses additional compliance error compensator to modify the prescribed Cartesian space trajectory in an offline manner. Being independent of the robot position controller, this approach is attractive for industrial applications. In robot-based machining applications, a method of symmetrical trajectory is normally used to realize trajectory modification assuming the robot stiffness model is linear (Depince and Hascoet, 2006, Gunnarsson et al., 2000). Although mathematically cheaper, its performance is limited in practice where strong non-linearities can appear in the robot stiffness model. In short, compensating robot compliance error by control software modification is always difficult to realize in real applications due to the inaccuracy in stiffness modelling and the inefficiency from offline trajectory modification.
1.2.3.2 Passive Compensation Based on Physical Decoupling

As mentioned in Section 1.2.2, Stokes et al. (2001) proposed a novel Stewart platform-based mechanism with actuators and encoders partially decoupled (Figure 1.12) which passively compensates for the load-cell compliance from the robot position measurement, although neither theoretical analysis nor results were given. Additional work is required to quantify the efficiency of this mechanism in compliance error mitigation. Furthermore, the proposed mechanism only considers compensating the compliance error from the load-cell and ignores the compliance error from the robot load frame which can easily propagate into the encoder measurement from the bottom coupling. In addition, the proposed mechanism results in a non-collocated sensor-actuator structure and therefore further study is required to be conducted on the control aspect of this structure.

![Figure 1.12. Zoomed section of the parallel robotic testing system developed by Stokes et al. shown in Fig 1.10, showing the unique top platform assembly design which decouples the upper ends of the actuators and encoders (Stokes et al., 2001)](image)

1.2.4 Backlash Error Compensation

Besides compliance, backlash or clearance in the robot active joints (e.g. ball screw actuators) is the other main cause of inaccuracy in the Stewart platform. Standard ball screw and lead screw configurations typically have backlash ranging from 50 micrometres to 0.25 millimetre which is significant compared to the required accuracy for biomechanical testing (Thomson Industries, Inc., 2013). The methods attempting to compensate for inaccuracy arising from mechanical backlash were generally reviewed and are described in this subsection.
1.2.4.1 General Industrial Solution

Anti-backlash ball screw actuators are available on the market. The main idea is to use an additional anti-backlash nut to mechanically force the ball nut and screw to engage. Most designs place a compression spring or other compliant member between the two nut halves to remove radial clearance. Preload between the nut halves must equal or exceed the applied axial load in the direction in which the assembly is loaded through the take-up mechanism to prevent lost motion. The result is a higher torque requirement which typically necessitates a larger motor. Moreover, the design significantly limits the load capacity of the actuator. As a result, a new proprietary anti-backlash nut was developed without introducing excessive drag force, however it greatly increases the complexity of the actuators which directly leads to larger actuator size and much higher cost.

Rather than eliminating backlash mechanically, addressing backlash inaccuracy via control technology is more general and cost-effective. Dual-loop PID control is a standard control form which has been widely used in industry for controlling mechanical systems with backlash for more than 20 years (Tal, 1998) due to its simplicity and effectiveness. Nowadays, most of the industrial servo motion controllers have this configuration. As shown in Figure 1.13, the method employs two sensors which are placed on both the motor and the load. The PID controller is then separated into two parts. The main loop is closed with the load sensor, and the PI terms are applied to that loop to eliminate the load position error. The D term of the controller, however, is applied to the motor encoder, resulting in a damping term which is based on the motor position. This configuration continuously compensates

![Figure 1.13. Block diagram showing the configuration of dual loop PID control (Tal, 1998)](image-url)
load position errors without suffering from backlash instabilities (with load sensor only) or backlash inaccuracies (with motor encoder only) commonly experienced in single loop control. However, the bandwidth of its closed loop system can be significantly limited when the preload between the motor and load is close to zero. Under such a circumstance, a high controller gain can cause the motor and the load to engage and disengage repeatedly (known as “buzz”) at a very high frequency which can easily stimulate the mechanical resonance of the system and as a result lead to system instabilities (e.g. limit cycles) and mechanical wear.

1.2.4.2 Backlash Model-based Control

Obviously, the dual-loop control scheme is not robust enough as it treats backlash as uncertainty. To better control a mechanical system with backlash, a large number of control methods were proposed by taking the backlash behaviour into consideration. Backlash is a common non-linearity in mechanical systems. Depending on the mechanical cause of the backlash, and the operating conditions, different mathematical models must be utilized to model the behaviour. In general, a two-mass model as shown in Figure 1.14 is used which assumes the transmission system between the motor and load is a mass-less shaft with a maximal backlash angle of $\alpha$.

Based on the two-mass model, the classical dead zone model was proposed with the assumption that the damping of the shaft is negligible (Tustin, 1947). The dead zone model is the most useful mathematical model which can be found in almost any basic

![Figure 1.14. Backlash in a two-mass system.](image)

Figure 1.14. Backlash in a two-mass system. $T_m$, $\omega_m$, and $J_m$ represent the torque, velocity, and inertia of the motor while $T_L$, $\omega_L$, and $J_L$ represent the torque/force, velocity, and inertia/mass of the load. The transmission system between the motor and load is modelled as a mass-less shaft. $\theta_s$ is the twist of the shaft (not explicitly shown in the figure), $\theta_d = \theta_m - \theta_L$ is the displacement angle between the motor angle $\theta_m$ and the load angle $\theta_L$, and $\theta_b = \theta_d - \theta_s$ is the backlash angle, with the restriction that $-\alpha \leq \theta_b \leq \alpha$. $k_s$, $c_s$ are the elasticity and damping coefficients, respectively, of the shaft, giving the shaft torque $T_s$. (Nordin and Gutman, 2002)
control course, for analysis and simulations of an elastic shaft with backlash. Besides the dead zone model, the exact model (Nordin, 1995), the describing function model (Thoms, 1954), and the hysteresis model (Tao and Kokotovic, 1996) were also often used to describe the behaviour of backlash. Following the appearance of these models, many linear and nonlinear model-based control methods occurred for addressing the backlash issue. Typical linear methods include standard PI, PD and PID control with a cascaded load observer (Brandenburg, 1986), controller design using quantitative feedback theory (Oldak et al., 1994), and variable structure controller design (Azenha and Tenreiro, 1996). Typical non-linear methods include fuzzy logic control (Lin et al., 1996) and non-linear PID control (Slotine and Li, 1991). In these works, both the control accuracy and the conditions for limit cycles were analysed. Results show that model-based methods can improve the overall system performance, however there is a trade-off between accuracy and stability like most non-linear control problems. In addition, it is always difficult to obtain an accurate model of the backlash in real industrial applications. Particularly after the mechanical system has been operated for a long duration, the backlash model can vary significantly due to the thermal expansion and mechanical wear. As backlash model-based control is not the focus of this thesis, the author only listed a small portion of the earlier work. If interested, the audience can find a comprehensive review of the backlash model-based control methods in the paper written by Nordin and Gutman (2002).

1.2.4.3 Preload Control

The previous backlash compensation methods are either mechanically complex or have limited bandwidth. Also these methods were designed for use in single actuator systems and thus are not very effective for a parallel robotic system with multiple joints. Consequently, researchers developed a new concept to address the backlash problem specifically in parallel robotic systems. The main idea is to generate interactive preloads between the multiple joints of the robot for the purpose of avoiding backlash.

As the pioneer in this field, Muller (2005) formulated an active control method which used actuation redundancy while considering dynamic effects for the purpose of backlash prevention along a specified path. Simulation results were given for a planar
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4RRR manipulator and a spatial heptapod. Although this general solution can be applied to all types of redundant parallel robots in theory, it requires an offline optimization process to solve and its performance is highly sensitive to model error due to its open-loop nature. These issues prevent the proposed approach working in real world applications.

Wei and Simaan (2010) proposed and analysed a passive method on the other hand using preloaded passive joints in order to eliminate backlash throughout a desired workspace when given norm-bounded external loads. A planar PRR parallel robot was used as a case study to present the method. Although much simpler than the active control method, the passive method is hard to realize on a parallel robot with more than three degrees of freedom (e.g. Stewart platform) and is not suitable for applications where large external loads in the order of 100N or greater are involved (e.g. biomechanical testing).

In addition, Choi et al. (1996) and Roberz et al. (2010) studied the use of dual motor control to replace the mechanical anti-backlash gear. The idea was to let the gear be driven by two motors with preloads of opposite signs such that the backlash gap is closed. The cost for this solution are increased energy usage in steady state, increased capital cost for purchasing double the number of motors and motion controllers for a parallel robot, and requirement of custom-built gear boxes.

1.2.5 Pure Force or Moment Control on Human Joints for Unconstrained Testing

In the field of joint biomechanics, unconstrained testing is largely used by applying a pure force or moment to human joints. Most of the testing machines can only be operated under position control and hence are not capable of unconstrained testing. This subsection reviews the methods developed for achieving pure force or moment on human joints.

1.2.5.1 Special Loading Apparatus

Many special loading apparatuses were proposed for use on the single axis testing machines (Panjabi et al., 1976, Schulz et al., 1996, Adams and Hutton, 1981, Bulter et al., 1980, Fukubayashi et al., 1982). Two typical examples are given here.
As shown in Figure 1.15, a torsion testing rig fitted to an Instron materials testing machine was developed by Latham et al. (1994) for spinal testing. This design used angled sliders acting on bearings, supported on a disc thrust bearing, to convert the linear motion of the crosshead into axial rotation. A double slider carriage—an X-Y table—was located between the torsion drive and the specimen to ensure that the specimen did not have an imposed fixed centre of rotation. In this way, pure torsion can be applied to the specimen without introducing coupled shears along x and y axes.

Hollis et al. (1991) developed various loading mechanisms used to apply either anterior-posterior force or varus-valgus moment to the knee. For anterior-posterior loading, the knee was mounted at fixed flexion angles while the remaining five DOF of knee motion were allowed as shown in Figure 1.16(a). In particular, axial tibial rotation was permitted by a rotary bearing connected to the tibia. The anterior-posterior load was applied perpendicular to the tibia in the anterior-posterior direction by means of a rod attached to the crosshead of the Instron testing machine. Varus-valgus moment was applied using a similar loading device with slight modification as
shown in Figure 1.16(b). As in the anterior-posterior test, five DOF motion were allowed but with the knee flexion angle fixed. The varus-valgus moment was applied to the tibia through a rotary bearing, pulley and cable system.

Obviously the use of special loading apparatus greatly increases the complexity of the mechanical system and different mechanisms are required to test different specimens or even to test the cases on different axes. Furthermore, the COR of the loading apparatus is fixed and consequently the specimen COR must align with the COR of the loading apparatus in set-up to ensure the fidelity of the testing which is not easy in practice.

1.2.5.2 6-DOF Systems with Independent Axes

It is easy to implement unconstrained testing in 6-DOF systems with independent axes, e.g. Instron Biopuls, ATMI Force-5, and custom-built systems developed by Wilke et al. (1994). In these systems, the movement of the specimen along each axis is controlled by each of the six independent actuators. As a result, the system can be reconfigured for unconstrained testing by simply replacing the original six position control loops to six force control loops closed by the feedback from the 6-DOF load-cell. Commanding one force control loop to follow the desired force or moment and
asking the other five force control loops to maintain zero force or moment allows pure force or moment to be applied to the specimen. However, few research groups use these systems to implement unconstrained testing mainly due to their high cost, fixed COR, and inflexibility of reconfiguration.

1.2.5.3 Robotic-based Methods

Robotic systems were often utilized to achieve unconstrained testing for two main reasons. Firstly, a robotic system has a movable COR which can be simply reconfigured in software. As a result, any specimen misalignment error resulting from mounting can be easily offset. Secondly, robotic systems are designed as reprogrammable-friendly devices for a wide range of applications. The users are allowed to write and modify their custom upper level program (e.g. force control algorithm) while the lower level factory program remains unchanged (e.g. position control which is essential for robot control) to realize position-based force control in all six DOF.

As the pioneers who explored the use of robotics technology to study human joints, Fujie et al. (1993, 1996) presented a method to control the force and moment applied to the joint as follows. An industrial robot/UFS system was set up as similar to the set-up in Figure 1.5 where the UFS measured the loads at the specimen. A compliance matrix of the specimen transformed the errors between the target loads and the measured loads to the desired displacements of the specimen required for minimizing the current load errors. The robot was then commanded to move to the desired position. This algorithm iteratively ran and incrementally regulated the position of the robot until the load errors on the specimen were within an acceptable tolerance. However neither the approach for calculating the compliance matrix of the specimen nor the results regarding the accuracy of the proposed force control method were clarified in their papers. Following on from this work, several similar methods were developed with slight modification. Gilbertson et al. (2000) proposed a pathSEEK algorithm which controls the displacement of the specimen about one rotational axis while minimizing the coupling of out of plane forces. Although satisfactory results were obtained, the method inherently ignored the loads on the other three DOF and therefore was no longer fully unconstrained. Tian and Gilbertson (2004) studied the
use of fuzzy logic instead of the pathSEEK algorithm to improve the control performance. However, the study was still implemented in a 3-DOF manner. Walker and Dickey (2007) studied the use of a Newton-based approach known as Broyden’s method to estimate the compliance matrix of the specimen in real-time and apply the proposed adaptive method on a custom-built parallel robot (Figure 1.9). However the resulting force control accuracies were very poor with up to ±25N force error and ±10 Nm moment error as Broyden’s method is very sensitive to inaccuracies from both measurement noise and the robot position control. As a result, the method was finally restricted to 3-DOF.

The previous position-based force control methods normally require a compliance matrix to be updated during each iteration. The requisite calculations are intensive which limits the control loop speed and continuity of the test. In addition, measurement errors due to system noise or other disturbances can result in excessive forces or instabilities. Goertzen et al. (2009) studied the use of velocity-based force control in robotic biomechanical testing. A constant force control gain was used to relate the force/moment error to the desired velocity of the robot on each axis. The algorithm was tested on a commercial parallel robot shown in Figure 1.7. Results showed that the method can continuously and effectively minimize the loads along all five unconstrained axes on a rabbit spine segment. Nevertheless, the success of testing a rabbit joint does not ensure the method can work properly on a human joint, since a human joint is much stiffer and has stronger non-linear couplings between its axes. In addition, the optimal tuning of the force control gain was not clarified in the paper and is required every time when testing a new specimen.

1.2.6 Reproducing In-vivo Kinematics on Human Joints

Previous studies of joint biomechanics used robotic systems to intuitively apply a simplified joint motion (e.g. pure axial rotation or flexion-extension combined with bending) under constrained testing and a pure joint force or moment under unconstrained testing. Although effective to study joint behaviour, these approaches did not truly simulate the physiological movement of the joint which is very complex during daily activities. Simulating the joint motion in a more physiological way enables greater understanding of joint behaviour under its natural movement, which is
1.2. Literature Review

essential to the study of the cause of joint disease as well as the development of improved joint replacement. This subsection reviews the methods that attempted to reproduce the *in-vivo* measured kinematics on cadaver joints using robotic systems.

Moore et al. (2006) developed and evaluated a method to more accurately reproduce the previously recorded kinematics of a knee joint using an industrial robot. They analysed the use of multiple registrations at different knee flexion-extension poses, robot calibration, and robot working volume reduction for the purpose of mitigating the inaccuracies from the industrial robotic system when reproducing the kinematics. Although results showed that multiple registrations improved the accuracy in position of the reproduced kinematics when compared to only a single registration, the resulting accuracy of the method was still very limited due to the inaccuracies of the industrial robot. Moreover, the paper did not clarify the approach for measuring the *in-vivo* kinematics of the knee where more errors can propagate into the final results.

Howard et al. (2007) developed a thorough technique for the robotic reproduction of previously measured subject-specific *in-vivo* motion based on the use of spatial referencing, a parallel robot, and custom software. Their paper discussed the measurement of the *in-vivo* kinematics, the special mechanical design for improving the accuracy of the reproduction, and the calculation of the robot motion from the measured specimen kinematics. Two sheep knee joints were used as an example to verify the concept and satisfied reproduction accuracies demanded from the experiment. However, the method was designed to reproduce subject-specific *in-vivo* kinematics and consequently required the living specimen to be sacrificed for research. This restricted the use of the technique to animal samples only.

1.2.7 Conclusion from the Literature Review

Biomechanical testing has been developed to study the mechanisms leading to human joint injury and degeneration. The invasiveness required for implementing biomechanical testing has led to *in-vitro* testing on human cadaver joints using 6-DOF testing machines with high load capacity and high accuracy.

Compared to commercial materials testing machines and custom-built testing systems, robotic-based testing systems are more cost-effective and flexible in use. Moreover, the use of a parallel robot (e.g. Stewart platform) is superior to the use of industrial
articulated robots in the aspects of weight-to-load ratio, stiffness and dexterity. However, to achieve the micrometre-order accuracy for joint testing, both errors arising from the compliance and the backlash of the Stewart platform are required to be further compensated.

The robot compliance error can usually be compensated using control software modification but it is always difficult to realize this in real applications due to the inaccuracy from the stiffness modelling and the inefficiency from offline trajectory modification. Physical decoupling between the actuation systems and the sensing systems is a potential way to compensate for compliance error passively from the Stewart platform. More analysis and evaluation are required to be implemented on both its structure and control.

The previous backlash compensation methods are either mechanically complex or have limited bandwidth. A new concept is proposed to address the backlash problem for parallel robotic systems by actively or passively causing interactive preloads between the robot joints. Both the active method and the passive method have drawbacks such as intensive computation in trajectory planning and limited robustness to model error and disturbance, which prevent the proposed methods from working on a Stewart platform in real applications.

In joint biomechanics, unconstrained testing is largely used by applying a pure force or moment to human joints. Many special loading apparatuses have been proposed for use with a single axis machine with the expense of involving complexities in both the mechanical system and the experimental set-up. Robotic-based methods are attractive due to the reconfigurable nature of the robotic systems. A few position-based force control methods are proposed, however these methods require intensive calculations while exhibiting poor load minimizing performance and even instabilities. A simple velocity-based force control method has been developed which shows satisfied performance when testing a compliant rabbit spine segment. More work is required to be conducted on the optimal control gain tuning to enable the method to work properly on a stiff human joint with strong couplings between its anatomical axes.

Reproducing the in-vivo measured kinematics on human cadaver joints allows the joint to be tested in a more physiological way. The previously developed methods focus on either addressing the inaccuracy of the robotic-based system when
reproducing given kinematics or reproducing the subject-specific \textit{in-vivo} kinematics on animal joints. A method is lacking to scientifically reproduce the general \textit{in-vivo} kinematics measured from live humans on a donated human cadaver joint.

1.3 Contribution of this Thesis

The main contribution of this thesis is to develop a novel Stewart platform-based robotic testing system for use in applications of biomechanical testing. In the development process, the four research gaps described in the literature review section (e.g. robot compliance compensation, robot backlash compensation, robotic-based pure force or moment control, and reproducing \textit{in-vivo} kinematics of human joints using robotic-based system) are addressed. Upon finishing, the developed system is required to be capable of implementing major testing on any large-scale biological specimen (e.g. joints and bones) with high fidelity and high flexibility. The specific contributions and objectives in addressing each of the research gaps are discussed in the following paragraphs.

Firstly, a novel Stewart platform-based mechanism was proposed in this thesis with a fully decoupled actuator-sensor arrangement for passively compensating for the structural compliance of the manipulator. The stiffness of the proposed manipulator was analysed and quantified via modelling and simulation. For minimizing the inaccuracies arising from robot compliance in biomechanical testing, the stiffness of the proposed manipulator was required to reach 100 times the stiffness of the testing samples (e.g. human joints). In addition, the control aspect of the proposed manipulator was studied theoretically and verified using experiments. Control issues arising from actuator-sensor non-collocation needed to be addressed to guarantee the accuracy of the manipulator tracking under a dynamic testing scenario. The proposed manipulator was not only limited to the applications of biomechanical testing but also applicable to all robotic-based applications, particularly the ones that necessitate ultra-high accuracy under high external forces/torques (e.g. machining and material testing).

Secondly, this thesis investigated combining the benefits of both active and passive preload control methods, using actuation redundancy to prevent backlash on a general Stewart platform. Early attempts described in Muller (2005) using offline optimization method only worked in theory where no model inaccuracy and
disturbance appear. Therefore, a novel online optimization algorithm combined with a feedback force control scheme was developed in this thesis to achieve a real-time method which was robust to both model inaccuracy and load disturbance. In addition, the design and modeling of the redundant manipulator were analysed to ensure a simplified but effective control realization. The efficiency of the method was verified by both simulation and experiments. The proposed Stewart platform-based preload control method can be utilized to achieve backlash elimination in a wide range of ultra-high accuracy applications (e.g. biomechanical testing, machining, military, etc.).

Thirdly, a novel adaptive velocity-based load control method was proposed in this thesis to more effectively achieve a pure force or moment on human joints under unconstrained testing. The force/moment control gains were designed to vary adaptively based on the tracking performance of the force/moment to make a compromise between load following and control stability. Although adaptive force control in velocity-commanded robots was an established technique, its application and extension to control the 6-DOF loads on biological joints was very challenging due to the highly nonlinear behaviour of the joints. Due to availability and similarity with human, sheep FSUs were used to experimentally validate the method on the custom-built Stewart platform-based manipulator. The proposed adaptive load control method can be used to test any biological specimen with a nonlinear stiffness.

Finally, this thesis developed a method to scientifically reproduce the general in-vivo kinematics measured from a living human on human cadaver wrist joints using the custom-built Stewart platform-based manipulator. The wrist kinematics measured from 12 patients were averaged and reproduced on 8 human cadaver wrists to validate the concept. The developed method can be applied to study a variety of human or animal joints under conditions that accurately reflect complex physiological motion.

In the technical aspects, the author undertook almost all the design and experimental work covered in this thesis except for the initial robot frame design, specimen fixation device design, and more biomedical related work (e.g. specimen preparation, surgery, X-ray and CT, and data analysis in biomechanics), which were undertaken by specialists in the respective fields.
1.4 Overview of this Thesis

Chapter 2 presents both the stiffness and control analysis on the proposed Stewart platform-based manipulator with decoupled sensor-actuator locations. In this chapter, the concept of the proposed mechanism is initially presented followed by the theoretical analysis on the stiffness and control aspects of the manipulator. The manipulator stiffness was validated based on the robot kinematic error model combined with finite element analysis (FEA) on both the top and bottom assemblies and was compared to the normal case where the sensors and actuators are collocated. A simple kinematics-based decoupled control algorithm was proposed to address the disturbance issue arising from the actuator-sensor non-collocation arrangement and a dynamics-based control scheme was also formulated for the manipulator. An experiment was established to validate the kinematics-based control effects on the manipulator with and without decoupling.

Chapter 3 presents the active preload control method proposed for eliminating backlash on a general Stewart platform using actuation redundancy. This chapter firstly describes the backlash free condition, which is the essential goal for preload control. Based on the backlash-free condition, the overall solution was formulated followed by four main problems to be further treated in the chapter. Firstly, the configuration of the redundant manipulator was analysed for the ease of control. Secondly, the inverse dynamics equation of the redundant manipulator was formulated for estimating the preloads on the robot actuators. Thirdly, an online preload optimization algorithm was developed to optimize the load trajectory on the redundant actuator based on the inverse dynamics equation and the backlash-free condition for the purpose of preventing backlash instabilities from the manipulator with minimum energy consumption. Fourthly, the force control on the redundant actuator was studied to improve the efficiency in active preload tracking. Simulations were used to validate the preload distribution efficiency of the proposed redundant manipulator configuration as well as quantify the performance of the proposed preload optimization algorithm and the force control scheme. Finally, a physical experiment was established with a large amount of design work to verify the effects of the proposed concept in real applications.
Chapter 1. Introduction

Chapter 4 presents the adaptive velocity-based load control method proposed for more effectively achieving pure forces or moments on human joints under robotic-based unconstrained testing. In this chapter, the overall concept of the method was initially described followed by the adaptive algorithm for updating the force control gains and other improvements of the control scheme (e.g. hyperbolic sine function, feed-forward control, and follower load preload). An ovine FSU as well as the custom-built Stewart platform-based testing system was used to experimentally assess the proposed robotic-based load control method under unconstrained testing on each of the 6-DOF.

Chapter 5 presents the scientific method proposed for reproducing the \textit{in-vivo} measured kinematics on human cadaver joints using the custom-built Stewart platform-based testing system. Human wrists were used as a typical example throughout this chapter to elaborate the theory of the method and to experimentally assess the fidelity of the method. The proposed method consists of four main phases. The first phase measured the \textit{in-vivo} wrist angles from the movement of living humans (e.g. hammering motion). The second phase prepared the cadaveric specimen and established coordinate systems on the wrist based on reflective markers and CT construction. The third phase transformed the \textit{in-vivo} measured angles to the 6-DOF kinematics of the wrist. The final phase sets up the cadaveric specimen on the custom-built Stewart platform-based testing system and transformed the wrist kinematics to the robot kinematics for reproduction. Experimental results demonstrate the accuracy of the proposed reproduction method, the load responses at the wrist joint centre, and the resulting carpal kinematics of the wrist.

Chapter 6 presents the robot control system design where the control hardware and software design, system functionality and GUI design, and robot specifications are discussed in sequence. In addition, this chapter presents three typical applications of the developed robotic testing system in biomechanics—assessment of impaction bone grafting, understanding of 3D lumbar intercerebral disc internal strain under repetitive loading, and assessment of primary stability of cementless tibial tray.

The thesis is concluded in Chapter 7, with potential future work discussed.
1.5 Publications and Awards Arising from this Thesis

Conference paper publications:


Journal paper publications:

Chapter 1. Introduction


Awards:

“Six Degree of Freedom Hexapod Robot for Biomechanical Research”

- 2012 South Australia Engineering Excellence Awards (Malcolm Kinnaird and Innovation Award—overall winner award; Best Research and Development Award)
- Finalist in the national 2012 Australian Engineering Excellence Awards
Chapter 2

Stiffness Analysis and Control of a Stewart Platform-based Mechanism with Decoupled Sensor-actuator Locations

2.1 Introduction

In a general robotic system, the compliance of the robot structure cannot be observed by its internal sensors, and therefore cannot be directly compensated using the robot control. This can significantly decrease the accuracy of the robot in applications where it is subjected to very high external forces/torques. A typical application where such an issue exists is the use of robotic systems to test the six degrees of freedom (6-DOF) properties of biological specimens (Fujie et al., 1993, Woo et al., 1998), for example a human spine intervertebral disc. As some biological specimens can have very high stiffnesses (e.g. 5000 N/mm axially for a human disc), the robotic system is required to have a very high load capacity to deform the specimen, as well as having a very high accuracy to measure and control the displacement of the specimen. This means that the stiffness of the robotic system must be several orders of magnitude higher than the biological specimen otherwise the compliance of the robot can significantly degrade the fidelity of the test results. Alternatively a separate metrology system is required, which leads to increased capital costs.
The Stewart platform, a 6-DOF parallel mechanism (Merlet, 2006), is well known to provide higher stiffness compared to a similar sized serial robot, and therefore has been used in biomechanical testing by several biomechanics research groups (Walker and Dickey, 2007, Howard, 2007, Goertzen and Kawchuk, 2008). However, none of these groups performed an analytical study on their system stiffness. A more general review shows that the stiffness of a general Stewart platform has been studied thoroughly (Gosselin, 2002, EI-Khasawneh and Ferreira, 1999, Svinin, 2001) but no studies ever attempted to offer an effective method to mitigate the overall compliance of a Stewart platform. Stokes et al. (2002) proposed a Stewart platform-based testing machine for the purpose of decoupling the force sensor compliance as well as part of the structure compliance from the robotic system, although neither theoretical analysis nor results were given.

In this chapter, to effectively address the robot compliance issue in biomechanical testing, the author proposes a Stewart platform-based manipulator with decoupled sensor-actuator locations based on an extension of the concept described by Stokes et al. (2002). In Section 2.3, an overview of the manipulator mechanism is given followed by the stiffness analysis of the manipulator. Section 2.3 also presents the control aspect of the manipulator, mainly the decoupling algorithm corresponding to the decoupled sensor-actuator locations. The effective stiffness of the robotic system as well as its load frame stiffness is quantified using the computational model combined with finite element analysis (FEA) in Section 2.4. Finally, Section 2.5 demonstrates the control accuracy of the manipulator using experiments.

2.2 Background Theory

This section covers the background theory behind the study in this chapter. Audiences who do not have knowledge of robotics and Stewart platform are encouraged to read through this section to assist in the understanding of the work done in this chapter.

2.2.1 Spatial Description and Transformation

In robotics, a description is used to specify certain attributes of various objects with which a manipulation system deals. These objects are parts, tools and the manipulator
2.2. Background Theory

![Figure 2.1. Description of a position and an orientation of frame {B} with respect to frame {A}](image)

The descriptions are positions, orientations and frames. For example, as shown in Fig. 2.1, a coordinate system frame \{B\} is attached to the robot end-effector to give a description of both the position and orientation of the robot end-effector relative to the global reference coordinate system frame \{A\} and can be formulated as follows:

\[
\mathbf{t}_B^A = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}, \quad \mathbf{R}_B^A = \begin{bmatrix} X_B \cdot X_A & Y_B \cdot X_A & Z_B \cdot X_A \\ X_B \cdot Y_A & Y_B \cdot Y_A & Z_B \cdot Y_A \\ X_B \cdot Z_A & Y_B \cdot Z_A & Z_B \cdot Z_A \end{bmatrix}
\]  

(2.1)

where \(\mathbf{t}_B^A\) represents the position of the origin of frame \{B\} relative to frame \{A\} which is described as a \(3 \times 1\) position vector whose elements are the projections of the vector on the three principle axes of frame \{A\}. \(\mathbf{R}_B^A\) represents the orientation of frame \{B\} relative to frame \{A\} which is described as a \(3 \times 3\) rotation matrix whose components are simply the projections of the unit vectors of the three principle axes of frame \{A\} onto the unit directions of the three principle axes of frame \{B\}.

In a great many of the problems in robotics, multiple coordinate system frames are required and the transformations between these frames are critical for solving the problems. A simple example is given in Fig. 2.2 where the position \(\mathbf{t}_C^C\) and the orientation \(\mathbf{R}_C^C\) of frame \{C\} relative to frame \{A\} can be obtained if the position \(\mathbf{t}_C^B\) and orientation \(\mathbf{R}_C^B\) of frame \{C\} relative to frame \{B\} and the position \(\mathbf{t}_B^A\) and orientation \(\mathbf{R}_B^A\) of frame \{B\} relative to frame \{A\} are known. The transformation can be described as:

\[
\mathbf{R}_C^A = \mathbf{R}_B^A \mathbf{R}_C^B, \quad (2.2)
\]

\[
\mathbf{t}_C^A = \mathbf{t}_B^A + \mathbf{R}_B^A \mathbf{t}_C^B. \quad (2.3)
\]
One method of describing the orientation of a moving frame relative to a reference frame is using roll ($\gamma$), pitch ($\beta$), yaw angles ($\alpha$). The method has various expressions and only the X-Y-Z fixed angles expression is considered here. As shown in Fig. 2.3, starting with coincident with a known reference frame {A}, the moving frame {B} is firstly rotated about $X_A$ by an angle $\gamma$, then is rotated about $Y_A$ by an angle $\beta$ and then rotate about $Z_A$ by an angle $\alpha$. The derivation of the equivalent rotation matrix is straightforward

$$R_{XYZ}(\gamma, \beta, \alpha)^B_A = R_Z(\alpha)R_Y(\beta)R_X(\gamma)$$

$$= \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cy & -sy \\ 0 & sy & cy \end{bmatrix}$$

(2.4)

where $c\alpha$ is shorthand for $\cos(\alpha)$ and $s\alpha$ for $\sin(\alpha)$. As the rotation matrix can be
described as a function of the roll, pitch, and yaw angles, the position and orientation of a frame \{A\} relative to a frame \{B\} can be described as a $6 \times 1$ pose vector

$$p^A_B = \begin{bmatrix} t^A_B \\ \gamma \\ \beta \\ \alpha \end{bmatrix}. \quad (2.5)$$

### 2.2.2 Inverse Kinematics

The inverse kinematics problem of the Stewart platform involves finding the six leg lengths based on the robot end-effector pose, and is essential for robot position control. The ball joints at the ends of the robot legs form a hexagon at the top and at the bottom of the device as shown in Fig. 2.4. The robot moving platform (end-effector) coordinate system \{E\} is attached to the centre of the top ball joints while the global coordinate system \{G\} is fixed at the centre of the bottom ball joints. Using the 4th leg as an example, the inverse kinematics solution can be described with equations

$$BA = GE + R(EA^m) - GB, \quad (2.6)$$

$$l_4 = \sqrt{(BA)^\top BA} \quad (2.7)$$

where $EA^m$ represents the position of the top ball joint (A) relative to \{E\} and $GB$ represents the position of the bottom ball joint (B) relative to \{G\}. $GE$ and $R$ represent the position and orientation of \{E\} relative to \{G\} respectively. $BA$ represents the directional vector of the 4th robot leg and $l_4$ represents the length of the 4th robot leg.

![Figure 2.4. Schematic showing the kinematics model of a general Stewart platform.](image-url)
Chapter 2. Stiffness Analysis and Control of a Stewart Platform-based Mechanism with Decoupled Sensor-actuator Locations

2.2.3 Direct Kinematics

The direct kinematics problem of the Stewart platform involves determining the pose of the robot end-effector, given the six robot leg lengths. Geometrically, it is equivalent to the problem of placing a rigid body such that six of its given points lie on six given spheres, which is a 40th degree polynomial with as many as 24 real solutions and is particularly challenging to solve. Minimization and root-finding methods are typically used to solve this problem (e.g. Newton-Raphson method). The main idea of these approaches is to iteratively search an end-effector pose that is able to make the difference between the estimated leg lengths and the measured leg lengths converge to a chosen tolerance. The approach can be described by the following equations for the \( n \)th iteration

\[
\mathbf{p}_i = \mathbf{p}_{i-1} + \mathbf{J}_{i-1} (\mathbf{L}_m - \mathbf{L}_{i-1}) \quad i \geq 1, \quad (2.8)
\]

\[
\mathbf{L}_i = \mathbf{L}_{i-1} + \mathbf{J}_{i-1}^{-1} (\mathbf{p}_i - \mathbf{p}_{i-1}) \quad i \geq 1 \quad (2.9)
\]

where \( \mathbf{p} \) represents the end-effector pose, \( \mathbf{J} \) represents the kinematic Jacobian matrix, \( \mathbf{L}_m \) represents the six measured leg lengths, and \( \mathbf{L} \) represents the estimated leg lengths. At each iteration, \( \mathbf{p}_{i-1} \) is calculated from the previous iteration via Eq. (2.8) except for the initial pose estimation \( \mathbf{p}_0 \), \( \mathbf{J}_{i-1} \) is determined by calculating the robot inverse kinematics Jacobian matrix \( \mathbf{J}_{i-1}^{-1} \) at \( \mathbf{p}_{i-1} \) (see Section 2.2.4 for more detail) and numerically inverting \( \mathbf{J}_{i-1}^{-1} \), \( \mathbf{L}_m \) is directly measured from the sensors, and \( \mathbf{L}_{i-1} \) is calculated from the previous iteration via Eq. (2.9) except for \( \mathbf{L}_0 \) which is calculated from \( \mathbf{p}_0 \) via inverse kinematics. The iterations drive the difference between \( \mathbf{L}_m \) and \( \mathbf{L}_i \) to approach zero and stops once the difference is below a chosen tolerance. The estimated end-effector pose at the final iteration \( \mathbf{p}_i \) is the direct kinematics solution. Due to its computational expense, the direct kinematics is specifically used for initial robot pose calibration.

2.2.4 Jacobians (Velocities and Statics)

The inverse kinematic Jacobian matrix of the Stewart platform relates the robot end-effector velocities to the actuated joint velocities (leg length variables)

\[
\dot{\mathbf{i}} = \mathbf{J}^{-1} \dot{\mathbf{p}} \quad (2.10)
\]
where \( \mathbf{p} \) represents the \( 6 \times 1 \) velocity vector of the end-effector, \( \mathbf{l} \) represents the \( 6 \times 1 \) velocity vector of the legs, and \( \mathbf{J}^{-1} \) represents the \( 6 \times 6 \) inverse kinematic Jacobian matrix. \( \mathbf{J}^{-1} \) is robot pose dependent and can be obtained as

\[
\mathbf{J}^{-1} = \begin{bmatrix}
(u_1)^T & (R(\mathbf{E}A_1) \times u_1)^T \\
(u_2)^T & (R(\mathbf{E}A_2) \times u_2)^T \\
(u_3)^T & (R(\mathbf{E}A_3) \times u_3)^T \\
(u_4)^T & (R(\mathbf{E}A_4) \times u_4)^T \\
(u_5)^T & (R(\mathbf{E}A_5) \times u_5)^T \\
(u_6)^T & (R(\mathbf{E}A_6) \times u_6)^T
\end{bmatrix}
\]  

(2.11)

where \( u_i = \mathbf{B}A_i/l_i \) \((i = 1:6)\). With reference to Fig. 2.4, \( \mathbf{B} \) and \( \mathbf{A} \) represent the direction vector and the length of the \( i \)th leg respectively, \( u_i \) represents the unit vector along the \( i \)th leg, \( \mathbf{E}A_i \) represents the position of the \( i \)th top ball joint relative to \{E\} and \( R \) represents the orientation of \{E\} relative to \{G\}. The task-space loads acting at the end-effector can be mapped to the equivalent forces along the robot legs via the transpose of the direct kinematic Jacobian matrix

\[
\mathbf{f} = \mathbf{J}^T \begin{bmatrix} F_e \\ M_e \end{bmatrix}
\]  

(2.12)

where \( F_e \) and \( M_e \) represent the three forces and three moments acting at the end-effector respectively, \( \mathbf{f} \) represents the six forces along the robot legs, and \( \mathbf{J}^T \) can be obtained by numerically inverting and transposing \( \mathbf{J}^{-1} \).

2.3 Theoretical Analysis

2.3.1 Mechanism Concept

Figure 2.5 shows the manipulator mechanism that forms the basis of this thesis. As with any Stewart platform design, the manipulator mainly consists of six linear legs, a moving platform assembly at the top, and a supporting frame at the bottom. Each of the six legs is a linear ballscrew actuator which is driven by a coupled servomotor. Via spherical joints, the upper ends of the legs are connected to a top platform while the lower ends of the legs are connected to three actuator pillars bolted on to a rigid base plate. The spherical joints form a hexagon at both the top and bottom. A 6-DOF load-cell is mounted beneath the centre of the top platform and is used to measure the forces and moments applied to the testing specimen. A height adjustable pillar is
Chapter 2. Stiffness Analysis and Control of a Stewart Platform-based Mechanism with Decoupled Sensor-actuator Locations

Figure 2.5. Left images: Inventor model showing the upper and lower decoupled sensor-actuator locations of the Stewart platform-based manipulator. Right image: Inventor model of the entire Stewart platform-based manipulator.

bolted at the centre of the base plate for mounting the bottom end of the specimen. The specimen pillar, together with the actuator pillars, allows both small specimens (e.g. spine disc), and large specimens (e.g. femur), to be fitted into the inner space of the manipulator. The main innovation of this mechanism is the use of sensors internal and separate from the load frame. Unlike the common design approach which has the sensors attached to the actuating frame, the current design decouples the sensor locations from the actuator locations for the purpose of increasing the effective stiffness of the manipulator. As shown in Figure 2.5, six linear encoders are mounted in parallel (when the device is at its nominal position) with the six actuators via spring loaded magnetic spherical joints. The encoder spherical joints are on the same plane as the actuator spherical joints, and thus also form a hexagon at both the top and bottom. A specimen fixation plate is attached beneath the load-cell for mounting the top section of the specimen and for mounting the upper ends of the linear encoders. Three encoder pillars sit on a hexagonal plate bolted between the upper part and lower part of the specimen pillar for mounting the lower ends of the linear encoders. As highlighted in Figure 2.5, the locations of the linear encoders are completely
decoupled from the robot load frame, and therefore a more accurate specimen
displacement can be measured, and as a result controlled, independent of the robot
load frame compliance.

2.3.2 Stiffness Analysis

In this subsection, the effective stiffness of the robotic system, as well as the load
frame stiffness, is analysed using the kinematic error model.

2.3.2.1 Stiffness Modelling Strategy

Figure 2.6 shows the kinematic model of the manipulator, which consists of an outer
Stewart platform (blue solid line) formed by the robot load frame and an inner Stewart
platform (black dash-dot line) formed by the sensing frame. These two platforms
share the same global coordinate system \( \{O\} \) located at the geometric centre of the
lower spherical joints and share the same moving platform coordinate system \( \{Op\} \)
located at the geometric centre of the upper spherical joints. \( A_i \) and \( B_i \) represent
the position vector of the lower actuator spherical joint (\( A_i \)) and the position vector of the
lower encoder spherical joint (\( B_i \)) with respect to \( \{O\} \) respectively. \( P_i^m \) and \( E_i^m \)
represent the position vector of the upper actuator spherical joint (\( P_i \)) and the position
vector of the upper encoder spherical joint (\( E_i \)) with respect to \( \{Op\} \) respectively.

The inverse kinematics of the two Stewart platforms are described as:

\[
\begin{align*}
    l_iu_i &= R P_i^m + t - A_i \quad \text{(2.13)} \\
    \lambda_i n_i &= R E_i^m + t - B_i \quad \text{(2.14)}
\end{align*}
\]
where \( l_i \) and \( \lambda_i \) are the length of the \( i \)th actuator and the length of the \( i \)th encoder respectively. \( u_i \) and \( n_i \) are the unit vectors along the \( i \)th actuator and along the \( i \)th encoder respectively. \( R \) is the rotation matrix describing the orientation of \( \{O_p\} \) with respect to \( \{O\} \). \( \mathbf{t} \) is the translation vector describing the position of \( \{O_p\} \) with respect to \( \{O\} \). Infinitesimal differences to Eq. (2.13) results in

\[
\Delta l_i \mathbf{u}_i + l_i \Delta \mathbf{u}_i = \mathbf{R} \Delta \mathbf{P}_i + \Delta \mathbf{R} \mathbf{P}_i + \Delta \mathbf{t} - \Delta \mathbf{A}_i .
\]

Assuming the perturbations are only along \( \mathbf{u}_i \) and therefore multiplying both sides of Eq. (215) by \( \mathbf{u}_i \) leads to

\[
\Delta \mathbf{R} \mathbf{P}_i \cdot \mathbf{u}_i + \Delta \mathbf{t} \cdot \mathbf{u}_i = \Delta l_i + \Delta \mathbf{A}_i \cdot \mathbf{u}_i - \mathbf{R} \Delta \mathbf{P}_i \cdot \mathbf{u}_i .
\]

We also know that

\[
\Delta \mathbf{R} \mathbf{P}_i = \Delta \mathbf{\theta} \times \mathbf{R} \mathbf{P}_i
\]

where \( \Delta \mathbf{\theta} \) is a vector that contains three task-space angle errors. Substituting Eq. (2.17) into Eq. (2.16) results in

\[
\left[(\mathbf{u}_i)^T \ (\mathbf{R} \mathbf{P}_i \times \mathbf{u}_i)^T\right] \frac{\Delta \mathbf{t}}{\Delta \mathbf{\theta}} = \Delta l_i + \Delta \mathbf{A}_i \cdot \mathbf{u}_i - \mathbf{R} \Delta \mathbf{P}_i \cdot \mathbf{u}_i .
\]

Finally, assembling the equations for all six legs leads to the following matrix form

\[
\mathbf{J}_{\text{out}}^{-1} \frac{\Delta \mathbf{t}}{\Delta \mathbf{\theta}} = \Delta \mathbf{l} + \mathbf{L}_u \Delta \mathbf{A} - \mathbf{L}_u \Delta \mathbf{P} 
\]

Equation (2.19) represents the kinematic error model of the outer Stewart platform (Masory et al., 1993, Li et al., 2002, Li et al., 2007). \( \mathbf{L}_u \) is a diagonal matrix, with \( (\mathbf{u}_i)^T \) on its diagonal terms which maps the actuator spherical joint position errors to the errors along the actuators. \( \mathbf{J}_{\text{out}}^{-1} \) is the inverse kinematics Jacobian of the outer Stewart platform, the inverse of which maps the total errors along the actuators to the total displacement error of the robot load frame in task-space. Applying the same process, the kinematic error model of the inner Stewart platform can be written as
2.3. Theoretical Analysis

\[
J_{in}^{-1} \left[ \frac{\Delta t}{\Delta \theta} \right]_{in} = \Delta \lambda + L_n \Delta B - L_n \Delta E ,
\]

(2.20)

where \(L_n\) maps the encoder spherical joint position errors to the errors along the encoders and the inverse matrix of \(J_{in}^{-1}\) maps the total errors along the encoders to the total displacement error of the robot sensing frame in task-space.

When testing stiff specimens, large forces and moments \( [F_e^T \ M_e^T]^T \) in all 6-DOF are exerted on the manipulator end-effector, and as a result the compliance of the manipulator results in deformation errors in the parallel link lengths and in the spherical joint positions which can be considered as kinematic errors in manipulator joint-space. Equations (2.19) and (2.20) transform the joint-space errors to the deformation error of the load frame and the deformation error of the sensing frame respectively in task-space. Given \( [F_e^T \ M_e^T]^T, [\Delta t^T \ \Delta \theta]^T_{out}, \) and \( [\Delta t^T \ \Delta \theta]^T_{lin} \) the stiffness of the load frame and the effective stiffness of the robotic system can be estimated.

2.3.2.2 Deformation Errors of Parallel Links

As each actuator is subjected to only axial (tension-compression) loads under static conditions, the deformation error of the \(i\)th actuator is governed by its axial stiffness

\[
f_i = k_i \Delta l_i ,
\]

(2.21)

where \(f_i\) is the axial force through the \(i\)th actuator, \(k_i\) is the axial stiffness of the \(i\)th actuator. Assembling the equations for all six legs result in

\[
f = K_l \Delta l
\]

(2.22)

where

\[
f = \begin{bmatrix} f_1 \\ \vdots \\ f_6 \end{bmatrix}, \quad K_l = \begin{bmatrix} k_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_6 \end{bmatrix}.
\]

The forces along the actuators can be derived from the task-space loads by

\[
f = J_{out}^T \begin{bmatrix} F_e \\ M_e \end{bmatrix}.
\]

(2.23)

Substituting Eq. (2.23) into Eq. (2.22) yields

\[
\Delta l = K_l^{-1} J_{out}^T \begin{bmatrix} F_e \\ M_e \end{bmatrix}.
\]

(2.24)
The axial stiffness of the $i$th actuator can be modeled as three serial components

$$k_i = k_p k_c k_t / \left(k_p k_c + k_c k_t + k_p k_t \right), \quad (2.25)$$

where $k_p$ is the stiffness of the ballscrew piston, $k_c$ is the stiffness of the ballscrew cylinder, and $k_t$ is the stiffness of the ballscrew transmission system.

The encoders are subjected to only their own weight, and therefore the deformation errors of the encoders are negligible, thus

$$\Delta \alpha = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T. \quad (2.26)$$

### 2.3.2.3 Deformation Errors of Platform and Framework

The position errors of the upper spherical joints ($\Delta A, \Delta B$) result from the deformation of the moving platform assembly, and the position errors of the lower spherical joints ($\Delta A, \Delta B$) result from the deformation of the supporting frame. Due to the structural complexity of the platform assembly and the supporting frame, it is extremely difficult to estimate the stiffnesses and thus the resulting spherical joint errors. Therefore, the platform assembly and the supporting frame were analyzed using FEA. More details about the FEA simulation are discussed in Section 2.4.

### 2.3.3 Control Analysis

This section analyses the control aspect of the manipulator, e.g. coordinate system transformation, sensor-actuator non-collocated control problem, and both the kinematics-based and dynamics-based control algorithms. For simplicity, the platform assembly, parallel links, and supporting frame have been assumed to be rigid bodies for the purpose of control.

#### 2.3.3.1 Coordinate System Transformation

Four coordinate systems used in this analysis are attached to the manipulator as shown in Fig. 2.7 In addition to the global coordinate system $\{O\}$ and the moving platform coordinate system $\{Op\}$ described in the last section, a specimen coordinate system $\{Sp\}$ is located at the specimen centre of rotation (COR) and a load-cell coordinate system $\{Lp\}$ is located at the measurement point of the load-cell. For coordinate transformation, the load-cell, the specimen fixation plate, and the specimen upper
2.3. Theoretical Analysis

and lower bodies are assumed to be rigid. Under such an assumption, the three moving coordinate systems \{Lp\}, \{Op\}, and \{Sp\} always have the same orientation and the offsets between the origins of the moving coordinate systems are always constant. Therefore we have

\[
\mathbf{t}_s = \mathbf{t} + \mathbf{Rt}_s^m
\]  
\tag{2.27}

\[
\begin{bmatrix}
F_s \\
M_s
\end{bmatrix}
= 
\begin{bmatrix}
-RF_i \\
-RM_i + Rt_s^m \times (RF_i)
\end{bmatrix}
\]  
\tag{2.28}

where \(\mathbf{t}\) is the vector describing the position of \{Op\} with respect to \{O\}, \(\mathbf{t}_s\) is the vector describing the position of \{Sp\} with respect to \{O\}, \(\mathbf{t}_s^m\) is the vector describing the position of \{Sp\} with respect to \{Op\}, \(\mathbf{t}_s^m\) is the vector describing the position of \{Sp\} with respect to \{Lp\}, \(\begin{bmatrix} F^T_s & M^T_s \end{bmatrix}^T\) represents the forces and moments measured by the load-cell relative to \{Lp\}, and \(\begin{bmatrix} F^T_s & M^T_s \end{bmatrix}^T\) represents the forces and moments at the specimen COR relative to \{O\}. Equation (2.27) allows the mapping between the manipulator pose and the specimen pose. Equation (2.28) transforms the forces and moments measured by the load-cell to the forces and moments at the specimen COR relative to the global coordinate system.
2.3.3.2 Sensor-Actuator Non-collocated Control Problem

As the encoder spherical joints are not collocated with the actuator spherical joints as shown in Figs. 2.5 to 2.7, there are differences between the encoder lengths and the actuator lengths. This difference is pose-dependent and can be described by the inverse kinematics Jacobians of the manipulator,

\[
\Delta l = J_{out}^{-1} \Delta \theta ,
\]

\[
\Delta \lambda = J_{in}^{-1} \Delta \theta .
\]

Combining Eqs. (2.29) and (2.30), we have

\[
\Delta l = (J_{out}^{-1} J_{in}) \Delta \lambda
\]

where \(J_{out}^{-1} J_{in}\) maps the encoder length errors to the actuator length errors.

2.3.3.3 Kinematics-based PID Control

Figure 2.8 shows the kinematics-based control scheme for controlling the displacements of the specimen. The initial pose of the manipulator \([t^T \theta^T]^T_0\) is calculated from the initial encoder lengths \(\lambda_0\) via direct kinematics of the inner Stewart platform. The direct kinematics of a Stewart platform can be solved by using minimization and root-finding methods such as the Newton-Raphson method (Dasgupta and Mruthyunjaya, 1998). Substituting \([t^T \theta^T]^T_0\) and the offset vector between the \{Sp\} and the \{Op\}, \(t^T_{0\text{Sp}}\), into Eq. (2.27), the initial pose of the specimen \([t^T \theta^T]^T_0\) is obtained. With reference to Figure 2.8, \(d_c\) is the 6-DOF displacement command of the specimen. The sum of \([t^T_0 \theta^T]^T_c\) and \(d_c\) gives the pose command of the specimen \([t^T \theta^T]^T_c\) which is then transformed to the pose command of the manipulator \([t^T \theta^T]^T_c\). Applying the inverse kinematics of the inner Stewart platform to \([t^T \theta^T]^T_c\), the command of the encoder lengths \(\lambda_c\) is obtained for control.

In addition, given \(\theta_c\) and the offset vector between the \{Sp\} and the \{Lp\}, \(t^T_1\), the forces and moments measured by the load-cell \([F^T_1 M^T_1]^T\) can be transformed to forces and moments at the specimen COR \([F^T_s M^T_s]^T\) via Eq. (2.28). \([F^T_s M^T_s]^T\) is essential for controlling the forces and moments on the specimen in biomechanical testing. Force control is out of the scope of this section and is discussed in Chapter 4.
2.3. Theoretical Analysis

Figure 2.8. Block diagram showing the kinematics-based control scheme

Figure 2.9. Block diagram showing the decoupled PID control algorithm

Figure 2.9 shows the PID-based control algorithm which forces the encoders to follow the given encoder lengths command $\lambda_c$. Comparing $\lambda_c$ and the actual encoder lengths $\lambda$ results in the encoder-length control errors $\Delta \lambda$ which is then transformed to the actuator-length control errors $\Delta l$ via Eq. (2.31). $J^{-1}_{out}J_{in}$ is calculated from the manipulator pose command $[t^T \ \theta^T]^T$. An individual dual loop PID controller is applied to each of the six actuators, with only the third controller shown in Fig. 2.9. The PI terms are applied to $\Delta l_3$ for the purpose of eliminating $\Delta \lambda$, while the D term is applied to the motor encoder position $m_3$ for stabilizing the movement of the actuator. The proposed algorithm decouples the sensor-actuator non-collocated problem as a simple 6 channel single-input-single-output (SISO) control problem and can be written as the following matrix form

$$\tau_m = K_P J^{-1}_{out}J_{in} \Delta \lambda + K_I \int (J^{-1}_{out}J_{in} \Delta \lambda) dt - K_D \dot{m}$$  

(2.32)
where \( K_p = \begin{bmatrix} K_{p1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{pn} \end{bmatrix} \), \( K_I = \begin{bmatrix} K_{I1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{In} \end{bmatrix} \), \( K_D = \begin{bmatrix} K_{D1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & K_{Dn} \end{bmatrix} \) are the controller gains, and \( \tau_m \) represents the motor control torques.

2.3.3.4 Dynamics-based PD Control

Assuming the encoder inertia is negligible compared to the platform assembly inertia and actuator inertia, the dynamics of the manipulator is simplified as the case of a general Stewart platform. Then the inverse dynamics of the manipulator in task-space can be written as (Do and Shahinpoor, 1998, Dasgupta and Mruthyunjaya, 1998, Ghabakhloo et al., 2006)

\[
\mathbf{J}_{\text{out}}^T (\mathbf{M}(\mathbf{p}) \ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}}) \dot{\mathbf{p}} + \mathbf{G}(\mathbf{p}) + \mathbf{N}_p) = \mathbf{f}
\]  

(2.33)

where \( p = [\mathbf{T} \quad \mathbf{\theta}^T]^T \) represents the pose of the manipulator, \( \mathbf{M}(\cdot) \) represents the inertia matrix, \( \mathbf{C}(\cdot) \) represents the Coriolis, centrifugal, and damping terms, \( \mathbf{G}(\cdot) \) represents the gravitational terms, \( \mathbf{N}_p \) represents the external forces and moments at the end-effector, and \( \mathbf{f} \) represents the resulting forces along the actuators. It is also known that

\[
\dot{\mathbf{p}} = \mathbf{J}_{\text{in}} \dot{\mathbf{\lambda}}
\]  

(2.34)

Differentiating Eq. (2.34) with respect to time results in

\[
\ddot{\mathbf{p}} = \mathbf{J}_{\text{in}} \ddot{\mathbf{\lambda}} + \mathbf{J}_{\text{in}} \dot{\mathbf{\lambda}}.
\]  

(2.35)

Substituting Eqs. (2.34) and (2.35) into Eq. (2.33), the inverse dynamics of the manipulator in joint-space can be written as

\[
\mathbf{J}_{\text{out}}^T \left[ \mathbf{M}(\mathbf{p}) \dot{\mathbf{\lambda}} + (\mathbf{M}(\mathbf{p}) \dot{\mathbf{\lambda}}_\text{in} + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}}) \dot{\mathbf{p}}) \dot{\mathbf{\lambda}} + \mathbf{G}(\mathbf{p}) + \mathbf{N}_p \right] = \mathbf{f}.
\]  

(2.36)

Based on Eq. (2.36), the control law is formed as

\[
\mathbf{f}_c = \mathbf{J}_{\text{out}}^T \left[ \mathbf{M}(\mathbf{p}_c) \dot{\mathbf{\lambda}} + (\mathbf{M}(\mathbf{p}_c) \dot{\mathbf{\lambda}}_\text{in} + \mathbf{C}(\mathbf{p}_c, \dot{\mathbf{p}}_c) \dot{\mathbf{p}}_c) \dot{\mathbf{\lambda}} + \mathbf{G}(\mathbf{p}_c) + \mathbf{N}_p \right]
\]  

(2.37)

where \( \mathbf{\alpha} = \dot{\mathbf{\lambda}}_c + \mathbf{K}_p (\mathbf{\lambda}_c - \mathbf{\lambda}) + \mathbf{K}_D (\dot{\mathbf{\lambda}}_c - \dot{\mathbf{\lambda}}) \).

In Eq. (2.37), \( \mathbf{f}_c \) represents the actuator control forces, \( \mathbf{p}_c \) represents the manipulator pose command, \( \mathbf{\lambda}_c \) and \( \mathbf{\lambda} \) represent the lengths command and the actual lengths of the encoder respectively, and \( \mathbf{K}_p \) and \( \mathbf{K}_D \) represent the matrix P gains and matrix D gains.
of the controller respectively. $J_{\text{out}}^T$ and $J_{\text{in}}$ are calculated from $p_c$. $N_p$ is derived via transforming the load-cell measured forces and moments to the forces and moments at the end-effector. Equation (2.37) is a standard computed torque control scheme consisting of a model-based portion (the last three terms) and a servo portion (the first term). The control scheme can be used as a basis to design more robust control algorithms (e.g. adaptive model-based control). On the other hand, the control scheme does not consider the model of the ballscrew backlash, as nonlinear modelling is out of the scope of this thesis. However, a method for addressing unknown system backlash in a general Stewart platform is discussed in Chapter 3.

### 2.4 Numerical Simulation on Stiffness

This subsection presents the geometrical and physical parameters of the manipulator that forms this study. The effective stiffness of the robotic system as well as the stiffness of the load frame is quantified via computational simulation combined with FEA simulation.

#### 2.4.1 Geometrical and Physical Parameters for Simulation

Tables 2.1 and 2.2 list the coordinates of the lower actuator spherical joints with respect to $\{O\}$ and the coordinates of the upper actuator spherical joints with respect to $\{O_p\}$ respectively. Tables 2.3 and 2.4 list the coordinates of the lower encoder spherical joints with respect to $\{O\}$ and the coordinates of the upper encoder spherical joints with respect to $\{O_p\}$ respectively. With reference to Fig. 2.5, the actuator piston used in this design is a solid cylinder with a diameter of 28.7mm and a length of 177.8mm. The actuator cylinder is a hollow cylinder with an outer diameter of 57.2mm, an inner diameter of 28.7mm, and a length of 300.7mm. For simplicity, the stiffness of the ballscrew transmission system $k_\epsilon$ is assumed as $4 \times 10^4 \text{N/mm}$ which is selected to emulate a very stiff actuated leg (8 times that of the spine disc). As the structures of the platform assembly and the supporting frame are complex, only the important dimensions are listed here. More detailed mechanical dimensions can be found from the drawings in electronic submissions. In the platform assembly, the load cell body is a 70mm $\times$ 70mm $\times$ 46mm cube. The specimen fixation plate is a 10mm thick hexagon plate. The top platform consists of a 20mm thick hexagonal plate, three
20mm thick trapezoid plates, and six upper spherical joint bearings with a diameter of 12.7mm and a length of 11mm. In the supporting frame, the base plate has a thickness of 75mm. The actuator pillar is a 75mm \times 75mm \times 350mm solid column. The lower spherical joint bearing has the same dimensions as the ones in the platform assembly. The specimen pillar consists of a lower round plate with a diameter of 340mm and a thickness of 85mm and an upper regular hexagonal column with a side length of 58mm and a height of 280mm. The hexagonal plate for supporting the encoder pillars has a thickness of 12mm. Table 2.5 lists the physical properties of the parts.

Table 2.1. Coordinates of \( A_i \) with respect to \{O\}

<table>
<thead>
<tr>
<th>( A_i )</th>
<th>( X ) (mm)</th>
<th>( Y ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>46.6</td>
<td>366.8</td>
</tr>
<tr>
<td>A2</td>
<td>341.0</td>
<td>-143.1</td>
</tr>
<tr>
<td>A3</td>
<td>294.4</td>
<td>-223.8</td>
</tr>
<tr>
<td>A4</td>
<td>-294.4</td>
<td>-223.8</td>
</tr>
<tr>
<td>A5</td>
<td>-341.0</td>
<td>-143.1</td>
</tr>
<tr>
<td>A6</td>
<td>-46.6</td>
<td>366.8</td>
</tr>
</tbody>
</table>

Table 2.2. Coordinates of \( P_i \) with respect to \{Op\}

<table>
<thead>
<tr>
<th>( P_i )</th>
<th>( X ) (mm)</th>
<th>( Y ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>166.3</td>
<td>130.6</td>
</tr>
<tr>
<td>P2</td>
<td>196.3</td>
<td>78.7</td>
</tr>
<tr>
<td>P3</td>
<td>30.0</td>
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</tr>
<tr>
<td>P4</td>
<td>-30.0</td>
<td>-209.3</td>
</tr>
<tr>
<td>P5</td>
<td>-196.3</td>
<td>78.7</td>
</tr>
<tr>
<td>P6</td>
<td>-166.3</td>
<td>130.6</td>
</tr>
</tbody>
</table>

Table 2.3. Coordinates of \( B_i \) with respect to \{O\}

<table>
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<td>B4</td>
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<td>-152.5</td>
</tr>
<tr>
<td>B6</td>
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<td>316.9</td>
</tr>
</tbody>
</table>

Table 2.4. Coordinates of \( E_i \) with respect to \{Op\}

<table>
<thead>
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<th>( E_i )</th>
<th>( X ) (mm)</th>
<th>( Y ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>125.8</td>
<td>82.0</td>
</tr>
<tr>
<td>E2</td>
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<tr>
<td>E3</td>
<td>8.1</td>
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</tr>
<tr>
<td>E4</td>
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<td>-150.0</td>
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<tr>
<td>E5</td>
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<td>68.0</td>
</tr>
<tr>
<td>E6</td>
<td>125.8</td>
<td>82.0</td>
</tr>
</tbody>
</table>

Table 2.5. Physical properties of the parts

<table>
<thead>
<tr>
<th></th>
<th>Loadcell body</th>
<th>Loadcell flange</th>
<th>Actuator</th>
<th>Robot frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Rubber</td>
<td>Aluminum</td>
<td>Steel</td>
<td>Steel</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>0.55</td>
<td>71</td>
<td>190</td>
<td>200</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.46</td>
<td>0.33</td>
<td>0.29</td>
<td>0.3</td>
</tr>
</tbody>
</table>
2.4.2 FEA Simulation on the Platform Assembly and the Supporting Frame

The platform assembly and the supporting frame were modeled using ANSYS Workbench 12.0 and the parameters discussed in the previous section. The forces exerted at the actuator spherical joint mounts are the reaction forces from the actuators, and therefore can be calculated via Eq. (2.23) given the forces and moments at the end-effector \((\{F_e^T \ M_e^T\}^T)\). Applying the resulting forces at the actuator spherical joint mounts, as well as setting the places for mounting the specimen as the fixed boundaries, the deformations of all the spherical joints due to \((\{F_e^T \ M_e^T\}^T)\) can be obtained. Figure 2.10 and 2.11 show the loading and boundary conditions on the platform assembly and on the supporting frame respectively when the manipulator is at its nominal pose \((\{\vec{r}^T \ \vec{\theta}^T\} = [0 \ 0 \ 500\text{mm} \ 0 \ 0 \ 0])\) and is under a large z-axis force \((\{F_e^T \ M_e^T\} = [0 \ 0 \ 20,000\text{N} \ 0 \ 0 \ 0])\). A 5 N force was applied at the location of each encoder spherical joint to simulate the gravity of each encoder. Subjected to these loading and boundary conditions, the deformations of the platform assembly and the supporting frame are plotted in Fig. 2.12 and 2.13 respectively. The deformations at the actuator spherical joint bearings (1.132mm at top and 0.070mm at bottom) are much higher than the deformations at the encoder spherical joint mounts (0.012mm at top and 0.002mm at bottom).

![Figure 2.10. Loading and boundary conditions on the platform assembly (\((\{\vec{r}^T \ \vec{\theta}^T\} = [0 \ 0 \ 500\text{mm} \ 0 \ 0 \ 0], \ (\{F_e^T \ M_e^T\} = [0 \ 0 \ 20,000\text{N} \ 0 \ 0 \ 0]))\)](image)
Chapter 2. Stiffness Analysis and Control of a Stewart Platform-based Mechanism with Decoupled Sensor-actuator Locations

2.4.3 Effective Robotic System Stiffness Vs Load Frame Stiffness

The procedure for quantifying the manipulator stiffness is listed below:

1. The deformation errors of the actuator spherical joints \((\Delta A, \Delta P)\) and the
2.4. Numerical Simulation on Stiffness

deformation errors of the encoder spherical joints ($\Delta B, \Delta E$) are obtained via FEA.

2. The deformation errors of the actuators ($\Delta I$) are calculated via Eq. (2.24).

3. Substituting the resulting joint-space deformation errors into Eq. (2.19) and (2.20), the deformation error of the load frame $[\Delta t_T^T \Delta \theta_T]^T_{out}$ and the deformation error of the sensing frame $[\Delta t_T^T \Delta \theta_T]^T_{in}$ are obtained.

4. Finally, relating $[F_e^T M_e^T]^T$ with the task-space deformation errors, the stiffness of the load frame and the effective stiffness of the robotic system are obtained.

In this study, the stiffness of the manipulator is analyzed at three robot poses (I: $[0 \ 0 \ 500 \text{mm} \ 0 \ 0 \ 0]$, II: $[0 \ 150 \text{mm} \ 500 \text{mm} \ 0 \ 0 \ 0]$, III: $[0 \ 0 \ 500 \text{mm} \ 20^\circ \ 0 \ 0]$). At each pose, the stiffness is calculated in all 6-DOF sequentially. The resulting effective stiffness of the robotic system, as well as the stiffness of the load frame, is shown in Table 2.6. Results show that the stiffness of the load frame is of the same order of stiffness as the biological specimen. By decoupling the sensor locations from the actuator locations, the stiffness of the manipulator is increased by 15 times along translational axes and 100 times about rotational axes.

Table 2.6. Effective robotic system stiffness versus the load frame stiffness (unit: N/mm and Nm/°)

<table>
<thead>
<tr>
<th>Pose</th>
<th>Axis</th>
<th>Robotic system stiffness</th>
<th>Load frame stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Fx/Tx</td>
<td>61958</td>
<td>3853</td>
</tr>
<tr>
<td></td>
<td>Fy/Ty</td>
<td>58343</td>
<td>3863</td>
</tr>
<tr>
<td></td>
<td>Fz/Tz</td>
<td>1621903</td>
<td>17773</td>
</tr>
<tr>
<td></td>
<td>Mx/Rx</td>
<td>49867</td>
<td>396</td>
</tr>
<tr>
<td></td>
<td>My/Ry</td>
<td>49867</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>Mz/Rz</td>
<td>87266</td>
<td>220</td>
</tr>
<tr>
<td>II</td>
<td>Fx/Tx</td>
<td>55617</td>
<td>3733</td>
</tr>
<tr>
<td></td>
<td>Fy/Ty</td>
<td>60241</td>
<td>2999</td>
</tr>
<tr>
<td></td>
<td>Fz/Tz</td>
<td>566038</td>
<td>16347</td>
</tr>
<tr>
<td></td>
<td>Mx/Rx</td>
<td>49867</td>
<td>399</td>
</tr>
<tr>
<td></td>
<td>My/Ry</td>
<td>49867</td>
<td>392</td>
</tr>
<tr>
<td></td>
<td>Mz/Rz</td>
<td>87266</td>
<td>219</td>
</tr>
<tr>
<td>III</td>
<td>Fx/Tx</td>
<td>63131</td>
<td>4026</td>
</tr>
<tr>
<td></td>
<td>Fy/Ty</td>
<td>60827</td>
<td>3861</td>
</tr>
<tr>
<td></td>
<td>Fz/Tz</td>
<td>1666666</td>
<td>14353</td>
</tr>
<tr>
<td></td>
<td>Mx/Rx</td>
<td>49867</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>My/Ry</td>
<td>58178</td>
<td>359</td>
</tr>
<tr>
<td></td>
<td>Mz/Rz</td>
<td>116360</td>
<td>231</td>
</tr>
</tbody>
</table>
2.5 Control Simulation and Experiments

This section shows the efficacy of using $J_{\text{out}}^{-1}J_{\text{in}}$ in the non-collocated control problem via simulation. Also shown is the performance of the kinematics-based PID control on the manipulator via experiments.

2.5.1 Effect of Non-collocated Sensor-actuator Mechanism

$J_{\text{out}}^{-1}J_{\text{in}}$ is calculated when the manipulator is at pose I, pose II, and pose III discussed in the previous section. The results are shown below

\[
(J_{\text{out}}^{-1}J_{\text{in}})_I = \begin{bmatrix}
1.197 & 0.066 & -0.048 & -0.168 & 0.049 & -0.097 \\
0.066 & 1.197 & -0.097 & 0.049 & -0.168 & -0.048 \\
0.049 & -0.097 & 1.197 & 0.066 & -0.048 & -0.168 \\
-0.168 & -0.048 & 0.066 & 1.197 & -0.097 & 0.049 \\
-0.048 & -0.168 & 0.049 & -0.097 & 1.197 & 0.066 \\
-0.097 & 0.049 & -0.168 & -0.048 & 0.066 & 1.197 
\end{bmatrix} \quad (2.38)
\]

\[
(J_{\text{out}}^{-1}J_{\text{in}})_II = \begin{bmatrix}
1.196 & 0.057 & -0.044 & -0.156 & 0.042 & -0.097 \\
0.078 & 1.197 & -0.104 & 0.052 & -0.169 & -0.056 \\
0.053 & -0.090 & 1.197 & 0.066 & -0.044 & -0.182 \\
-0.182 & -0.044 & 0.066 & 1.197 & -0.090 & 0.053 \\
-0.056 & -0.169 & 0.052 & -0.104 & 1.197 & 0.078 \\
-0.097 & 0.042 & -0.156 & -0.044 & 0.057 & 1.196 
\end{bmatrix} \quad (2.39)
\]

\[
(J_{\text{out}}^{-1}J_{\text{in}})_III = \begin{bmatrix}
1.198 & 0.032 & -0.014 & -0.159 & 0.043 & -0.105 \\
0.072 & 1.211 & -0.110 & 0.057 & -0.177 & -0.053 \\
0.046 & -0.078 & 1.181 & 0.091 & -0.073 & -0.169 \\
-0.169 & -0.073 & 0.091 & 1.181 & -0.078 & 0.046 \\
-0.053 & -0.177 & 0.057 & -0.110 & 1.211 & 0.072 \\
-0.105 & 0.043 & -0.159 & -0.014 & 0.032 & 1.198 
\end{bmatrix} \quad (2.40)
\]

Equations (2.38)-(2.40) shows that within the robot movement range, $J_{\text{out}}^{-1}J_{\text{in}}$ is a fully populated matrix with off-diagonal terms as high as 16% of the diagonal terms. Results also show that when the robot is at a different pose, the individual elements in the matrix can see a change as much as 3.4%. This means the actuator-sensor non-collocated mechanism is a $6 \times 6$ multiple input multiple output system and therefore $J_{\text{out}}^{-1}J_{\text{in}}$ is essential in the control algorithm to decouple the system to enable SISO control.
2.5.2 Experimental Method (Kinematic-based PID Control)

2.5.2.1 Mechanical Design

As shown in Fig. 2.14, the manipulator assembly was manufactured based on the geometries given in Section 2.4.1. The robot frame was manufactured from 304 stainless steel, which was chosen over mild steel for its corrosion resistance in biomechanical applications, where body fluids and saline liquids are present. Six EDRIVE VT209-07 linear actuators were chosen as the robot legs, each driven by an Aerotech BM250 DC brushless servomotor using a toothed belt with a 2:1 drive ratio. The actuator motor assembly is capable of generating 4kN of thrust and generating a maximum linear velocity of 200mm/s. The six linear encoders were custom-made. Each consists of a MicroE Systems B36678 glass scale and a LDM-54 optical read head, with a resolution of 0.5µm. AMTI MC3A-6-1000 was chosen as the load-cell.

2.5.2.2 Control Hardware Configuration

Figure 2.15 shows a schematic of the control hardware configuration of the manipulator. A host PC runs a custom-built LabVIEW GUI for operating the manipulator. Communicating with the host PC via Ethernet, a NI PXI-8106 real-time
controller runs the kinematics based control algorithm as shown in Fig. 2.8 in a 1kHz loop. Two NI PXI-7852R FPGA boards communicate with the RT controller via DMA and run the decoupled PID control algorithms as shown in Fig. 2.9 at 10kHz sampling rate. At the lowest level, six Aerotech Soloist CP20 servo amps run six current control loops at 20kHz which regulate the currents in the motors to achieve the required motor torque commands. More detailed technical aspects of the robot control system are discussed in Chapter 6.

2.5.2.3 Experimental Protocol

To validate the performance of the proposed control algorithm on the manipulator during biomechanical testing, a high density polymer cylindrical specimen (81 mm diameter and 132 mm exposed height) was mounted between the specimen fixation plate and the specimen pillar via two mounting cups as shown in Fig. 2.14. Both ends of the polymer specimen were fixed into the mounting cups by six screws (Fig. 2.16). The high density polymer is a form of polyurethane and has an 82A Shore hardness reading according to ASTM D2240 (Standard Test Method for Rubber Properties). The polymer specimen was chosen over a biological specimen mainly due to its higher flexibility which allows the manipulator to travel in a larger range without tripping the measurement limitations of the load-cell. The manipulator was commanded to move about the polymer specimen COR with a combined sinusoidal displacement of ±3 degrees lateral bending (Rx), and ±2 degrees axial torsion (Rz) at a 1Hz cycle rate. Such a testing protocol was chosen to cause the robot to move dynamically with large acceleration and deceleration under high reaction moments, such that both dynamic errors and disturbances on the manipulator are present during control. Two kinematic-based based control strategies were considered. One included
2.5. Control Simulation and Experiments

the use of $J_{out}^{-1}J_{in}$ in the control algorithm and the other ignored $J_{out}^{-1}J_{in}$ by replacing it with the identity matrix. The later is equivalent to applying SISO control on the manipulator without proper decoupling. The PID gains were initially auto-tuned by the Soloist controller and were then further adjusted based on trials. PID gains in the equivalent continuous time domain were finally set as $K_p = 12$ bit/μm, $K_i = 700$ bit/(μm · sec), $K_d = 0.1$ bit · sec /μm (Input units: half micrometre for $P$ and $I$ terms and 1/8 micrometre for $D$ term; output units: 16-bit, ±10V analog output) for both circumstances in the experiments.

2.5.3 Results on the Kinematics-based PID Control

Figure 2.17 illustrates the moments at the specimen COR during testing. The peak-to-peak moments on lateral bending ($M_x$) and axial torsion ($M_z$) reached 70Nm and 25Nm respectively. Figure 2.18(a) and Figure 2.19(a) show the tracking errors on three translations and three rotations respectively at COR under SISO control. Figure 2.18(b) and Figure 2.19(b) show the counterpart under decoupled SISO control. As expected, the tracking errors under SISO control were larger than their counterpart under decoupled SISO control. By comparing the RMS of the tracking errors on all six axes between SISO and decoupled SISO control, it was found that the decoupled SISO can improve the tracking accuracy of the manipulator by approximately 25% as shown in Table 2.7. On the two command tracking axes ($R_x$ and $R_z$), the RMS of the
Chapter 2. Stiffness Analysis and Control of a Stewart Platform-based Mechanism with Decoupled Sensor-actuator Locations

Figure 2.17. Three moments at specimen COR during testing

Figure 2.18. Tracking errors on three translations at specimen COR under kinematics-based control

Figure 2.19. Tracking errors on three rotations at specimen COR under kinematics-based control
Table 2.7. RMS errors on six axes under SISO and decoupled SISO control

<table>
<thead>
<tr>
<th>Axis</th>
<th>SISO control</th>
<th>Decoupled SISO</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx</td>
<td>0.0100 mm</td>
<td>0.0098 mm</td>
<td>2%</td>
</tr>
<tr>
<td>Ty</td>
<td>0.0196 mm</td>
<td>0.0068 mm</td>
<td>65%</td>
</tr>
<tr>
<td>Tz</td>
<td>0.0035 mm</td>
<td>0.0026 mm</td>
<td>26%</td>
</tr>
<tr>
<td>Rx</td>
<td>0.0482 degree</td>
<td>0.0347 degree</td>
<td>28%</td>
</tr>
<tr>
<td>Ry</td>
<td>0.0018 degree</td>
<td>0.0015 degree</td>
<td>16%</td>
</tr>
<tr>
<td>Rz</td>
<td>0.0322 degree</td>
<td>0.0231 degree</td>
<td>28%</td>
</tr>
</tbody>
</table>

tracking errors were kept within 1.2% of the command amplitude under decoupled SISO control. The results were very satisfactory considering the speed of the testing.

2.6 Conclusion

In this chapter, a Stewart platform-based manipulator with decoupled sensor-actuator locations was proposed for applications in biomechanical testing. Studies were performed on both the stiffness and control aspects of the manipulator. Results show that the unique sensor-actuator non-collocated mechanism is able to increase the stiffness of a general Stewart platform testing system by a factor of 15 over a common collocated design, and therefore significantly improve the static accuracy of the manipulator when subjected to large reaction forces and moments. Results also show that using the proposed decoupled control algorithm on the manipulator improves the dynamic accuracy of the manipulator by 25%.

The concept behind the passive compliance compensation method is to obtain an accurate specimen displacement measurement for control, for example in this thesis an inner Stewart platform has been used as the sensing mechanism. In theory, there are many ways in which to replace the inner Stewart platform (sensing frame) to measure the true specimen displacement which would be independent of load frame compliance (e.g. laser, kinect, motion capture system). However, these systems measure the displacement in the task space, which can cause difficulties in feedback control of the robot as measurements are normally obtained from the robot joint space for easier control realization (Merlet, 2006). The idea of the inner Stewart platform (sensing frame) measures the true specimen displacement in the form of inner robot
joint (sensor) space, which has a simple and easy computational relationship with the outer robot joint (actuator) space. This leads to a simple but effective control solution with minimal modification from a general Stewart platform as already discussed in the above sections.
Chapter 3

Active Preload Control of a
Redundantly Actuated Stewart
Platform for Backlash Prevention

3.1 Introduction

Parallel robots are well known for their advantages in providing higher rigidity and
stiffness, being more compact in structure, and having greater payload capacity than
their serial counterparts. As a result, they are often used in applications where
precision of the order of micrometres is required from the robot (e.g. biomechanical
testing, manufacturing). However, joint clearances or backlash can largely degrade the
accuracy of parallel robots in these applications (Khalil et al., 2011, Briot and Bonev,
2008) as well as severely limiting bandwidth. Many linear and non-linear control
methods have been proposed to mitigate backlash inaccuracies on a single actuated
joint (Nordin and Gutman, 2002). These methods often require a highly accurate
backlash model which is difficult to approximate in practice. Flexure joints have been
developed to remove backlash at the expense of limited range of motion (McInroy,
2002, Kang et al., 2005).

Recent research found it was possible to achieve backlash prevention for parallel
robots by controlling the preloads on their actuated joints. Preload control can be
further divided into two categories: the active method and the passive method. The
active method uses actuation redundancy while considering dynamic effects for the
Chapter 3. Active Preload Control of a Redundantly Actuated Stewart Platform for Backlash Prevention

purpose of backlash prevention along a specified path (Muller, 2005, Boudreau et al., 2011). This approach requires an offline optimization process and its performance is highly sensitive to model error, which prevents the proposed approach working in many real-time applications. The passive method on the other hand uses preloaded passive joints in order to eliminate backlash throughout a desired workspace when given norm-bounded external loads (Wei and Simaan, 2010). Although much simpler than the active method, the passive method is hard to realize on a parallel robot with more than three degrees of freedom and is not feasible with large external loads in the order of 100N or greater.

In this chapter, the author investigates combining the benefits of both active and passive preload methods, using actuation redundancy to prevent backlash on a six degree of freedom Stewart platform. Rather than using the offline optimization method reported in Muller (2005), an online optimization algorithm is developed combined with a feedback force control scheme to achieve a real-time method which is robust to both model inaccuracy and load disturbance. In addition, the design and modeling of the redundant manipulator are analysed to ensure a simplified but effective control realization. The proposed approach is ideal for applications where the Stewart platform is required to implement a high-precision task under large external loads, e.g. biomechanical testing, machining, etc. Section 3.3 presents the backlash free condition, which is the essential goal for preload control. Based on the backlash free condition, the overall solution is formulated, followed by four main problems to be further treated in Section 3.4: the configuration of the redundant manipulator (Subsection 3.4.1), the inverse dynamics equation (Subsection 3.4.2), the preload optimization algorithm (Subsection 3.4.3), and the force control scheme on the redundant actuator (Subsection 3.4.4). Section 3.5 presents the simulation results on the custom-built Stewart platform-based manipulator developed in this study for biomechanical testing, followed by Section 3.6 which uses physical experiment to further verify the results.

3.2 Background Theory

This section covers the background theory behind the study in this chapter. Audiences who do not have the knowledge of system dynamics are encouraged to read through this section for a better understanding of the work done in this chapter.
3.2 Background Theory

3.2.1 Inertia Tensor

In three-dimensional space, the inertia moment of an object can be described as a 3 × 3 inertia matrix often described as the inertia tensor. As for the example in Fig. 3.1, the inertia tensor relative to frame {A} is expressed as

\[
 I_A = \begin{bmatrix}
 l_{xx} & -l_{xy} & -l_{xz} \\
 -l_{xy} & l_{yy} & -l_{yz} \\
 -l_{xz} & -l_{yz} & l_{zz}
 \end{bmatrix}
\] (3.1)

where the mass moments of inertia are given by

\[
 l_{xx} = \iiint (y^2 + z^2) \rho(dv), \quad l_{yy} = \iiint (x^2 + z^2) \rho(dv), \quad l_{zz} = \iiint (x^2 + y^2) \rho(dv)
\] (3.2)

and the mass products of inertia are given by

\[
 l_{xy} = \iiint xy \rho(dv), \quad l_{xz} = \iiint xz \rho(dv), \quad l_{yz} = \iiint yz \rho(dv)
\] (3.3)

where \( \rho \) represents the density of the object and \( dv \) presents the differential volume element of the object. Thus, the set of six quantities depends on the position and orientation of the frame in which they are defined.

3.2.2 Velocity and Acceleration on a Rigid Body

Besides displacement, the velocity and acceleration are very important parameters when describing the dynamics of a moving rigid body. The case may become complex when the rigid body is moving linearly and rotationally at the same time.
Figure 3.2. Free-body diagram shows two fixed points (C and B) on a moving rigid body where a frame \{B\} is attached to point B and a global reference frame \{A\} is spatially fixed.

With reference to Fig. 3.2, points C and B are two fixed points on a moving rigid body where a frame \{B\} is attached to point B to describe the displacement, velocity, and acceleration of point B relative to frame \{A\}. Then the velocity and acceleration of point C relative to frame \{A\} can be obtained by

\[
\mathbf{\dot{t}}_C^A = \mathbf{\dot{t}}_B^A + \omega_B^A \times \mathbf{R}_B^A \mathbf{t}_C^B, \tag{3.4}
\]

\[
\mathbf{\ddot{t}}_C^A = \mathbf{\ddot{t}}_B^A + \omega_B^A \times (\omega_B^A \times \mathbf{R}_B^A \mathbf{t}_C^B) + \dot{\omega}_B^A \times \mathbf{R}_B^A \mathbf{t}_C^B, \tag{3.5}
\]

where \(\dot{t}_C^A\) and \(\dot{t}_B^A\) represent the linear velocities of point C and point B relative to frame \{A\} respectively, \(\omega_B^A\) and \(\dot{\omega}_B^A\) represent the angular velocity and angular acceleration of the rigid body (frame \{B\}) relative to frame \{A\} respectively, and \(\ddot{t}_C^A\) and \(\ddot{t}_B^A\) represent the linear accelerations of point C and point B relative to frame \{A\} respectively.

### 3.2.3 Newton-Euler Equations

Newton-Euler equations are useful for building the dynamics equations of a rigid body. With reference to Fig. 3.2, assuming point C is the centre of gravity of the rigid body, gives the following Newton-Euler equations

\[
\sum \mathbf{F}_i = m \mathbf{\ddot{t}}_C^A, \tag{3.6}
\]

\[
\sum \mathbf{M}_{Bi} = \mathbf{I}_A \dot{\omega}_B^A + \omega_B^A \times (\mathbf{I}_A \omega_B^A) + m \mathbf{R}_B^A \mathbf{t}_C^B \times \mathbf{\ddot{t}}_C^A, \tag{3.7}
\]
where \( \mathbf{F} \) represents the external forces acting on the rigid body relative to frame \( \{A\} \), \( \mathbf{M}_{Bi} \) represents the external moments acting at point \( B \) on the rigid body relative to frame \( \{A\} \), \( m \) represents the mass of the rigid body, and \( \mathbf{I}_A \) represents the inertia tensor of the rigid body relative to frame \( \{A\} \).

### 3.2.4 Backlash-free Condition

The backlash behaviour can be simplified by the dead-zone model (Nordin and Gutman 2002) as shown in Fig. 3.3, assuming an elastic shaft with zero damping and inertia connected between the motor and load (Fig. 1.14). When the magnitude of the angle difference between the motor and the load \( |\theta_d| \) is larger than the maximum backlash angle \( \alpha_b \), the backlash shaft torque \( T_s \) becomes a non-zero value which is linearly proportional to \( \theta_d - \alpha_b \). In another words, if the magnitude of \( T_s \) is always kept larger than zero and its sign remains fixed during the task, a linear system can be obtained with the absence of backlash. Nevertheless, it is hard to directly control the exact value of \( T_s \) due to the complexity and sensitivity of the backlash dynamics. The relationship between the actuator control torque and the shaft torque is described as

\[
T_m + (\phi + \Omega) = T_s \tag{3.8}
\]

where \( T_m \) represents the actuator control torque, \( \phi \) represents the dynamics terms corresponding to the motor and actuator transmission system, and \( \Omega \) represents the uncertainty in modeling. Assuming the magnitude of the actuator control torque is always higher than the sum of the dynamics terms and the model uncertainty and the sign of the actuator control force remains constant, we have

\[
T_m > |\phi + \Omega|_{\text{max}}, \quad \text{or} \quad T_m < -|\phi + \Omega|_{\text{max}}. \tag{3.9}
\]

Substituting Eq. (3.9) into Eq. (3.8) gives

\[
0 \leq |\phi + \Omega|_{\text{max}} + (\phi + \Omega) < T_s,
\]

or

\[
0 \geq -|\phi + \Omega|_{\text{max}} + (\phi + \Omega) > T_s. \tag{3.10}
\]

which satisfies the condition that the magnitude of \( T_s \) is always kept larger than zero and its sign remains fixed. Therefore, it is possible to eliminate backlash from the system by ensuring that the magnitude of the actuator control torque is always higher than a threshold, equal or larger than \( |\phi + \Omega|_{\text{max}} \), while keeping the control torque...
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Figure 3.3. Backlash dead-zone model where the vertical axis represents the value of the backlash shaft torque $T_s$, the horizontal axis represents the value of the angle difference between the motor and the load $\theta_d$, and $\alpha_b$ represents the maximum backlash angle (damping is ignored in the dead-zone model).

sign fixed during the task. Such a threshold can be found via experimental testing.

3.3 Problem Statement

A general Stewart platform mechanism consists of six linear actuators, which are connected via universal joints to a fixed base below and via spherical joints to a moving platform above. Ballscrews driven by rotary motors are often used as the linear actuators. The backlash in the ballscrew actuators is dominant compared to all other sources. The backlash-free condition for a linear actuator is shown in Fig. 3.4, where $\tau_c$ represents the actuator control forces, $\varepsilon$ represents the backlash-free threshold, and $\sigma$ represents the actuator payload limit. The backlash-free condition physically means the magnitude of the actuator control force must remain above a certain level and its sign must remain fixed during the period of the task for backlash prevention (Muller, 2005, Wei and Simman, 2010). Its mathematical expression is:

$$\sigma > |\tau_c(t)| \geq \varepsilon, \quad \text{sign}(\tau_c(t)) = \text{constant}, \quad t \in [0, T].$$

(3.11)

Figure 3.4. Backlash-free condition for a linear actuator
In order to prevent backlash on a Stewart platform, all of its six ball screw actuators must satisfy the backlash-free condition. For achieving this, a new preload control method is proposed with a redundant linear actuator attached to the moving platform (Fig. 3.5). Overall, the concept is to use the redundant actuator to regulate the preloads on the original position-controlled ballscrews for the purpose of ensuring the control forces remain in the backlash-free region. As the solution is not unique, this forms an optimal force control problem in which the redundant actuator is required to generate minimum internal preloads to satisfy the backlash-free condition with lowest cost. Therefore, a preload optimization algorithm is used to search for the desired preload on the redundant actuator \( f_r \) based on the backlash-free condition, the inverse dynamics equation, and the varying parameters (e.g. external forces and moments, and the end-effector trajectory). In series with the optimization algorithm, a feedback force control scheme is used to drive the redundant actuator to achieve the desired preload. A kinematics-based dual loop PID control scheme as described in Section 1.2.4.1 and Section 2.3.3.3 is used to control the original six ballscrews for accurately positioning the robot end-effector. The use of dual loop PID control not only ensures the global accuracy of the positioning (load position) when backlash is not present but also guarantees the stability of the plant in the case backlash is not eliminated effectively.

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**Figure 3.5.** Schematics showing the preload control method where \( F_{ex} \) and \( M_{ex} \) represents the external forces and moments, \( p, \dot{p}, \) and \( \ddot{p} \) represents the end-effector trajectory, velocity, and acceleration, \( f_r \) represents the desired preload on the redundant actuator, and \( \tau_r \) and \( \tau_c \) represent the control forces for driving the redundant actuator and the original six ballscrews respectively.
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3.4 Theoretical Analysis

Four main problems are treated in this section for achieving the proposed preload control method in practice. Firstly, the configuration of the redundant actuator in the Stewart platform is analysed for ease of control. Secondly, a simplified inverse dynamics equation is derived for the redundant manipulator configuration. Thirdly, an online optimization algorithm is proposed to determine the preload requirement on the redundant actuator in real-time. Finally, the force control on the redundant actuator is investigated for the purpose of accurate tracking and disturbance rejection.

3.4.1 Redundant Manipulator Configuration

The redundant actuation of parallel robots has been widely studied due to the advantages of eliminating singularities, increasing manipulator stiffness, payload and acceleration, and reducing power consumption (Wang et al., 2011, Nahon and Angles, 1989). These aspects are not the focus of this study. Instead, the redundant actuation of the Stewart platform is used to regulate the preloads assigned on the ballscrews for the purpose of backlash prevention. From a controllability point of view, this is difficult as all six ballscrews must satisfy the backlash-free condition with the regulation from only one actuator. There is no doubt that the configuration of the redundant manipulator is fundamental to the success of the proposed method.

As a Stewart platform is symmetrical in its nominal configuration, an external preload along the centre vertical axis of the manipulator (z-axis of the global coordinate system) can effectively cause preloads on all six legs. Therefore, the redundant actuator is configured to align with this centre vertical axis at the robot nominal pose (Fig. 3.6). The top end of the actuator is connected via a spherical joint to a rigid support frame whose mounting point is on the centre vertical axis while the bottom end is attached via a spherical joint to the centre of the moving platform. Although misalignment between the redundant actuator and the centre vertical axis occurs during motion of the moving platform, effective preloads can still be achieved on all six legs within the envelope of motion of a typical Stewart platform. With a sufficiently long redundant actuator assembly, it is possible to apply all compressions or all tensions on the six legs, and therefore largely decrease the overall control difficulty. Moreover, a passive element (mass-spring-damper system) is introduced into the redundant actuator assembly to achieve a moderately compliant coupling with
3.4. Theoretical Analysis

Figure 3.6. Configuration of the redundant manipulator where BSP represents the ball screw piston, M1 and M2 represent the upper mass and bottom mass of the mass-spring-damper system respectively, FS represents the force sensor, and SJ represents the lower spherical joint.

the Stewart platform. This inherently increases the disturbance rejection and force resolution of the system (Schutter and Brussel, 1988), and thus makes the implementation of force control much easier. A force sensor is attached to the redundant actuator to measure its preload. Details of parameter selection for the passive element will be discussed in Section 3.4.4. In this study, the redundant leg is placed at the upper space of the robot assuming that the inner space is used for implementing tasks which is often the case in applications involving large interaction forces (e.g. biomechanical testing). If the upper space of the robot is required for task implementation, the redundant leg can be placed inside the inner space.

3.4.2 Inverse Dynamics Equation

The inverse dynamics model of a general Stewart platform has been presented in detail in Do and Shahimpoor (1988), Dasgupta and Mruthyunjaya (1998), and Harib and Srinivasan (2003). In the proposed preload control method, the inverse dynamics is used to predict the actuator control forces for optimizing the preload on the redundant actuator. To ease the computation expense in optimization, a simplified inverse dynamics model is derived for the redundant manipulator with the following assumptions: 1) As the motion range of the Stewart platform is limited around its nominal pose in high precision and high load applications, the centre of gravity of
each leg is always fixed at the point which is the equivalent centre of gravity when the
leg is at its nominal length (mid stroke). 2) A universal joint is used at the stationary
end of each leg (includes the redundant leg) and therefore there is no rotational
movement about the longitudinal axis of the leg. 3) Friction is not considered. 4)
Motor dynamics and actuator transmission system dynamics are not considered.

Fig. 3.7 shows the free-body diagram of one leg and the moving platform. Each leg
(actuator) consists of a cylinder and a piston. As the moving platform and each leg are
connected via a frictionless spherical joint, there is no moment but a single force
exerted at \( a_i \) which can be decomposed as a force along the longitudinal axis of the
leg \( \mathbf{F}_{ui} \) and a force normal to the longitudinal axis \( \mathbf{F}_{ni} \). \( \mathbf{F}_{ui} \) results from the
actuator control force and \( \mathbf{F}_{ni} \) is caused by the rotational dynamics of the leg. In
order to solve \( \mathbf{F}_{ui}, \mathbf{F}_{ni} \) must be solved first. Considering the moments acting on \( i \)th
leg about the rotation centre of the leg \( b_i \), Euler’s equation gives:

\[
l_i \mathbf{u}_i \times \mathbf{F}_{ni} + m_c l_c \mathbf{u}_i \times \mathbf{g} = I_i \dot{\mathbf{\omega}}_i + \mathbf{\omega}_i \times (l_i \mathbf{\omega}_i) + m_c l_c \mathbf{u}_i \times \mathbf{a}_{ci} \tag{3.12}
\]

Figure 3.7. Free-body diagram of one leg and the moving platform, where \( a_i \) represents the fixed joint
centre of leg \( i \), \( p_i \) represents the moving joint centre of leg \( i \), \( c_i \) represents the gravity centre of leg \( i \), \( o \)
represents the end-effector, \( q \) represents the gravity centre of the moving platform, and \( e \) represents the
point where the external loads are exerted on the moving platform. The end-effector coordinate system
frame \( \{o\} \) is attached to \( o \), a leg inertia coordinate system frame \( \{a_i\} \) is attached to \( a_i \) and rotates in
coincidence with leg \( i \), and a global coordinate system frame \( \{O\} \) is fixed for reference. \((i=1:7)\).
3.4. Theoretical Analysis

where \( \mathbf{u}_i \) is the unit vector along the leg, \( l_i \) is the leg length, \( l_c \) is the distance between the leg rotation centre and the leg centre of gravity, \( m_c \) is the leg mass, \( \mathbf{I}_i \) represents the inertia tensor of leg, \( \mathbf{\omega}_i \) and \( \mathbf{\dot{\omega}}_i \) are the angular velocity and the angular acceleration of the leg respectively, \( \mathbf{a}_{ci} \) is the acceleration of the gravity centre of the leg, and \( \mathbf{g} \) is the gravitational vector. The global basis \( \mathbf{I}_i \) can be obtained via the following equation

\[
\mathbf{I}_i = \mathbf{R}_{ai} \mathbf{I}_a (\mathbf{R}_{ai})^T
\]  

(3.13)

where \( \mathbf{I}_a \) is the inertia tensor of the leg relative to the leg inertia coordinate system \( \{a_i\} \) and remains as a constant, \( \mathbf{R}_{ai} \) is the rotation matrix describing the orientation of \( \{a_i\} \) relative to the global coordinate system \( \{O\} \).

By assuming there is no rotation moment about the longitudinal axis of the leg (i.e. \( \mathbf{u}_i \cdot \mathbf{\omega}_i = 0 \) and \( \mathbf{u}_i \cdot \mathbf{\dot{\omega}}_i = 0 \)), the kinematics of the leg can be written as Eqs. (3.14)-(3.18) (Dasgupta and Mruthyunjaya, 1998),

\[
l_i = \|\mathbf{R} \mathbf{p}_i^m + \mathbf{r}_i\|, \quad \mathbf{u}_i = (\mathbf{R} \mathbf{p}_i^m + \mathbf{r}_i)/l_i
\]  

(3.14)

where \( \mathbf{t} \) represents the end-effector position, \( \mathbf{R} \) represents the end-effector orientation, \( \mathbf{p}_i^m \) represents the position of the \( i \)th spherical joint in the end-effector coordinate system \( \{o\} \), and \( \mathbf{r}_i \) represents the position of the \( i \)th universal joint.

\[
\dot{l}_i = \mathbf{u}_i \cdot (\mathbf{\omega}_o \times \mathbf{R} \mathbf{p}_i^m + \dot{\mathbf{t}})
\]  

(3.15)

where \( \dot{l}_i \) represents the elongation speed of leg \( i \), and \( \mathbf{\omega}_o \) and \( \dot{\mathbf{t}} \) represents the angular velocity and the linear velocity of the end-effector respectively. The angular velocity and acceleration of the leg respectively are given by:

\[
\mathbf{\omega}_i = \mathbf{u}_i \times (\mathbf{\omega}_o \times \mathbf{R} \mathbf{p}_i^m + \dot{\mathbf{t}})/l_i,
\]  

(3.16)

\[
\mathbf{\dot{\omega}}_i = (\mathbf{u}_i \times (\mathbf{\omega}_o \times \mathbf{R} \mathbf{p}_i^m + \mathbf{\omega}_o \times (\mathbf{\omega}_o \times \mathbf{R} \mathbf{p}_i^m) + \dot{\mathbf{t}}) - 2\dot{l}_i \mathbf{\omega}_i)/l_i
\]  

(3.17)

where \( \mathbf{\omega}_o \) and \( \ddot{\mathbf{t}} \) represents the angular and linear acceleration of the end-effector respectively. The acceleration of the centre of gravity of leg \( i \) can be written as

\[
\mathbf{a}_{ci} = \mathbf{\dot{\omega}}_i \times (l_c \mathbf{u}_i) + \mathbf{\omega}_i \times (\mathbf{\omega}_i \times l_c \mathbf{u}_i).
\]  

(3.18)

By substituting Eqs. (3.13)-(3.18) into Eq. (3.12), \( \mathbf{F}_{ni} \) can be solved. Then considering the dynamics of the moving platform, Newton’s equation for the moving platform gives
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\[ m_p \mathbf{g} + \mathbf{RF}_{ex}^m - \sum_{i=1}^7 (F_{ui}) - \sum_{i=1}^7 (F_{ni}) = m_p (\dot{\omega}_o \times \mathbf{Rq}^m + \omega_o \times (\omega_o \times \mathbf{Rq}^m) + \ddot{\mathbf{r}}), \]

(3.19)

where \( m_p \) represents the moving platform mass, \( \mathbf{F}_{ex}^m \) represents the external forces acting on the platform in the end-effector coordinate system, and \( \mathbf{q}^m \) represents the position vector of the gravity centre of the moving platform in the end-effector coordinate system. Considering the moments acting on the moving platform about \( o \), Euler’s equation gives

\[ m_p \mathbf{Rq}^m \times \mathbf{g} + \mathbf{Re}^m \times (\mathbf{RF}_{ex}^m) + \mathbf{RM}_{ex}^m - \sum_{i=1}^7 (\mathbf{Rp}_i^m \times F_{ui}) - \sum_{i=1}^7 (\mathbf{Rp}_i^m \times F_{ni}) = \]

\( \mathbf{I}_p \dot{\omega}_o + \omega_o \times (\mathbf{I}_p \omega_o) + m_p \mathbf{Rq}^m \times (\dot{\omega}_o \times \mathbf{Rq}^m + \omega_o \times (\omega_o \times \mathbf{Rq}^m) + \ddot{\mathbf{r}}), \]

(3.20)

where \( \mathbf{M}_{ex}^m \) represents the external moments acting on the platform in the end-effector coordinate system, \( \mathbf{e}^m \) represents the position vector of the external force exerting point in the end-effector coordinate system, and \( \mathbf{I}_p \) represents the inertia tensor of the moving platform and can be obtained via

\[ \mathbf{I}_p = \mathbf{RI}_o (\mathbf{R})^T \]

(3.21)

where \( \mathbf{I}_o \) is the inertia tensor of the moving platform relative to end-effector coordinate system \( \{o\} \) and remains as a constant. The dynamics equation of the moving platform can be written in matrix form:

\[ \mathbf{J}^{-T} \mathbf{f}_u = \mathbf{M}_p \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\omega}_o \end{bmatrix} + \mathbf{C}_p + \mathbf{G}_p + \mathbf{H}_n + \mathbf{J}_r \mathbf{f}_r + \mathbf{N}_p \]

(3.22)

with \( \mathbf{J}^{-T} = \begin{bmatrix} \ldots & \mathbf{u}_i & \ldots & \mathbf{Rp}_i^m \times \mathbf{u}_i & \ldots \end{bmatrix}_{i=1:6} \), \( \mathbf{f}_u = \begin{bmatrix} \dot{\mathbf{r}} \\ \mathbf{f}_{ui} \end{bmatrix}_{i=1:6} \),

\[ \mathbf{M}_p = \begin{bmatrix} -m_p & m_p \mathbf{Rq}^m \\ -m_p \mathbf{Rq}^m & -\mathbf{I}_p + m_p (\mathbf{Rq}^m \times \mathbf{Rq}^m)^2 \end{bmatrix}, \]

\[ \mathbf{C}_p = \begin{bmatrix} -m_p \omega_o \times (\omega_o \times \mathbf{Rq}^m) \\ -\omega_o \times (\mathbf{I}_p \omega_o) - m_p \mathbf{Rq}^m \times (\omega_o \times (\omega_o \times \mathbf{Rq}^m)) \end{bmatrix}, \]

\[ \mathbf{G}_p = \begin{bmatrix} m_p \mathbf{g} \\ m_p \mathbf{Rq}^m \times \mathbf{g} \end{bmatrix}, \quad \mathbf{H}_n = \begin{bmatrix} -\sum_{i=1}^7 (F_{ni}) \\ -\sum_{i=1}^7 (\mathbf{Rp}_i^m \times F_{ni}) \end{bmatrix}, \]

\[ \mathbf{J}_r = \begin{bmatrix} -\mathbf{u}_7 \\ -\mathbf{Rp}_7^m \times \mathbf{u}_7 \end{bmatrix}, \quad \mathbf{N}_p = \begin{bmatrix} \mathbf{RF}_{ex}^m \\ \mathbf{Re}^m \times (\mathbf{RF}_{ex}^m + \mathbf{RM}_{ex}^m) \end{bmatrix}, \]
where \( \mathbf{J} \) represents the 6 \( \times \) 6 kinematics Jacobian matrix of a general Stewart platform, \( \mathbf{f}_u \) represents the preloads on six original legs \( \mathbf{u}_i \mathbf{f}_{ui} = \mathbf{F}_{ui} \), \( \mathbf{M}_p \) represents the inertia matrix of the moving platform, \( \mathbf{C}_p \) represents the centrifugal and Coriolis terms of the moving platform, \( \mathbf{G}_p \) represents the gravity vector of the moving platform, \( \mathbf{H}_n \) represents the terms generated from the dynamics of the legs, \( \mathbf{J}_r \) represents the statics vector of the redundant leg, \( \mathbf{f}_r \) represents the axial preload on the redundant leg \( \mathbf{u}_r \mathbf{f}_r = \mathbf{F}_{u7} \), and \( \mathbf{N}_p \) represents the external loads. In Eq. (3.22), if \( \mathbf{f}_r \), the external loads, and the trajectory of the end-effector are known, then \( \mathbf{f}_u \) can be calculated.

Finally, considering the dynamics of actuator piston, the actuator control forces can be derived:

\[
\tau_{ci} = -\mathbf{f}_{ui} - m_{pis} \mathbf{g} \cdot \mathbf{u}_i + m_{pis} \ddot{l}_i, \quad i = 1:6
\]  

(3.23)

where \( m_{pis} \) is the mass of the actuator piston, and \( \ddot{l}_i \) is the elongation acceleration of leg \( i \) which can be written as

\[
\ddot{l}_i = \mathbf{u}_i \cdot (\dot{\omega}_o \times \mathbf{R}_i^m + \omega_o \times (\omega_o \times \mathbf{R}_i^m)) + \dot{\mathbf{i}} + l_i \omega_i \cdot \omega_i.
\]  

(3.24)

Although simplified, the inverse dynamics model of the redundant manipulator is still difficult to solve in real-time. The model can be further simplified by eliminating all the Coriolis and centrifugal terms. In applications where the motion of the manipulator is slow and external loads are large, the dynamics of the legs and pistons can be ignored and therefore the inverse dynamics model can be finally simplified as a closed form:

\[
\tau_c = -\mathbf{J}^T \left[ \mathbf{M}_p \left[ \begin{array}{c} \dot{\mathbf{i}} \\ \omega_o \end{array} \right] + \mathbf{G}_p + \mathbf{G}_l + \mathbf{J}_r \mathbf{f}_r + \mathbf{N}_p \right] - \mathbf{G}_u \mathbf{g} - \mathbf{f}_u
\]  

(3.25)

with

\[
\tau_c = \left[ \begin{array}{c} \tau_{ci} \\ \vdots \end{array} \right]_{i=1:6}, \quad \mathbf{G}_u = m_{pis} \left[ \begin{array}{c} \vdots \\ (\mathbf{u}_i)^T \end{array} \right]_{i=1:6},
\]

\[
\mathbf{G}_l = \left[ \begin{array}{c} -\sum_{i=1}^{7} ((m_c l_i / l_i) \mathbf{u}_i \times (\mathbf{u}_i \times \mathbf{g})) \\ -\sum_{i=1}^{7} ((m_c l_i / l_i) \mathbf{R}_i^m \times \mathbf{u}_i \times (\mathbf{u}_i \times \mathbf{g})) \end{array} \right]
\]

where \( \tau_c \) represents the control forces for the original six actuators, \( \mathbf{G}_u \) represents the gravity matrix of the pistons, and \( \mathbf{G}_l \) represents the gravity vector of all seven legs.
3.4.3 Preload Optimisation

3.4.3.1 Optimisation Problem Formulation

Even if assuming the trajectories of the end-effector and the external loads are known, the solution for \( f_r \) in Eq. (3.25) is not unique in satisfying the backlash-free condition as described in Eq. (3.11). This problem can be solved by minimizing the total internal preloads acting on the seven actuators \([f_u]^T f_r\). The lower the total internal preloads means the lower the energy consumption of the system. Therefore, the backlash prevention optimal control problem can be formed as:

\[
\begin{align*}
L(t) &\rightarrow \min \\
\rho &> f_r(t) > -\rho \\
(\sigma &> |\tau_{ci}(t)| \geq \varepsilon, \ \text{sign}(\tau_{ci}(t)) = \text{constant}_{i=1:6}) \\
&\text{subject to } t \in [0, T] \quad (3.26)
\end{align*}
\]

where \( L = \left\| \left( J^T \left( M_p \left[ \ddot{\omega}_a \right] + G_p + G_l + J_r f_r + N_p \right) \right)^T f_r \right\| \),

\[
\tau_c = -J^T \left( M_p \left[ \ddot{\omega}_a \right] + G_p + G_l + J_r f_r + N_p \right) - G_u g,
\]

where \( L \) represents the 2-norm of the total internal preloads at time \( t \), \( \rho \) represents the maximum allowed preload on the redundant leg. As \( f_r \) is the only unknown in \( L \) and all six \( \tau_{ci} \) need to satisfy the backlash prevention condition, Eq. (3.26) is a one-dimensional quadratic optimization problem subject to seven inequality constraints. Furthermore, there are \( 2^6 = 64 \) possible combinations of signs of \( \tau_{ci} \), each of which has to be considered independently in Eq. (3.26) and therefore the required computational time for solving Eq. (3.26) is enlarged 64 times. As mentioned in Section 3.4.1, with the redundant actuator configuration, it is easy to add all compression (positive preloads) or all tension (negative preloads) on the six original legs and thus is far more feasible to cause all positive or all negative \( \tau_{ci} \) rather than the other cases. This reduces the possible combinations of signs from 64 to 2 and the optimal control problem is simplified as only two possible cases:

\[
\begin{align*}
L(t) &\rightarrow \min \\
\rho &> f_r(t) > -\rho \\
(\sigma &> \tau_{ci}(t) \geq \varepsilon)_{i=1:6}
\end{align*}
\]

or

\[
\begin{align*}
L(t) &\rightarrow \min \\
\rho &> f_r(t) > -\rho \\
(-\varepsilon &\geq \tau_{ci}(t) > -\sigma)_{i=1:6}
\end{align*}
\]
3.4.3.2 Optimisation Algorithm

Problem (3.27) can be solved by offline optimization methods if a prescribed trajectory of the end-effector and a prescribed trajectory of the external loads are both given. However, in real-time applications, the external loads are caused by the interaction between the robot and environment, and thus the prescribed trajectory of external loads is generally unpredictable. This prevents the offline optimization methods from working in applications where the external loads are dominant. In order to address this issue, the author developed an online optimization algorithm. This approach requires the external loads to be measured by a 6-DOF load sensor which normally exists in most biomechanical testing robotic systems. With the external load feedback, \( \mathbf{r}_c \) can be observed and predicted for determining \( f_r \) at each discrete time \( t_k \).

As the proposed optimization algorithm is based on online feedback measurement rather than offline processing, \( \mathbf{r}_c \) may slip into the backlash-free condition before a control decision is made. Furthermore, when tracking the determined \( f_r \) on the redundant leg under force control, force tracking errors must appear and cause preload errors on the six original legs. This can also lead \( \mathbf{r}_c \) to stray into the backlash problem region. Therefore, the backlash free-condition in Eq. (3.27) is redefined for compensating the delays in measuring external loads and controlling \( f_r \):  

\[
(\sigma - \mu > \tau_{cl}(t) \geq \varepsilon + \mu)_{i=1:6}, \quad \text{or} \quad (-\varepsilon - \mu \geq \tau_{cl}(t) > -\sigma + \mu)_{i=1:6}, \tag{3.28}
\]

where \( \mu \) represents a safety margin, which is equal to the magnitude of the maximum preload error caused by force control delay. The margin narrows the original backlash-free condition. If Eq. (3.28) is satisfied, \( \mathbf{r}_c(t) \) are in the safe zone, where \( \mathbf{r}_c(t) \) not only satisfy the backlash-free condition but also are away from the backlash problem region. If Eq. (3.28) is not satisfied, \( \mathbf{r}_c(t) \) are in the danger zone, and \( \mathbf{r}_c(t) \) are either very close to or already in the backlash problem region. With the definition of Eq. (3.28), a decision can be made before the backlash problem actually occurs.

The flow chart of the proposed algorithm is shown in Fig. 3.8. At each discrete time \( t_k \), the external forces \( \{F_{ex(k)}^m\} \) and moments \( \{M_{ex(k)}^m\} \) are measured from the sensor. \( \mathbf{p}_k \) and \( \dot{\mathbf{p}}_k \) represent the current desired end-effector pose and acceleration respectively. \( f_{r(k-1)} \) represents the preload requirement for the redundant leg calculated at the last discrete time \( t_{k-1} \). Using the inverse dynamics Eq. (3.25), the current control forces of the six position-controlled actuators \( \mathbf{r}_{ck} \) can be
approximated as well as the current total internal preloads index $L_k$. Then $\tau_{ck}$ are checked in Eq. (3.28). If $\tau_{ck}$ are in the safe zone, $f_{rk}$ is regulated within its range to minimize the total internal preloads index in the range of the safe zone. In order to guarantee the smoothness of $f_r$ and decrease the computation burden, only the two points ($f_{rk-p}, f_{rkn}$) around $f_{r(k-1)}$ with a small increment $d$ are considered. The total internal preloads index $L_{kp}$ and $L_{kn}$ for these two points are calculated and compared. The smaller one is then compared with the current index $L_k$. If $L_k$ is smaller, the preload requirement at $\tau_k (f_{rk})$ remains the same as $f_{r(k-1)}$. Otherwise, the control forces under the new preload ($\tau_{ck-p}$ or $\tau_{ck-n}$) are calculated via Eq. (3.25) and checked.
in Eq. (3.28). If $\tau_{ckp}$ or $\tau_{ckn}$ are in the safe zone, $f_{rk}$ is equal to $f_{rKP}$ or $f_{rKn}$. If not, $f_{rk}$ remains the same as $f_{r(k-1)}$. In the case when $\tau_{ck}$ are in the danger zone, $f_{rk}$ is regulated in its range to quickly move the control forces into the safe zone. This is achieved by iteratively searching $f_{rk}$ along both positive and negative directions simultaneously from $f_{r(k-1)}$. At each iteration $j$, $f_{rKP}^j$ and $f_{rKn}^j$ are increased in their directions with an increment of $d$. Then the corresponding control forces $\tau_{rKP}^j$ and $\tau_{rKn}^j$ are calculated via Eq. (3.25) and checked in Eq. (3.28). If none of $\tau_{rKP}^j$ and $\tau_{rKn}^j$ are in the safe zone, the next iteration starts. Otherwise, the iteration ends and the rest of the code simply ensures the discrete increment between $f_{rk}$ and $f_{r(k-1)}$ is below a maximum allowed value $\gamma$.

### 3.4.4 Force Control

In order to achieve the optimised preload trajectory on the redundant leg, an accurate force control is required. This subsection investigates the redundant leg dynamics as well as the control algorithm for preload tracking.

#### 3.4.4.1 Dynamics Model of the Redundant Leg

Figure 3.9 shows a simplified schematics of the redundant leg, where the system is modelled as a linear three-mass system under two assumptions. Firstly, we assume the connection between motor and ballscrew piston is infinitely rigid compared to the mass-spring-damper (MSD) system. Secondly, by assuming the ballscrew backlash is infinitely small compared to the MSD system displacement, backlash non-linearity is

![Figure 3.9. Simplified schematic diagram of the redundant actuator](image)
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ignored. Analysing the torque balance on the motor, we have:

\[ J_m \ddot{\theta}_m = \tau_m - \tau_r / \eta - b_m \dot{\theta}_m \quad (3.29) \]

where \( J_m \) is the motor moment of inertia, \( b_m \) is the viscous motor friction, \( \theta_m \) is the motor rotational angle, \( \tau_m \) is the motor driving torque, \( \tau_r \) is the control force for driving the ballscrew piston, and \( \eta \) is the torque to force ratio of the ballscrew. As the ballscrew piston and MSD cylinder are bolted together, the differential equation describing their dynamics can be written in the form of a collective mass,

\[ m_{pc} \ddot{x}_{pc} = \tau_r + m_{pc} \mathbf{g} \cdot \mathbf{u}_r - k_s (x_{pc} - l_r) - b_d (\dot{x}_{pc} - \dot{l}_r) - b_{pc} \ddot{x}_{pc}, \]

with \( x_{pc} = \lambda \theta_m \),  

\[ (3.30) \]

where \( m_{pc} \) is the total mass of the ballscrew piston and MSD cylinder, \( b_{pc} \) is the viscous friction of the ballscrew piston, \( x_{pc} \) is the displacement of the ballscrew piston, the displacement of the MSD piston is equal to the displacement of the redundant leg length \((l_r)\), \( k_s \) and \( b_d \) are the spring stiffness and damping of the MSD system respectively, and \( \lambda \) is the angle to displacement ratio (also known as lead) of the ballscrew. Analysing the force balance on the MSD piston, we have:

\[ m_{mp} \ddot{l}_r = k_s (x_{pc} - l_r) + b_d (\dot{x}_{pc} - \dot{l}_r) + m_{mp} \mathbf{g} \cdot \mathbf{u}_r + f_r \quad (3.31) \]

where \( m_{mp} \) is the mass of the MSD piston, \( f_r \) is the preload on the redundant leg. With Eqs. (3.29) to (3.31), the block diagram of the redundant actuator dynamics can be found in Fig. 3.10. Clearly, the preload we want to control \( (f_r)\) is subject to the acceleration term \((l_r m_{mp} s^2)\) and gravity term \((m_{mp} \mathbf{g} \cdot \mathbf{u}_r)\) of the MSD piston, and the stiffness term \(((x_{pc} - l_r) k_s)\) and damping term \(((x_{pc} - l_r) b_d s)\) of the MSD system. The acceleration term and gravity term of the MSD piston are the disturbances in \( f_r \) as they are not controllable via \( \tau_m \). Therefore, the MSD piston mass \( m_{mp} \) is ideally made as small as possible to minimize such disturbances. The stiffness term of the MSD system is the major term in \( f_r \) which can be controlled by regulating \( x_{pc} \) using a position control loop. The selection of an appropriate MSD system stiffness \( k_s \) is critical. Very high \( k_s \) can lead to low disturbance rejection. This physically means any disturbance or error in spring movement can lead to large force errors in \( f_r \). Conversely, a very low \( k_s \) can decrease the bandwidth of force control if the actuator slew rate is limited. The stability of force control is subject to the MSD
3.4. Theoretical Analysis

3.4.4.2 Force Control Algorithm

As \( f_r \) is mainly governed by the relative displacement between the ballscrew piston and MSD piston, a position-based explicit control algorithm (Schutter, 1988) is applied to control \( f_r \). Fig. 3.11 shows the algorithm in the form of a block diagram, where superscript \( d \) represents the desired value and superscript \( s \) represents the real value. An outer force control loop is placed around an inner position control loop. The force loop calculates the desired relative displacement \( (x_{fd}) \) between the ballscrew piston and MSD piston for minimizing the force error \( (\Delta f_r) \) between the desired force \( (f_{rd}) \) and the measured force \( (f_{rs}) \). \( f_{rd} \) is derived from the preload optimisation.

Figure 3.10. Block diagram of the redundant actuator dynamics

Figure 3.11. Block diagram of position-based force control

damping term. There is a trade-off in selecting the MSD damping \( b_d \). A low \( b_d \) can cause control instability, while a high \( b_d \) can lead to a large time constant and therefore decrease the bandwidth.
algorithm while \( f_r^s \) is measured from the sensor. The absolute displacement of the leg length \( l^d_r \) is calculated via inverse kinematics from the end-effector desired pose \( \mathbf{p}^d \) and is used to compensate the impact of the displacement of the MSD piston \( (l_r) \) on \( f_r \). The sum of \( l^d_r \) and \( x^d_f \) gives the total desired displacement of the ballscrew piston \( (x^d_{pc}) \), while the real displacement of the ballscrew piston \( (x^s_{pc}) \) is obtained from the motor rotary encoder. The internal position loop calculates the motor torque \( \tau_m \) based on the displacement error \( (\Delta x_{pc}) \) between \( x^d_{pc} \) and \( x^s_{pc} \) for driving the ballscrew piston to achieve the desired displacement. A PID controller is applied to the position control loop. The force controller consists of a pure integral term and a low pass filter:

\[
g_c(s) = k_s^{-1} k_f s^{-1} \left( N_f s + 1 \right)^{-1}
\]

where \( k_s \) is an approximation of the spring stiffness, \( k_f \) is the integral gain, and \( N_f \) is the low pass filter time constant. Integral control is commonly used in position-based force control and yields good accuracy. \( k_f \) is normally set as half of the position-loop bandwidth with the resulting bandwidth of the force loop half the position-loop bandwidth. With integral control, the force error is proportional to the desired velocity of the actuator and therefore any discontinuity in force error can result in discontinuity in actuator motion. In order to ensure smooth actuator motion, a low pass filter is used in series.

### 3.5 Numerical Simulation

This section uses the Stewart platform-based manipulator developed from this study as an example to verify the preload control method with the assumption that an additional leg consisting of a ballscrew (same as the original leg ballscrews) and a mass-spring-damper system is mounted at the top of the manipulator. Simulations are implemented on a high fidelity model of this system in the aspects of the redundant manipulator configuration, the preload optimization algorithm, and the force control strategy respectively.

#### 3.5.1 Preload Distribution Efficiency of the Redundant Manipulator

This subsection assesses the proposed redundant manipulator configuration in terms of its efficiency to distribute active preloads on the six position-controlled legs. The
Table 3.1. Geometrical parameters of the additional leg and the robot

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values (mm)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^T_7$</td>
<td>[0 0 136.5]^T</td>
<td>Position of the additional leg spherical joint in ${o}$</td>
</tr>
<tr>
<td>$r_\gamma$</td>
<td>[0 0 1573.1]^T</td>
<td>Position of the additional leg universal joint in ${O}$</td>
</tr>
<tr>
<td>$t^n_0$</td>
<td>[0 0 −100]^T</td>
<td>Position of the specimen centre of rotation in ${o}$</td>
</tr>
<tr>
<td>$[t^T_0 \theta^T_0]^T$</td>
<td>[0 0 490.7 0 0 0]^T</td>
<td>Initial robot pose</td>
</tr>
</tbody>
</table>

geometrical parameters of the original manipulator can be found in Table 2.1-2.4 and the additional parameters required for simulation are listed in Table 3.1. Given these geometrical parameters, the active preloads distributed on the six original legs arising from the unit preload of the additional leg can be calculated. In order to obtain the overall distribution efficiency of the redundant manipulator within the workspace of the robot, such a relationship is quantified during the movement of the robot along three translational axes and three rotational axes about a virtual specimen centre of rotation. The results corresponding to each of the degree of freedoms are shown in Fig. 3.12 to 3.17, where the solid lines represent the preload ratio of forces between the $i$th leg and the redundant leg and the pink dashed line represents a preload ratio boundary (0.05). The boundary is defined as approximately one quarter of the preload ratio at the robot nominal pose and is regarded as marginally effective to assign the preload on the leg. Thus, preload ratios above the boundary means that effective preload can be distributed to the corresponding leg, whilst ratios below the boundary leads to low distribution efficiency, in which circumstance the preload on the corresponding leg is

Figure 3.12. Preload distribution efficiency on x-axis translation.
Chapter 3. Active Preload Control of a Redundantly Actuated Stewart Platform for Backlash Prevention

Figure 3.13. Preload distribution efficiency on y-axis translation.

Figure 3.14. Preload distribution efficiency on z-axis translation.

Figure 3.15. Preload distribution efficiency on x-axis rotation.
3.5. Numerical Simulation

difficult to control. The worst case scenario is when any of the ratios goes to negative. When this occurs, the hypothesis in Eq. (3.27) that the redundant leg can cause all tensions or all compressions on the original six legs is no longer valid. From the figures, we can see that the preload ratios are larger than 0.05 when the robot motion is restricted to $\pm 70\text{mm}$ on $T_x$, $\pm 75\text{mm}$ on $T_y$, $\pm 22$ degrees on $R_x$, and $\pm 18$ degrees on $R_y$. This range of motion is close to the full range of motion of the Stewart platform and is normally sufficient for high precision applications (e.g. biomechanical testing, machining). Consequently, the proposed mechanism exhibits acceptable preload distribution efficiency within its workspace, which ensures acceptable preload controllability for all six position-controlled legs.

Figure 3.16. Preload distribution efficiency on y-axis rotation.

Figure 3.17. Preload distribution efficiency on z-axis rotation.
Table 3.2. Geometrical and physical parameters for preload optimization simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{sp}$</td>
<td>[80 80 500 6 6 4]^T</td>
<td>Linear stiffness of the specimen (N/mm, Nm/degree)</td>
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<tr>
<td>$q^m$</td>
<td>[0 0 60]^T</td>
<td>Position of the platform centre of gravity in ${o}$ (mm)</td>
</tr>
<tr>
<td>$e^m$</td>
<td>[0 0 −100]^T</td>
<td>Position of the specimen centre of rotation in ${o}$ (mm)</td>
</tr>
<tr>
<td>$l_c$</td>
<td>240</td>
<td>Length between leg rotation centre and gravity centre (mm)</td>
</tr>
<tr>
<td>$m_p$</td>
<td>20</td>
<td>Platform mass (kg)</td>
</tr>
<tr>
<td>$m_{pis}$</td>
<td>2</td>
<td>Actuator piston mass (kg)</td>
</tr>
<tr>
<td>$m_c$</td>
<td>5</td>
<td>Leg mass (kg)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>70</td>
<td>Backslash-free threshold (N)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>4000</td>
<td>Actuator payload limit (N)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>4000</td>
<td>Redundant actuator payload limit (N)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>10</td>
<td>Safety margin (N)</td>
</tr>
<tr>
<td>$d$</td>
<td>2</td>
<td>Preload searching resolution (N)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>20</td>
<td>Preload discrete increment limit (N)</td>
</tr>
<tr>
<td>$f_{r0}$</td>
<td>100</td>
<td>Initial preload on the additional leg (N)</td>
</tr>
</tbody>
</table>

3.5.2 Preload Optimization Simulation

The proposed preload optimization algorithm is assessed in this subsection under the following two assumptions. 1) The robot tests a stiff specimen within its inner space and therefore undergoes large 6-DOF external loads. For simplicity, the specimen is assumed to have a linear stiffness matrix with diagonal terms only. 2) As the simulated motion is slow and the resulting external loads are large, the end-effector acceleration term and the leg dynamics terms are negligible and thus are ignored in Eq. (3.25) during optimization. The geometrical and physical parameters required for preload optimization are listed in Table 3.2. The backlash-free condition ($\xi$, $\sigma$, $\rho$ and $\mu$) for the robot ballscrews are obtained from trials. The loop running the preload optimization algorithm has a loop rate of 100Hz. The initial preload on the redundant leg is defined as 100N (a positive value means compression) for initially moving the control forces of all the original ballscrews into the backlash-free region, such that all six control forces are positive. Simulations are implemented by deforming the specimen about its centre of rotation in shear (x and y axis translation), axial loading (z axis translation), bending (x and y axis rotation), and torsion (z axis rotation). A sinusoidal waveform with +/-3mm (+/-10 degrees for rotation) amplitude and 0.1Hz is
applied on each of the above six degrees of freedom sequentially for three cycles. Besides the major movement, a sinusoidal waveform with +/-0.1mm amplitude and 0.1Hz is superposed to all three translational axes for simulating the coupled forces arising from the movement of the specimen centre of rotation.

Figures 3.18, 3.20, 3.22, 3.24, 3.26, and 3.28 show the control forces on the six position-controlled legs under external loads on each of the degrees of freedom without preload control. Obviously, the control forces on the six legs cross the backlash problem region in all external loading cases. The conditions of some legs are particularly serious where the control force on the leg remains in the backlash problem region the entire time (e.g. 3rd leg and 4th leg in Fig. 3.26). There is no doubt that the control bandwidth and accuracy are both dramatically limited on that leg in a normal Stewart platform. On the other hand, Figures 3.19, 3.21, 3.23, 3.25, 3.27, 3.29 show the cases under the proposed preload control, where subplot (a) shows the optimized control forces on the six position-controlled legs (solid lines) and the backlash-free threshold (pink dash line) and subplot (b) shows the required preload on the redundant leg. The results demonstrate that the proposed algorithm is able to restrict the control forces on the six position-controlled legs to the backlash-free region by generating a consistent desired preload trajectory on the redundant leg. As we can further see from the plots, as soon as the control force on any of the six legs approaches the margin of the danger zone (defined as 80N in simulation), the algorithm enlarges the desired preload on the redundant leg in order to move the control forces on the six legs away from the backlash problem region. When the control forces on the six legs are in the safe zone, the algorithm gradually decreases the total internal preloads on all seven legs. Results also demonstrate that the proposed method is robust to large external loads on six degrees of freedom (e.g. ±240N shear, ±1500N compression/tension, ±60Nm bending, ±40Nm torsion).
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Figure 3.18. Control forces on the six legs under ±240N x-axis shear without preload control

Figure 3.19. Optimized control forces and desired preload under ±240N x-axis shear

(a) Optimized actuator control forces on the six legs

(b) Desired preload on the redundant leg

Figure 3.19. Optimized control forces and desired preload under ±240N x-axis shear
3.5. Numerical Simulation

Figure 3.20. Control forces on the six legs under ±240N y-axis shear without preload control

![Control forces on the six legs](image1)

Figure 3.21. Optimized control forces and desired preload under ±240N y-axis shear

(a) Optimized actuator control forces on the six legs

![Optimized actuator control forces](image2)

(b) Desired preload on the redundant leg

![Desired preload](image3)
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Figure 3.22. Control forces on the six legs under ±1500N z-axis loading without preload control

Figure 3.23. Optimized control forces and desired preload under ±1500N z-axis loading

(a) Optimized actuator control forces on the six legs

(b) Desired Preload on the redundant leg
3.5. Numerical Simulation

Figure 3.24. Control forces on the six legs under ±60Nm x-axis bending without preload control

(a) Optimized actuator control forces on the six legs

(b) Desired preload on the redundant leg

Figure 3.25. Optimized control forces and desired preload under ±60Nm x-axis bending
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Figure 3.26. Control forces on the six legs under ±60Nm y-axis bending without preload control

Figure 3.27. Optimized control forces and desired preload under ±60Nm y-axis bending

(a) Optimized actuator control forces on the six legs

(b) Desired preload on the redundant leg

Figure 3.27. Optimized control forces and desired preload under ±60Nm y-axis bending
3.5. Numerical Simulation

Figure 3.28. Control forces on the six legs under ±40Nm z-axis torsion without preload control

Figure 3.29. Optimized control forces and desired preload under ±40Nm z-axis torsion

(a) Optimized actuator control forces on the six legs

(b) Desired preload on the redundant leg

Figure 3.29. Optimized control forces and desired preload under ±40Nm z-axis torsion
Chapter 3. Active Preload Control of a Redundantly Actuated Stewart Platform for Backlash Prevention

Table 3.3. Model parameters for force control simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.404</td>
<td>Lead of the ballscrew actuator (mm/rad)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2215</td>
<td>Torque to force ratio of the ballscrew actuator (N/Nm)</td>
</tr>
<tr>
<td>$J_m$</td>
<td>0.0001</td>
<td>Motor moment of inertia (kgm^2)</td>
</tr>
<tr>
<td>$b_m$</td>
<td>0.005</td>
<td>Motor viscous friction (Nm/(rad/s))</td>
</tr>
<tr>
<td>$b_{pc}$</td>
<td>5</td>
<td>Viscous friction of the ballscrew piston (N/(mm/s))</td>
</tr>
<tr>
<td>$m_{pc}$</td>
<td>2</td>
<td>Total mass of the ballscrew piston and MSD cylinder (kg)</td>
</tr>
<tr>
<td>$m_{mp}$</td>
<td>1</td>
<td>Mass of the MSD piston (kg)</td>
</tr>
<tr>
<td>$b_d$</td>
<td>5</td>
<td>Damping coefficient of the MSD system (N/(mm/s))</td>
</tr>
<tr>
<td>$k_s$</td>
<td>100</td>
<td>Spring stiffness of the MSD system (N/mm)</td>
</tr>
<tr>
<td>$K_p$</td>
<td>1.2</td>
<td>Proportional gain of the position PID controller (Nm/mm)</td>
</tr>
<tr>
<td>$K_i$</td>
<td>2.4</td>
<td>Integral gain of the position PID controller (Nm/mm/s)</td>
</tr>
<tr>
<td>$K_d$</td>
<td>0.004</td>
<td>Derivative gain of the position PID controller (Nm/(mm/s))</td>
</tr>
<tr>
<td>$k_f$</td>
<td>10</td>
<td>Integral gain of the force controller (1/s)</td>
</tr>
<tr>
<td>$N_f$</td>
<td>0.01</td>
<td>Low pass filter time constant of the force controller</td>
</tr>
</tbody>
</table>

3.5.3 Force Control Simulation

The force control of the redundant leg was simulated in Matlab Simulink 7.6.1. The redundant leg dynamics were modelled as the simplified system shown in Fig. 3.10, where the numerical values used for simulation are listed in Table 3.3. The parameters corresponding to the actuator dynamics ($\lambda$, $\eta$, $J_m$, $b_m$, $b_{pc}$ and $m_{pc}$) were obtained from the Aerotech BM250 motor and EDRIVE VT209-07 actuator manuals. The parameters corresponding to the MSD system dynamics ($m_{mp}$, $b_d$ and $k_s$) were selected to achieve a high bandwidth force control as well as good disturbance rejection. For example, the maximum backlash in the actuator is about 0.05mm which can only result in 5N disturbance with the selected spring stiffness. Therefore, ballscrew backlash of the redundant leg is negligible in simulation, as is the tracking error of the robot end-effector, which is normally within 0.05mm when the control forces on the position-controlled legs remain in the backlash-free region. The integral based force control algorithm described in Fig. 3.11 runs in a 100Hz loop. The parameters of the force controller and the position controller in the equivalent continuous time domain are listed in Table 3.3.
3.5. Numerical Simulation

The simulation has been undertaken on the steepest preload trajectory (Fig. 3.19(b)) obtained above. As the desired preload on the redundant leg is periodic, only the section between the 11th second and 20th second which corresponds to the 2nd preload optimization cycle is displayed. Fig. 3.30 shows the simulated force tracking results. The maximum tracking error is approximately 45N. This physically means the maximum force error assigned on each of the six position-controlled legs is about 8N which is lower than the safety margin defined (10N). Thus, a backlash-free condition is ensured even with a delay in the control so long as the safety margin, obtained by trial and error, is sufficiently large to tolerate the force control error.

![Figure 3.30. Simulated force control performance on the redundant leg](image)

(a) Force tracking performance in a cycle

(b) Zoomed-in section showing the maximum tracking error

Figure 3.30. Simulated force control performance on the redundant leg
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3.6 Physical Experiment

This section further assesses the proposed preload control method via a physical experiment. An additional framework and a redundant leg assembly were developed for use on the custom-built Stewart platform-based manipulator. The performance of the resulting redundantly actuated manipulator under preload control is compared to the performance of the normal manipulator with six legs only.

3.6.1 Experiment Method

This subsection illustrates the experiment preparation involving additional framework design, redundant leg assembly design, control hardware configuration, and experimental protocol.

3.6.1.1 Additional Framework Design

As the inner space of the robot has been used for mounting the specimens, an additional framework is required to suspend the redundant leg at the top space of the robot. The design of the framework is not a simple exercise since it must ensure the framework is sufficiently stiff to sustain the large preload on the redundant leg, while has sufficient open space for mounting the specimen and taking X-rays. The Inventor model of the framework design is shown in Fig. 3.31. The whole frame consists of three separate parts—a top part, a front part and a back part which are bolted together for easy assembly and disassembly. Each part is made of steel RHS beams and solid plates. The very top plate is used to mount the spherical joint of the redundant leg. The very bottom four RHS beams match the four sides of the robot base plate and are bolted to the base plate via 12 high stress bolts. In order to leave enough space for mounting the specimen into the robot, the front of the frame is designed to remain as open as possible. The front side corner is also opened which allows a clear view of the specimen for the X-ray machine. In addition, space is left at the sides and back to avoid potential collision between the manipulator and the framework.

The framework model was input into ANSYS Workbench 12.0 for static analysis during the design process. For simplicity and computational efficiency, the framework was modelled as one part and all the bolting holes are ignored. The surfaces matching the robot base were set as the fixed boundaries and the centre of the top plate was set as the loading point. Due to the function of the spherical joint, only a force can be
3.6. Physical Experiment

Figure 3.31. Inventor model of the additional framework for mounting the redundant leg

applied to the framework. The analysis was implemented at the worst loading condition, where maximum force that can be generated by the redundant leg (4000N) was applied. Depending on the pose of the robot, the possible maximum tension/compression and shears acting on the frame were about 4000N and 1000N respectively. Fig. 3.32 shows the deformations of the framework under extreme loading cases, which peak at approximately 0.07mm. Vibration analysis was also implemented on the framework in Workbench. The lowest vibration frequency of the framework was about 87Hz.

3.6.1.2 Redundant Leg Assembly Design

The Inventor model of the redundant leg assembly is shown in Fig. 3.33(a), while the model of the fully assembled redundant manipulator is shown in Fig. 3.33(b). Spherical joints are placed at both ends of the legs to allow sufficiently large rotational movement of the leg. Each spherical joint consists of a rod eye, a limit bolt, two gap sleeves and two support shoulders and is able to rotate at least 20 degrees about each of its three axes. The top two shoulders are directly mounted to the framework top plate and the bottom two shoulders are bolted to an adapter plate which is coupled to the robot top platform. The actuator is the same as the ones used
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Figure 3.32. Deformations of the additional framework under extreme loading conditions

(a) Case under 4000N z-axis tension
(b) Case under 1000N x-axis shear
(c) Case under 1000N y-axis shear
3.6. Physical Experiment

Figure 3.33. Inventor model of redundantly actuated manipulator

for positioning the robot. A NET motorbike shock absorber (Fig. 3.34) was selected as the MSD system. The lower spherical joint of the shock absorber is replaced as an adapter to couple to the Novatech F214 load stud. Another adapter is fixed at the upper assembly of the shock absorber via both a limit bolt and Devcon adhesive for coupling the shock absorber to the actuator piston. The location of the load stud shown in Fig. 3.33 allows direct measurement of the interaction force (preload) between the redundant leg and the moving platform.

The static performance of the shock absorber (Fig. 3.35) was directly measured using an Instron model 8511 material testing machine. Equipped with a compliant compression spring, the shock absorber only exhibits compliant behaviour under compression force and can only sustain a maximum 1000N compression force within its travel stroke. The behaviour of the shock absorber is approximately linear and has an average stiffness of about 30N/mm and a damping constant of about 8N/(mm/s). The non-linear region at the first 2mm displacement represents the dynamics of the rebound damper which is used to absorb the rebound force of the spring. Therefore, the shock absorber only exhibits a desired linear performance under a compression
force between 150N and 1000N. In another words, the preload on the redundant leg must be restricted to this region to ensure good force control performance.

The framework parts and the redundant leg parts were manufactured from powder coated steel and were assembled on the existing Stewart platform-based manipulator as shown in Fig. 3.36(a). The assembly of the redundant leg is highlighted in Fig. 3.36(b). The mechanical drawings of the framework parts and the redundant leg parts can be found in Appendix A.
3.6.1.3 Control Hardware Configuration

Figure 3.37 shows the overall control hardware configuration for the preload control experiment. As the configuration for positioning the robot has been described in Chapter 2 (with reference to Section 2.5.2.2), only the configuration for controlling the preload is discussed here. An AMTI MSA-6 strain gauge amplifier converts the AMTI load-cell analog signal to a digital form and is sent serially over RS232 in order to minimize the noise arising from the motor servo amplifiers. The converted RS232 signal is then input into the PXI real-time controller via the only serial port on the controller and is decoded using the built-in National Instrument Virtual Instrument Software Architecture (NI VISA). In this way, the external forces and moments arising from the specimen deformation is obtained at a 200Hz sampling rate and the maximum noise in the obtained signal is about ±6N and ±0.3Nm. For the same reason, a custom-built strain gauge amplifier is used to digitize the Novatech load stud signal and sent via RS232. With only a single serial port on the PXI chassis, the digitized signal is input into the PXI FPGA board and a custom program is written to decode the serial signal. The program runs at the maximum board clock rate (40MHz)
and samples the measured preload on the redundant leg at 1kHz, which is then sent to the PXI controller via DMA. Unfortunately the obtained preload signal contains noise as high as $\pm 70$N which arises from the large measurement range of the load-cell ($\pm 15000$N) and is far beyond the acceptable range for this application. To further reduce the noise, a smaller load-cell with lower capacity is required, however this would reduce the stiffness of the redundant leg and consequently degrade its dynamic performance. Considering the limited duration and budget of this project, an alternative solution—estimating the preload from the deformed displacement of the shock absorber—is applied to avoid directly measuring the preload. The deformed displacement of the shock absorber can be obtained by comparing the difference between the total leg travel length (from the 7th motor encoder) and the true length of the leg (from the robot pose). Then a linearised function approximating the relationship between the deformed displacement of the shock absorber and the force response on the shock absorber (as shown in Fig. 3.35) is used to estimate the preload on the redundant leg. The preload optimization and the force control algorithm as described in Fig. 3.8 and Fig. 3.11 run at 100Hz on the PXI controller. As the hardware was purchased based on running the controllers of the original six legs, there is no additional memory on the FPGA boards and motor amplifier for controlling the redundant leg. Therefore, a Maxon EPOS2 70/10 position controller is used to run the inner position loop (as a form of velocity control with a 10kHz loop rate) described in Fig. 3.11. As the LabVIEW real-time controller does not support the Maxon LabVIEW driver (which only works under LabVIEW windows), the velocity
3.6. Physical Experiment

command of the redundant motor calculated from the force control loop has to be sent to the Maxon controller indirectly via the host PC at 50Hz sampling rate.

3.6.1.4 Experimental Protocol

As shown in Fig. 3.36(a), the polymer specimen was mounted on the redundantly actuated manipulator to emulate the robot testing a human joint and undergoing large external loads from deforming the joint. Most of the control parameters for the experiment were defined as the same as the values defined for the simulation or if otherwise are stated in Table 3.4. The backlash-free threshold $\varepsilon$ and the safety margin $\mu$ were increased to 80N and 20N respectively to compensate the delay and error arising from the limitations of the control hardware set-up. The payload generated by the redundant leg was restricted between 150N and 1000N to ensure that the shock absorber remains within its linear range. For the redundant leg, the force control gains were selected from trial and error and position control gains were tuned by the Maxon EPOS2 controller auto-tuning system.

The robot was commanded to deform the polymer specimen along each of the 6-DOF under two circumstances. In the first circumstance, the robot was controlled without the redundant leg but with a dead mass preload (180N) on top of the robot as usual. Under the second circumstance, the robot was controlled with the redundant leg using the proposed active preload control method. To obtain comparable results, all the common parameters (e.g. control gains of the position-controlled legs) and testing

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^m$</td>
<td>$[0 \ 0 \ -160]^T$</td>
<td>Position of the specimen centre of rotation in ${\alpha}$ (mm)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>80</td>
<td>Backlash-free threshold (N)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>20</td>
<td>Safety margin (N)</td>
</tr>
<tr>
<td>$\rho_{\text{max}}$</td>
<td>1000</td>
<td>Redundant actuator payload upper limit (N)</td>
</tr>
<tr>
<td>$\rho_{\text{min}}$</td>
<td>150</td>
<td>Redundant actuator payload lower limit (N)</td>
</tr>
<tr>
<td>$k_s$</td>
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<td>Spring stiffness of the shock absorber (N/mm)</td>
</tr>
<tr>
<td>$k_f$</td>
<td>2</td>
<td>Integral gain of the force controller (1/s)</td>
</tr>
<tr>
<td>$N_f$</td>
<td>0.016</td>
<td>Low pass filter time constant of the force controller</td>
</tr>
</tbody>
</table>
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protocols were defined as the same for both circumstances. The testing protocols included shearing the specimen by 1mm along the x and y axis, compressing the specimen by 0.4mm along the z axis, bending the specimen by 6 degrees about the x and y axis, and twisting the specimen by 6 degrees about the z axis. For shearing, compression and torsion testing, the displacements were applied as a form of haversine waveform at 0.02Hz for three cycles. For bending testing, the displacements were applied as a form of haversine waveform at 0.01Hz for three cycles. These protocols were chosen for the following reasons. Firstly, the displacements were selected to ensure the resulting external loads on the robot were within the allowable range, which can be addressed by the force capacity (150N to 1000N) of the shock absorber. Secondly, the testing speed was defined in a very slow manner to minimize the error from preload estimation, where only static force was considered and to also tolerate the delay in the control hardware set-up. Finally, backlash instabilities normally occurred at slow test speeds which meant that the actuators spent considerable time in the backlash region during zero crossings of actuator load. Furthermore a slow motion allowed the limit cycles arising from backlash instabilities to become dominant and obvious within the overall robot dynamic tracking inaccuracies.

3.6.2 Experimental Results

Figures 3.38-3.43 show the experimental results corresponding to the tests along each of the 6-DOF, respectively, where subfigures (a) and (b) represent the three forces and three moments arising from deforming the specimen respectively, subfigures (c) and (d) represent the three translational and three rotational tracking errors of the robot respectively, and subfigure (e) represents the preloads on the six position-controlled legs. All the plots on the left hand side illustrate the results under the dead mass preload method, while all the plots on the right hand side illustrate the results under the active preload control method using the redundant leg.

As shown in subfigures (a) and (b), the forces and moments arising from deforming the specimen were approximately the same under the two preload methods which ensured the results obtained under the two methods were comparable. The very small differences (e.g. offsets) were due to specimen relaxation and robot motor relaxation
3.6. Physical Experiment

Figure 3.38. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to shear the specimen by 1mm along x-axis using a have-sine waveform at 0.02Hz for three cycles. (Fig. 3.38 continues on next page)

a) Three forces arising from deforming the specimen

b) Three moments arising from deforming the specimen

c) Three translational tracking errors of the robot

Figure 3.38. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to shear the specimen by 1mm along x-axis using a have-sine waveform at 0.02Hz for three cycles. (Fig. 3.38 continues on next page)
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Figure 3.38. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to shear the specimen by 1mm along x-axis using a haver-sine waveform at 0.02Hz for three cycles.

when shifting between the dead mass and the redundant leg. Under the dead mass preload method, the preloads on the position-controlled legs were inevitably moved into the backlash-problem region as shown in subfigures (e). As soon as this happened, the stability margin of the corresponding leg was narrowed and consequently limit cycles arose from backlash instabilities as shown in subfigures (c) and (d). The magnitude of the limit cycles was subject to the number of legs in the backlash-problem region and also the robot configuration. In the most extreme case (compression) the preloads on all six position-controlled legs were moved into the backlash-problem region, causing the magnitude of the limit cycles to reach as high as 0.08mm and 0.03 degrees as shown in Fig. 3.40(c) and Fig. 3.40(d) respectively which consequently resulted in errors of approximately 100N and 4Nm in the
3.6. Physical Experiment

Figure 3.39. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to shear the specimen by 1mm along y-axis using a haver-sine waveform at 0.02Hz for three cycles. (Fig. 3.39 continues on next page)

a) Three forces arising from deforming the specimen

b) Three moments arising from deforming the specimen

c) Three translational tracking errors of the robot

Figure 3.39. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to shear the specimen by 1mm along y-axis using a haver-sine waveform at 0.02Hz for three cycles. (Fig. 3.39 continues on next page)
Chapter 3. Active Preload Control of a Redundantly Actuated Stewart Platform for Backlash Prevention

Figure 3.39. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to shear the specimen by 1mm along y-axis using a haver-sine waveform at 0.02Hz for three cycles.

measured forces and moments as shown Fig. 3.40(a) and Fig. 3.40(b) respectively. Such high frequency limit cycles can be harmful to the ballscrews and other mechanical components of the robot. By contrast, under the active preload control method, the preloads on all six position-controlled legs were consistently kept in the backlash-free region as shown in subfigures (e). Under such circumstances, the non-linear dynamics of the backlash was eliminated in the leg dynamics and consequently backlash instabilities disappeared as shown in subfigures (c) and (d). As a result, the robot tracking abilities and load measuring accuracies were significantly improved. The RMS tracking errors of the robot for the dead mass preload (DMP) method and for the active preload control (APC) method were computed and compared in Table 3.5. The RMS errors arising from APC were within 5 µm on translational axes and
3.6 Physical Experiment

Figure 3.40. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to compress the specimen by 0.4mm along z-axis using a haversine waveform at 0.02Hz for three cycles. (Fig. 3.40 continues on next page)

a) Three forces arising from deforming the specimen

b) Three moments arising from deforming the specimen

c) Three translational tracking errors of the robot

Figure 3.40. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to compress the specimen by 0.4mm along z-axis using a haversine waveform at 0.02Hz for three cycles. (Fig. 3.40 continues on next page)
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Figure 3.40. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to compress the specimen by 0.4mm along z-axis using a haver-sine waveform at 0.02Hz for three cycles.

5 arc-second on rotational axes which are about 2 to 15 times smaller than the counterparts arising from DMP. This proved the efficacy of the proposed active preload control method.
3.6. Physical Experiment

Figure 3.41. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to bend the specimen by 6 degree about x-axis using a haver-sine waveform at 0.01Hz for three cycles. (Fig. 3.41 continues on next page)
Figure 3.41. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to bend the specimen by 6 degree about x-axis using a haver-sine waveform at 0.01Hz for three cycles.

d) Three rotational tracking errors of the robot

e) Preloads on the six position-controlled legs

Figure 3.41. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to bend the specimen by 6 degree about x-axis using a haver-sine waveform at 0.01Hz for three cycles.
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Figure 3.42. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to bend the specimen by 6 degree about y-axis using a haver-sine waveform at 0.01Hz for three cycles. (Fig. 3.42 continues on next page)
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Figure 3.42. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to bend the specimen by 6 degree about y-axis using a haver-sine waveform at 0.01Hz for three cycles.

d) Three rotational tracking errors of the robot

e) Preloads on the six position-controlled legs

Figure 3.42. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to bend the specimen by 6 degree about y-axis using a haver-sine waveform at 0.01Hz for three cycles.
3.6. Physical Experiment

Figure 3.43. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to twist the specimen by 6 degree about z-axis using a haver-sine waveform at 0.02Hz for three cycles. (Fig. 3.43 continues on next page)

a) Three forces arising from deforming the specimen

b) Three moments arising from deforming the specimen

c) Three translational tracking errors of the robot
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Figure 3.43. Comparison between dead mass preload (left figures) and active preload control using the redundant leg (right figures). The robot was commanded to twist the specimen by 6 degree about z-axis using a haver-sine waveform at 0.02Hz for three cycles.

Table 3.5. A comparison between the RMS tracking errors of the robot under dead mass preload and under active preload control

<table>
<thead>
<tr>
<th>Method</th>
<th>Axis</th>
<th>Shear (x-axis)</th>
<th>Shear (y-axis)</th>
<th>Compression (z-axis)</th>
<th>Bending (x-axis)</th>
<th>Bending (y-axis)</th>
<th>Torsion (z-axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>DMP</td>
<td>APC</td>
<td>DMP</td>
<td>APC</td>
<td>DMP</td>
<td>APC</td>
</tr>
<tr>
<td>Tx (µm)</td>
<td></td>
<td>6.40</td>
<td>2.04</td>
<td>2.51</td>
<td>1.90</td>
<td>18.68</td>
<td>1.97</td>
</tr>
<tr>
<td>Ty (µm)</td>
<td></td>
<td>4.92</td>
<td>1.19</td>
<td>6.40</td>
<td>2.61</td>
<td>18.28</td>
<td>2.05</td>
</tr>
<tr>
<td>Tz (µm)</td>
<td></td>
<td>1.87</td>
<td>0.63</td>
<td>1.32</td>
<td>0.84</td>
<td>9.87</td>
<td>0.73</td>
</tr>
<tr>
<td>Rx (m°)</td>
<td></td>
<td>6.02</td>
<td>1.27</td>
<td>4.17</td>
<td>2.03</td>
<td>18.64</td>
<td>1.69</td>
</tr>
<tr>
<td>Ry (m°)</td>
<td></td>
<td>4.08</td>
<td>0.89</td>
<td>3.44</td>
<td>2.14</td>
<td>27.44</td>
<td>1.80</td>
</tr>
<tr>
<td>Rz (m°)</td>
<td></td>
<td>6.23</td>
<td>1.70</td>
<td>3.52</td>
<td>1.85</td>
<td>17.20</td>
<td>1.86</td>
</tr>
</tbody>
</table>

e) Preloads on the six position-controlled legs
3.7 Discussion and Conclusion

This chapter studied the use of actuation redundancy to eliminate backlash inaccuracy for a general 6-DOF Stewart platform. A novel redundancy arrangement with a refined active preload control method was proposed for real-time control applications. Simulation results demonstrated an acceptable preload distribution efficiency of the redundancy arrangement within the workspace of the robot. Simulation results also demonstrated that the proposed method can effectively achieve backlash-free positioning of the manipulator under large 6-DOF external loads. Because of the hardware limitations, the experiment was restricted to low speed tests, however, based on simulation results, it is expected that using improved hardware, the bandwidth of testing could increase. The experimental results further demonstrated that the proposed method can eliminate backlash instabilities from control and consequently higher bandwidth control can be achieved on the robot with higher accuracy compared to the same system without the redundant leg.

In order to make the proposed active preload control method fully applicable in industry, further design and research work is required. Firstly, the design of the redundant leg assembly is critical. A bicycle shock absorber is not ideal not only because of its single-direction load capacity but also due to the instabilities in its transversal directions. Therefore, a more sophisticated mass-damper-spring system needs to be designed to allow a single DOF compliant motion along its longitudinal axis only. As the redundant leg controls the preloads on all six position-controlled legs, the load capacity of the redundant leg is required to be 4 to 6 times higher than the position-controlled leg to ensure the controllability of the system. Secondly, the location of the redundant leg needs further investigation to reach the optimal preload distribution efficiency of the redundant configuration. It is recommended to place the redundant leg into the robot inner space. This configuration avoids a perpendicular relation occurring between the legs, and consequently ensure the overall preload distribution efficiency of the structure. Thirdly, the control of the redundantly actuated manipulator requires further improvements. A more intelligent control algorithm is necessary to improve the efficiency, speed and ease of robot control by using a higher fidelity model (which incorporates backlash dynamics, manipulator acceleration and tracking errors).
Chapter 3. Active Preload Control of a Redundantly Actuated Stewart Platform for Backlash Prevention
Chapter 4

Adaptive Velocity-based Load Control of Human Joints for Unconstrained Testing—A Robotic-based Method

4.1 Introduction

Robots, particularly parallel robots, have great potential to further the field of biomechanics with their accuracy, repeatability, load capacity and programmability. The unconstrained testing (or flexibility testing) protocols, which apply a pure force or moment to human joints about one of their 6 DOF and leave the remaining 5 DOF unconstrained, are essential for the biomechanical evaluation of joints and their repair. A few robotic-based force control methods have been developed to enable unconstrained testing of human joints. Most of the methods are stepwise or quasistatic which incrementally regulates the robot position to track the specified load target on one axis while minimizing the loads on the remaining five axes (Fujie et al., 1993, Gilbertson et al., 2000, Walker and Dickey, 2007). These methods require an upper level force control algorithm to adaptively and iteratively compute an instantaneous stiffness matrix of the specimen that is then used to calculate the robot displacement command for minimizing the load errors on the specimen. Consequently, the algorithm is computationally intensive and is inherently sensitive to measurement errors (e.g. sensor noises), disturbances (e.g. robotic system dynamics), and robot inaccuracies and non-linearities. Recently, Goertzen and Kawhuk (2009) proposed the use of velocity-based force control for unconstrained testing. The method simply
applies a velocity-based force control algorithm on each of the 6-DOF and uses a constant proportional gain to relate the load error with the desired velocity of the axis while limiting the maximum axis velocity to ensure stability. Although satisfactory results were obtained when testing a rabbit spine, there remained some question—can the method work effectively on a stiffer sample (e.g. a human joint)? How is the force control gain optimized for testing different samples?

In this chapter, an improved velocity-based load control method is developed for use in robotic unconstrained testing. The main innovation of the method is its capability to adaptively optimize the force control gain based on the force tracking performance at higher dynamic frequencies. This is discussed in Section 4.3 as well as the other improvements of the algorithm for improving the control or physiological performance of the unconstrained testing. An ovine lumbar functional spinal unit (FSU) was used to assess the proposed method due to it being readily available and its general similarity to a human lumbar spine (Wilke et al., 1997). The cadaver preparation, experimental hardware configuration, testing protocol, and experimental results are presented in Section 4.4. Although adaptive velocity-based force control was an established technique, its extension to biomechanics has never been attempted.

4.2 Background Theory

This section covers the background theory behind the study in this chapter. Audiences who do not have a strong knowledge of joint biomechanics and force control are encouraged to read through this section to assist in understanding the work done.

4.2.1 Nonlinear Behaviour of Human Joints

Human joints are complex mechanisms and consequently their mechanical behaviour is fully nonlinear. As an example, the idealized representation of the load-displacement behaviour of a lumbar FSU along a single axis is shown in Fig. 4.1 where NZ represents the neutral zone and EZ represents the elastic zone. The FSU has a low stiffness in the neutral zone and a high stiffness in the elastic zone. Furthermore, the FSU exhibits hysteresis behaviour due to its viscoelastic properties. These lead to a nonlinear, multiphasic behaviour in a single DOF. When expanded to 6-DOF, the behaviour becomes more complex, which can be described by an instantaneous
4.2. Background Theory

Figure 4.1. Idealized representation of the load-displacement behaviour of a lumbar FSU. In the NZ, large changes in displacement can occur with little or no change in applied load. In the EZ, large changes in load can be produced by small changes in applied displacement. (Gilbertson et al., 2000)

The stiffness matrix as shown in Eq. (4.1). The 6 x 6 matrix relates the three forces and three moments at the estimated joint COR to the three translations and three rotations about the estimated joint COR. The diagonal terms of the matrix govern the behaviour of the joint on each of the 6-DOF. As the true joint COR is floating during movement, strong coupling can occur between two individual DOF. As a result, K is a fully populated time-varying matrix which is very difficult to estimate in practice.

### 4.2.2 Velocity-based Force Control

Velocity-based force control is often used in a robot-environment interaction. Figure 4.2 shows a very standard velocity-based force control scheme where subscript d and r represent the desired value and the real value respectively and superscript t and j represent the value in robot task space and the value in robot joint space respectively. The scheme compares the desired contact forces and moments $W_d$ and the measured contact forces and moments $W_r$, and feeds the resulting errors through a force controller $K_g$ to compute the desired task-space velocities of the robot $v_d^t$ for
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Figure 4.2. Block diagram showing the velocity-based force control scheme where subscript d and r represent the desired value and the real value respectively and superscript t and j represent the value in robot task space and the value in robot joint space respectively.

minimizing the load errors. $\mathbf{v}_d^r$ is then transformed to the desired joint-space velocities $\mathbf{v}_d^j$ via the robot inverse kinematics Jacobian matrix $\mathbf{J}^{-1}$. Then the robot joint motions are controlled using a velocity control loop based on the errors between the desired velocities $\mathbf{v}_d^j$ and the measured velocities $\mathbf{v}_r^j$. The rest of the flow shows the transfer between $\mathbf{v}_r^j$ and the penetration of the robot into the environment, where $\mathbf{v}_r^t$ represents the real task-space velocities, $\mathbf{p}_r^t$ represents the real task-space robot pose, and $\mathbf{p}_r^e$ represents the pose of the equilibrium contact point between the robot and the environment. The force controller, $\mathbf{K}_g$, is normally defined as a $6 \times 6$ constant matrix with zeros on the off-diagonal terms and the force control gains on the diagonal terms, given by

$$
\mathbf{K}_g = \begin{bmatrix}
    k_1^g & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & k_6^g \\
\end{bmatrix}
$$

where $k_i^g/k_i^0$ ($i = 1: 6$) represents the independent control gains on each of the 6-DOF. $k_i^g$ and $k_i^0$ respectively represent a pre-adjustable gain and an estimation of the environment stiffness on that DOF. With such a form, the controller assumes the contact environment is a fully decoupled system with a linear time-invariant stiffness on each DOF. This assumption suits most of the cases when the robot interacts with an isotropic material object.
4.3 Theoretical Analysis

4.3.1 Overall Concept

The overall concept of the improved velocity-based load control method is shown in Fig. 4.3, where the subscripts d and r represent the desired value and the real value respectively and superscript s represents the value at the specimen COR. A specimen coordinate system \{Sp\} is attached at the virtual COR of the specimen, which can be estimated according to the dimensions of the specimen (Pearcy and Bogduk, 1988). It is assumed that the estimated joint COR is fixed during movement. With such an assumption, the 6-DOF loads measured by the load-cell in the load-cell coordinate system \{Lp\} $W_r^l$ can be transformed to the loads at the specimen COR relative to the global coordinate system $W_r^s$ using Eq. (2.28). The desired loads at the specimen COR $W_d^s$ is the superposition of the constrained testing commands $W_d^a$ and the follower load preload $W_d^p$ simulating the human body weight. The scheme applies a force feedback controller and a force feedforward controller in parallel to calculate the desired velocities of the specimen $v_d^s$, which is then integrated to the desired specimen displacements $d_d^s$ for robot position control (with reference to Fig. 2.8). Similar to a standard force controller, the force feedback controller involves a $6 \times 6$ matrix $K_g$ which only contains six control gains on its diagonal terms and each of control gains is adaptively updated based on the force tracking performance on the

![Block diagram showing the improved velocity-based force control scheme where subscript d and r represent the desired value and the real value respectively and superscript s represent the value at the specimen COR.](image)
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corresponding DOF. This implicitly allows the force controller to adapt to unknown system nonlinearities arising from specimen viscoelasticity, couplings between the specimen DOFs, and robot dynamics. The feedback controller also involves a saturation on each of the 6-DOF to clamp the maximum velocities of the robot which avoids involving high dynamic loads and consequently ease off the difficulties in control. In addition, a hyperbolic sine function is used to replace the linear control gain near the steady-state load error region for the purpose of overcoming instabilities arising from the measurement noise. More details about the algorithm of the adaptive feedback controller and several other improvements in the overall algorithm are discussed in the following two sections.

4.3.2 Adaptive Estimation of Force Control Gains

The six control gains on the diagonal terms of $K_6$ are represented as $k_i^R/k_i^0$ $(i = 1:6)$ where $k_i^R$ and $k_i^0$ respectively represent a pre-adjustable gain and an estimation of the environment stiffness in the corresponding DOF. It is possible to adaptively estimate $k_i^0$ based on the measured load change at the specimen and the measured specimen displacement change over a short preceding period, however large inaccuracies can occur in the estimated stiffness due to measurement error and position control error. Therefore, rather than estimating the absolute stiffness of the specimen, the author developed an algorithm which incrementally adjusts the six control gains $k_t = (k_t^R/k_t^0)$ based on the load tracking errors $\mathbf{W}_d^s$ over the previous second. $\mathbf{W}_d^s$ is a very good index to observe the force control performance (e.g. accuracy and stability) and one second of data gives sufficient up-to-date information for the controller to make a correct decision. As the control gain on each of the 6-DOF is independent in computation, only the algorithm on a single DOF is given in Fig. 4.4 and the other five DOFs follow the same logic. The control gain optimization algorithm runs iteratively at each discrete time $t_f$. At the beginning of the iteration, a load error vector $\mathbf{e}_t$ is acquired, which contains 10 elements sampled at 0.1Hz over the previous second,

$$\mathbf{e}_t = [e_1 \ldots e_{10}]. \quad (4.3)$$

Initially, the algorithm calculates the root mean square value of the load error vector
4.3. Theoretical Analysis

Figure 4.4. Flow chart showing the force control gain optimization algorithm on one DOF at discrete time $t_j$

\[
e_{\text{rms}} = \sqrt{\frac{1}{10} \sum_{i=1}^{10} (e_i)^2} \tag{4.4}
\]

which is compared to a predefined threshold $\alpha$. An error $e_{\text{rms}}$ equal to or smaller than $\alpha$ means the overall force tracking error is controlled within the acceptable range by the current control gain $k_{j-1}$ and consequently the new control gain $k_j$ remains the same as the current value. On the other hand, if $e_{\text{rms}}$ is larger than $\alpha$ the current control gain is not able to achieve the desired force control performance. Then the algorithm must make a decision to either decrease the control gain to ensure control stability or increase the control gain to ensure the tracking accuracy. This is recognized by a single index $n$ which represents the numbers of force error sign change (NSC) in $e_f$ and can be obtained via the following equation

\[
n = \sum_{i=1}^{9} \text{XOR}(\text{sign}(e_i) > 0, \text{sign}(e_{i+1}) > 0). \tag{4.5}
\]

Equation (4.5) uses an “exclusive or” logic to detect the change in sign between the two adjacent samples (e.g. the samples with the same signs give 0 and with different signs give 1) and sums the total number of sign changes to get $n$. $n \geq 3$ means the
load error frequently changes its direction which is very possibly caused by instabilities such as limit cycles. Consequently, the control gain is decreased by a predefined constant \( b \) to increase the stability of the controller. In the case \( n = 2 \), the system has the potential to reach the instability region and thus the control gain is slowly decreased with an decrement of \( b/2 \). If \( n = 1 \) or 0, the large value of \( e_{\text{rms}} \) must arise from the force tracking lag and the control gain is required to increase. The amount of increment is proportional to the weighted average of \( e_{f} \), which is given by

\[
e_{wa} = \frac{1}{1023} \sum_{i=1}^{10} 2^{i-1} e_i
\]  

where a more recent sample has higher weight compared to an older one. \( c \) is a predefined constant describing the incremental speed. Finally the new control gain is clamped into a reasonable range to avoid windup appearing.

### 4.3.3 Other Improvements in the Algorithm

#### 4.3.3.1 Use of Hyperbolic Sine Function in the Feedback Controller

Measurement noise is always an issue in force control. Responding to the measurement noise at steady-state, often leads to limit cycle behaviour in the system. To address this issue, a hyperbolic sine function is used combined with the adaptive control gain and the saturation function. Such an algorithm in a single DOF can be expressed as the following equation:

\[
v_{f} = \begin{cases} 
\phi, & k_{j}e_{c} \geq \phi \\
k_{j}e_{c}, & k_{j}\delta \leq |k_{j}e_{c}| < \phi \\
\sinh \left( \text{asinh} \left( k_{j}\delta \frac{e_{c}}{\delta} \right) \right), & |e_{c}| < \delta \\
-\phi, & k_{j}e_{c} \leq -\phi 
\end{cases}
\]  

(4.7)

where the desired velocity arising from the feedback controller \( v_{f} \) is a nonlinear function of the current load error \( e_{c} \), the current force control gain \( k_{j} \), a predefined maximum allowed velocity \( \phi \), and a predefined load error threshold \( \delta \). Eq (4.7) is explained graphically in Fig. 4.5. The nonlinear feedback control function restricts the desired velocity within \( \pm \phi \) and splits the function into two regions—a linear control region and a sinh function region which is divided by \( \delta \). When \( e_{c} \) is located into the
Figure 4.5. Nonlinear feedback control function consisting of an adaptive force control gain, a hyperbolic sine function, and a saturation

linear control region, $v_f$ is simply proportional to $e_c$ and thus the feedback controller acts as a normal proportional controller. However, when $e_c$ is located into the sinh function region, $v_f$ is calculated via a hyperbolic sine function in which circumstance the slope of the function quickly drops to zero to prevent the system from responding to broad band measurement noise. Thus, the control law filters the information of the same order of magnitude of the noise, consequently improving the stability.

4.3.3.2 Feed-forward Control

In parallel with the feedback controller, a feedforward controller reacts to the speed of the load command $W_0^\delta$ by compensating for the delay in the commanding DOFs. The controller $K_f$ is a $6 \times 6$ constant matrix with six predefined feedforward gains on its diagonal terms. Care is needed when selecting the feedforward gains. If the gain is too large, the control effort arising from the feedforward term can overtake the control effort arising from the feedback term and therefore results in large disturbances in the feedback gain optimization algorithm. This can largely degrade the performance of the overall algorithm. On the other hand, if the gain is set too small, there is insufficient control effort arising from the feedforward term to compensate for the delay.
4.3.3.3 Follower Load Preload

It has been described in Patwardhan et al. (2003) that the presence of a follower load preload—a constant preload along the longitudinal axis of the joint upper body (z-axis of the specimen coordinate system {Sp}), changes the stiffness of the spinal joint. There is no doubt this is also the case for other human joints such as knee and hip joints. Therefore, a follower load preload is required to be applied to the specimen under unconstrained testing. This can be implicitly realized by commanding the specimen to follow the equivalent desired loads in the global coordinate system which can be obtained by

\[
W_d^P = \begin{bmatrix}
R \\
0 \\
f_z/T \\
0 \\
0 \\
0
\end{bmatrix}
\]

(4.8)

where \(f_z\) represents a constant predefined preload and \(R\) represents the rotation matrix of the robot end-effector. As the joint upper body is rigidly fixed to the end-effector, \(R\) can be used to describe the orientation of \(\{Sp\}\) and transform the preload in \(\{Sp\}\) to the forces in the global coordinate system.

4.3.3.4 Partially Unconstrained Testing

In some testing scenarios, it is less physiological or more difficult to make all six DOFs of the specimen unconstrained. For example, the knee joint has a very large neutral zone about its flexion/extension axis, where a very small load perturbation can result in a very large displacement. As a result, the flexion/extension axis of the knee joint is normally constrained using position control under testing. Furthermore, it is difficult to apply a pure shear force to the joint since strong coupling moments are always produced under shear. To minimize the resulting coupling moment via load control, the specimen can be rotated to an unrealistic pose which is no longer physiologically possible for the joint. Therefore, the rotational axis (e.g. lateral bending) which strongly couples to the shear axis (e.g. lateral shear) is normally constrained under shear testing. These partially unconstrained tests can be easily achieved by a hybrid position-force control scheme which applies position control on the DOF requiring constraint while applying load control on the remaining DOFs.
4.4 Physical Experiment

This section assesses the efficacy of the improved velocity-based load control method via experiments. Unconstrained testing along each of the 6-DOF is implemented on an ovine FSU using the custom-built Stewart platform-based testing machine.

4.4.1 Experimental Method

This subsection illustrates the methods involving ovine FSU cadaver preparation and set-up, control hardware configuration and experimental protocol.

4.4.1.1 Ovine FSU Cadaver Preparation and Set-up

To test the algorithm in a biomechanics context, an ovine lumbar FSU was flensed of all non-ligamentous tissue and had its vertebral bodies cut parallel to the midline of the disc. The sheep FSU was selected due to its availability and similarity to the human lumbar spine (Wilke et al., 1997). As shown in Fig. 4.6(a), Wood’s Metal was used to pot the superior and inferior vertebra into the custom-built fixable bath cups due to its higher Young’s modulus compared to Poly Methyl Methacrylate (PMMA) and dental stone (Kim et al., 2006). The FSU was submerged in a protease inhibitor bath kept at $37^\circ$C by a water jacket to simulate a physiological environment, since testing conditions have been shown to affect results (Costi et al.)

![Figure 4.6](image)

(a) FSU and the upper fixable bath cup  
(b) Overall set-up on the testing machine

Figure 4.6. Left figure showing the inferior vertebra of the FSU potted into the lower fixable bath cup and the right figure showing the overall experiment set-up on the Stewart platform-based machine.
Figure 4.7. Control hardware configuration for the unconstrained testing experiment

2002). Figure 4.6(b) shows the overall experiment set-up on the Stewart platform-based testing machine, where the FSU is not visible since it resided in the water bath cups. The whole fixation system was designed to place the FSU as close as possible to the robot end-effector to minimize measurement error in loads arising from load transformation.

4.4.1.2 Control Hardware Configuration

Figure 4.7 shows the control hardware configuration for the unconstrained testing experiment. The lower level position control algorithm was arranged using the previous configuration as described in Chapter 2 with a kinematics based control (1Hz) running on the real-time controller, decoupled PID control (10kHz) running on the FPGA, and current control (20kHz) running on the Soloist servo amp. The upper lever load controller described in Fig. 4.3 was programmed on the real-time controller and was implemented at 100Hz loop rate. The 6-DOF loads measured by the load cell were input into the real-time controller via a serial RS232 port.

4.4.1.3 Experimental Protocol

The FSU was tested along each of its 6-DOF sequentially under unconstrained testing. For example, when testing the flexion/extension axis, the load command for the remaining five axes was set as 0 and therefore left such axes unconstrained. The axis under test was commanded to follow a haver-sine waveform load (e.g. flexion/extension). The haver-sine waveform was used over a sine waveform due to the asymmetric behaviour of the FSU (Wilke et al., 1998). More information of the waveform command is provided in Table 4.1, where the waveform magnitude was
Table 4.1. Command waveform along each of the 6-DOF for the unconstrained testing

<table>
<thead>
<tr>
<th>Testing Axis</th>
<th>Waveform Type</th>
<th>Magnitude</th>
<th>Frequency</th>
<th>Unconstrained type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression</td>
<td>Haver-sine</td>
<td>200N</td>
<td>0.01Hz</td>
<td></td>
</tr>
<tr>
<td>Posterior shear</td>
<td>Haver-sine</td>
<td>100N</td>
<td>0.01Hz</td>
<td>Partial (flexion/extension constrained)</td>
</tr>
<tr>
<td>Lateral shear</td>
<td>Haver-sine</td>
<td>100N</td>
<td>0.01Hz</td>
<td>Partial (lateral bending constrained)</td>
</tr>
<tr>
<td>Flexion</td>
<td>Haver-sine</td>
<td>8Nm</td>
<td>0.01Hz</td>
<td>Full</td>
</tr>
<tr>
<td>Lateral bending</td>
<td>Haver-sine</td>
<td>8Nm</td>
<td>0.01Hz</td>
<td>Full</td>
</tr>
<tr>
<td>Axial rotation</td>
<td>Haver-sine</td>
<td>8Nm</td>
<td>0.01Hz</td>
<td>Full</td>
</tr>
</tbody>
</table>

determined to avoid damaging the sheep FSU (Costi et al., 2002). Shear testing was implemented in a partially unconstrained manner to achieve a more physiological testing condition and easing the difficulty in control. In addition, a follower load preload generating a typical 0.2MPa intradiscal pressure (Sasaki et al., 2001) was applied to the FSU to mimic the in-vivo loading condition. According to the dimensions of the FSU, this is equivalent to applying an 89N compressive preload aligned with the longitudinal axis of the upper vertebra disc. The location of the FSU COR was estimated using the equations formulated by Pearcy and Bogduk (1988) assuming symmetry about the sagittal plane. This gave the position of the FSU COR in the robot end-effector coordinate system $\mathbf{t}^m_s$ and in the load-cell coordinate system $\mathbf{t}^l_s$ respectively, as shown in Table 4.2, which are critical for load and position transformation. Table 4.2 also lists the pre-defined control parameters for the experiment. $k_{\text{max}}$ and $k_{\text{min}}$ were defined based on the possible stiffness range of human and animal joints which involves the range for ovine FSU (Costi, 2002). $a$ and $\delta$ were determined according to the measurement noise range and the acceptable error tracking error range. The selections of $b$ and $c$ aimed to achieve a fast but fine enough change of the control gain. Since the main purpose of the experiment was to assess the adaptability of the proposed method, $k_f$ was minimized to avoid conflict with the feedback gain optimization algorithm. Finally, $\varphi$ was defined to clamp the maximum robot motion speed.
Chapter 4. Adaptive Velocity-based Load Control of Human Joints for Unconstrained Testing—A Robotic-based Method

Table 4.2. Physical and control parameters for implementing the unconstrained testing experiment

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_s^m$</td>
<td>[0.5 3.4 −67]$^T$</td>
<td>Vector describing the position of FSU COR in the end-effector coordinate system (mm)</td>
</tr>
<tr>
<td>$t_s^l$</td>
<td>[0.4 2.4 −109.9]$^T$</td>
<td>Vector describing the position of FSU COR in the load-cell coordinate system (mm)</td>
</tr>
<tr>
<td>$k_{\text{max}}$</td>
<td>[50 50 20 900 900 900]$^T$</td>
<td>Maximum allowed value of the feedback control gain along 6-DOF ($\times 10^{-3}\text{mm/s/N}$)</td>
</tr>
<tr>
<td>$k_{\text{min}}$</td>
<td>[1 1 0.2 20 20 20]$^T$</td>
<td>Minimum allowed value of the feedback control gain along 6-DOF ($\times 10^{-3}\text{mm/s/N}$)</td>
</tr>
<tr>
<td>$a$</td>
<td>[5 5 5 0.3 0.3 0.3]$^T$</td>
<td>Load RMS error threshold along 6-DOF (N and Nm)</td>
</tr>
<tr>
<td>$b$</td>
<td>[0.1 0.1 0.05 2 2 2]$^T$</td>
<td>Decrement of the feedback control gain along 6-DOF ($\times 10^{-3}\text{mm/s/N}$)</td>
</tr>
<tr>
<td>$c$</td>
<td>[0.02 0.02 0.01 2 2 2]$^T$</td>
<td>Increment ratio of the feedback control gain along 6-DOF ($\times 10^{-3}\text{mm/s/N}$)</td>
</tr>
<tr>
<td>$k_f$</td>
<td>[1 1 0.2 20 20 20]$^T$</td>
<td>Feedforward control gain along 6-DOF ($\times 10^{-3}\text{mm/N}$)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>[5 5 5 0.3 0.3 0.3]$^T$</td>
<td>Load error threshold dividing the sinh region and linear region along 6-DOF (N and Nm)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>[2 2 2 3 3 3]$^T$</td>
<td>Maximum allowed control velocity along 6-DOF (mm/s and degree/s)</td>
</tr>
<tr>
<td>$f_z$</td>
<td>-89</td>
<td>Value of the follower load preload (N)</td>
</tr>
</tbody>
</table>

4.4.2 Results

The results corresponding to each of the 6-DOF testing are displayed in Fig. 4.8-4.13 where subplot (a) shows the tracking performance on the testing axis, and subplots (b) and (c) respectively show the force tracking errors on three translational axes and the moment tracking errors on three rotational axes. From the plots, we can see that the load tracking errors arising from all the tests are successfully minimized within $\pm 15$N and $\pm 1$Nm except for the case on the constrained axis under shear testing (e.g. flexion/extension axis under posterior shear testing). The RMS values of the load tracking errors are listed in Table 4.3. The table shows a maximum RMS force error of 7.16N and a maximum RMS moment error of 0.45Nm which are very satisfactory when compared to the noise range in the load feedback ($\pm 6$N and $\pm 0.3$ Nm).
4.4. Physical Experiment

Figure 4.8. Results under compression (Fz) testing with the remaining 5-DOF fully unconstrained

(a) Tracking performance on the compression axis

(b) Force errors on three translational axes

(c) Moment errors on three rotational axes

Figure 4.8. Results under compression (Fz) testing with the remaining 5-DOF fully unconstrained
Figure 4.9. Results under posterior shear testing (Fx) with the remaining 5-DOF partially unconstrained

(a) Tracking performance on the posterior shear axis

(b) Force errors on three translational axes

(c) Moment errors on three rotational axes

Figure 4.9. Results under posterior shear testing (Fx) with the remaining 5-DOF partially unconstrained
4.4. Physical Experiment

(a) Tracking performance on the lateral shear axis

(b) Force errors on three translational axes

(c) Moment errors on three rotational axes

Figure 4.10. Results under lateral shear testing (Fy) with the remaining 5-DOF partially unconstrained
Figure 4.11. Results under flexion testing (Mx) with the remaining 5-DOF fully unconstrained

(a) Tracking performance on the flexion axis

(b) Force errors on three translational axes

(c) Moment errors on three rotational axes

Figure 4.11. Results under flexion testing (Mx) with the remaining 5-DOF fully unconstrained
4.4. Physical Experiment

Figure 4.12. Results under lateral bending testing (My) with the remaining 5-DOF fully unconstrained

(a) Tracking performance on the lateral bending axis

(b) Force errors on three translational axes

(c) Moment errors on three rotational axes

Figure 4.12. Results under lateral bending testing (My) with the remaining 5-DOF fully unconstrained
Figure 4.13. Results under axial rotation testing (Mz) with the remaining 5-DOF fully unconstrained.

(a) Tracking performance on the axial rotation axis

(b) Force errors on three translational axes

(c) Moment errors on three rotational axes

Figure 4.13. Results under axial rotation testing (Mz) with the remaining 5-DOF fully unconstrained
4.5 Discussion and Conclusion

An adaptive velocity-based load control method was proposed for use on the robotic system to implement unconstrained testing on human joints. The control method is not only self-adaptive to the unknown behaviour of the joint but also computationally efficient. The efficiency of the method was demonstrated using experiments and could be further improved if the following limitations in the system could be overcome. Measurement noise in the load feedback is the main limitation which prevents the adaptive controller from making the right decision. A possible solution is to use a more advanced load-cell amplifier having better noise rejection. In addition, the accuracy and stability of the higher level force control is highly dependent on the accuracy and stability of the lower level position control. As a result, any nonlinearity (e.g. backlash) degrading the position control performance can directly degrade the performance of the force control. The current solution was to ease off the position control gain to ensure control stability, although this can sacrifice the control accuracy. A potential solution is to involve the redundant leg method (introduced in Chapter 3) into the experiment but this is beyond the timeline of this project.

Although the results obtained were degraded by the limitations of the system, the accuracy of the force/moment tracking was still satisfactory compared to the previous robotic-based unconstrained control attempts. As results can vary dramatically when testing specimens from different species (e.g. rabbit and human spines), only the results obtained from human and equivalent size animal specimens (e.g. sheep and pig) are compared here. In Fujie et al. (1996), a mean force/moment tracking accuracy of 10N on translational axes and 1.5Nm on rotational axes (approximated from their

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Table 4.3. RMS load errors on all six axes under unconstrained testing along each of the 6-DOF

<table>
<thead>
<tr>
<th>Testing Axis</th>
<th>Fx (N)</th>
<th>Fy (N)</th>
<th>Fz (N)</th>
<th>Mx (Nm)</th>
<th>My (Nm)</th>
<th>Mz (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression</td>
<td>3.48</td>
<td>3.49</td>
<td>5.11</td>
<td>0.21</td>
<td>0.15</td>
<td>0.04</td>
</tr>
<tr>
<td>Posterior shear</td>
<td>4.31</td>
<td>4.37</td>
<td>3.94</td>
<td>Constrained to zero</td>
<td>0.23</td>
<td>0.06</td>
</tr>
<tr>
<td>Lateral shear</td>
<td>4.23</td>
<td>5.10</td>
<td>4.80</td>
<td>0.26</td>
<td>Constrained to zero</td>
<td>0.07</td>
</tr>
<tr>
<td>Flexion</td>
<td>4.58</td>
<td>4.32</td>
<td>5.57</td>
<td>0.41</td>
<td>0.20</td>
<td>0.07</td>
</tr>
<tr>
<td>Lateral bending</td>
<td>6.12</td>
<td>6.04</td>
<td>7.71</td>
<td>0.45</td>
<td>0.36</td>
<td>0.15</td>
</tr>
<tr>
<td>Axial rotation</td>
<td>7.16</td>
<td>5.05</td>
<td>5.98</td>
<td>0.31</td>
<td>0.16</td>
<td>0.23</td>
</tr>
</tbody>
</table>
plots) were achieved when testing a human knee using their custom designed algorithm on an industrial articulated robot. In Walker and Dickey (2007), a Newton-based approach was proposed as well as a custom-built parallel robot for spinal testing, which resulted in ±25N force errors on three translational axes, and ±1Nm, ±7Nm and ±10Nm moment errors on flexion-extension, lateral bending and axial rotation respectively when testing a porcine lumbar spine.
Chapter 5

A Scientific Method for Reproducing the *In-vivo* Measured Kinematics on Human Cadaver Joints Using a Stewart Platform-based Manipulator

5.1 Introduction

Previous studies of joint biomechanics have used robotic systems to intuitively apply a simplified joint motion (e.g. pure axial rotation or flexion-extension combined with bending) under constrained testing and a pure joint force or moment under unconstrained testing. Although very effective to study joint behaviour, these approaches did not truly simulate the physiological movement of the joint, which is very complex during daily activities. There is no doubt that simulating joint motion in a more physiological way is very important in joint biomechanics. Consequently, several methods have been proposed to reproduce the *in-vivo* measured joint kinematics under *in-vitro* cadaver testing (Moore et al., 2006, Howard et al., 2007), however these methods were either ambiguous in measuring the *in-vivo* kinematics or limited to use on animal specimens only. In this chapter, the author develops a method to scientifically reproduce the *in-vivo* measured kinematics on human cadaver joints using the custom-built Stewart platform-based manipulator. Human wrists are used as a typical example throughout this chapter to elaborate the theory of the method and to assess the fidelity of the method. In addition, wrist carpal kinematics is measured.
from the reproduced *in-vitro* wrist motion and is analysed from a biomechanics point of view.

### 5.2 Background Theory

This section covers the background theory behind the study in this chapter. Human wrist anatomy is briefly reviewed as well as the techniques for assessing wrist kinematics. In addition, this chapter introduces the transformation matrix for describing the wrist kinematics.

#### 5.2.1 A Review on Wrist Anatomy and Assessment Techniques

The wrist is a complex joint that bridges the hand to the forearm. It is actually a collection of multiple bones and joints. As shown in Fig. 5.1, the bones comprising the wrist include the distal ends of the radius and ulna, 8 carpal bones, and the proximal portions of the 5 metacarpal bones. All of these bones participate in complex articulations that allow variable mobility of the hand. Relative to the forearm, the hand is capable of 3 main DOF—flexion and extension, pronation-supination, and radio-ulnar deviation (Kijima and Viegas, 2009). The wrist also has a complex

![Figure 5.1. Bones of the human wrist joint (Phillips and Schmidt, 2013)]
configuration of ligaments to maintain mobility without sacrificing stability (not shown in Fig. 5.1 due to complexity). In order to understand the wrist, it is essential to understand the kinematics of the carpal bones (Gardner et al., 2006). However, many factors make measurement of carpal kinematics difficult (Garcia-Elias et al., 1989). The carpal bones are surrounded by multiple tendinous and ligamentous structures; they are small, of irregular shapes; and they undergo complex rotational and translational motions of small amplitudes. As a consequence, to date there is no clear consensus on the kinematics of carpal bones.

There exist a number of techniques currently being used or that have previously been used to assess carpal kinematics. These include techniques such as: dissection-based methods, ionising and non-ionising radiation imaging-based methods, and motion capture-based methods (McLean et al., 2006, Galley et al., 2007, Nuttall et al., 1998, Nakamura et al., 1999). Within the literature there exist two major flaws with the methods. Firstly, imaging-based in-vivo methods use a series of static images taken at different postures. While these methods are non-invasive, they are not a true representation of the dynamic behaviour of the joint. Rather, the continuous motion presented in the literature is an interpolation between the points of the static measurements, which may not account for tissue responses to the loads associated with dynamic motion. Secondly, studies of in-vitro kinematics generally move the wrist through a single DOF in isolation, e.g. flexion-extension or radio-ulna deviation. This does not account for the complex, cross-planar motions that we know occur at the wrist. Therefore, it is essential to develop a novel method to reproduce the physiological motion of the human wrist for more accurately assessing its carpal kinematics.

5.2.2 Transformation Matrix and Anatomical System Definition

As described earlier in Chapter 2, the pose of one coordinate system (e.g. frame \( \{B\} \)) relative to another coordinate system (e.g. frame \( \{A\} \)) can be described as a position vector \( \mathbf{t}_B^A \) and a rotation matrix \( \mathbf{R}_B^A \). For simplicity, the position vector and the rotation matrix are normally combined into a \( 4 \times 4 \) homogeneous matrix
Chapter 5. A Scientific Method for Reproducing the In-vivo Measured Kinematics on Human Cadaver Joints Using a Stewart Platform-based Manipulator

\[ T^A_B = \begin{bmatrix} R^A_B & t^A_B \\ 0 & 1 \end{bmatrix}. \]  

(5.1)

The homogeneous matrix \( T^A_B \) is often called the transformation matrix which describes the pose of frame \{B\} relative to frame \{A\} and follows the following arithmetic to achieve compound transformation

\[ T^A_C = T^A_B T^B_C \]  

(5.2)

where the transformation matrix of frame \{C\} relative to frame \{A\} \( T^A_C \) is equal to the linear combination of \( T^A_B \) and the transformation matrix of frame \{C\} relative to frame \{B\} \( T^B_C \). The transformation matrix concept is used throughout this chapter to achieve compound transformation between multiple coordinate systems.

In order to describe the wrist motion, two anatomical coordinate systems (ACS) are defined and attached to the forearm and hand respectively as shown in Table 5.1. The pose of the forearm ACS relative to the global coordinate system (GCS) is expressed as \( T^F_{ForeA} \) and the pose of the hand ACS relative to the GCS is expressed as \( T^H_{HandA} \).

The global wrist angles are the rotations of the hand ACS with respect to the forearm ACS, following a ZYX Euler sequence. Wrist flexion-extension (FE) \( \theta_{FE} \), pronation-supination (PS) \( \theta_{SP} \), and radio-ulnar deviation (RUD) \( \theta_{RUD} \) are the rotations about the \( Z \) (ZF), \( Y \) (YH), and \( X \) (XH) axes respectively. The carpal bones ACS are defined as initially aligned with the forearm ACS when all three relative rotations are zero. Rotation sequences and angle definitions are the same as for the global wrist angles.

<p>| Table 5.1. Definition of anatomical coordinate systems for the forearm and hand |</p>
<table>
<thead>
<tr>
<th>Forearm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_F )</td>
</tr>
<tr>
<td>( X_F )</td>
</tr>
<tr>
<td>( Y_F )</td>
</tr>
<tr>
<td>( Z_F )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hand</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_H )</td>
</tr>
<tr>
<td>( X_H )</td>
</tr>
<tr>
<td>( Y_H )</td>
</tr>
<tr>
<td>( Z_H )</td>
</tr>
</tbody>
</table>
5.3 Methods

5.3.1 Summary

The study received ethics approval from institutional human research ethics committees. The first stage of the study was *in-vivo* data collection. This work was performed by Dr Francois Fraysse at the University of South Australia. Twenty-one healthy volunteers were asked to hammer on a rubber block at 1Hz, and hammering kinematics were recorded using an optoelectronic motion capture system. The second stage of the study aimed at reproducing the above-recorded *in-vivo* kinematics on cadaveric specimens. Eight fresh upper limb specimens were used. K-wires (bone pins) were inserted into the forearm and carpal bones, and retroreflective markers were fixed on the bone pins. CT scans of each specimen were taken and reconstructed by Dr Francois Fransyse. The specimens were then mounted on the custom-built Stewart platform-based testing system, and passively moved through a series of wrist motions including the aforementioned hammering movement, as well as flexion-extension and radio-ulnar deviation. During *in-vitro* testing, the wrist motion and carpal kinematics were recorded using an optoelectronic motion capture system and the loads at the wrist COR were collected using the load-cell in the robotic testing system.

5.3.2 *In-vivo* Data Collection

Twenty-one healthy volunteers (12 male, 9 female, age 29.9 ± 9.5 years, height= 1.80 ± 0.10 m, mass= 76.4 ± 16.8 kg) participated. Written informed consent was obtained before data collection. Fifteen retroreflective markers were placed on the upper arm, forearm and hand and used to define the forearm ACS and hand ACS (Table 5.1). Participants were asked to strike a rubber block using a standard hammer. Hammering frequency was set at 1Hz and controlled with a metronome. The experimenter gave a verbal instruction to start hammering, and once the subject achieved a steady-state hammering rhythm, recording was started. Five, 10-second trials were recorded per subject. Marker trajectories were recorded at 100Hz using a 12-camera optoelectronic motion capture system (Optitrack®, Natural Point, USA) and low-pass filtered at 6Hz using a 4th order, zero-lag Butterworth filter. The wrist joint angles were computed over each cycle, and the mean for all subjects and all
Chapter 5. A Scientific Method for Reproducing the In-vivo Measured Kinematics on Human Cadaver Joints Using a Stewart Platform-based Manipulator

5.3.3 Specimen Preparation and CT Construction

Eight fresh cadaveric upper limb specimens were used. Bone pins coupled with retroreflective markers were inserted by an orthopaedic surgeon into the distal ulna and radius, 3rd metacarpal, scaphoid, lunate, triquetrum and capitate as shown in Fig. 5.3A. Pin positioning was initially confirmed using plain film radiographs and later by CT scans. Skin incisions were made on the back of the hand to help locating the carpal bones. All ligaments and other soft tissues were left intact. CT scans of the cadavers were then taken (slice thickness 0.5mm, no slice gap). The wrists were then individually packed to avoid any movement of the pins and refrozen.
5.3. Methods

For each prepared specimen, three-dimensional surface models of the bones were reconstructed from the CT data using ScanIP (Simpleware, Exeter, UK) as shown in Fig. 5.3B and exported to Matlab (R2011b, The Matworks Inc.) For each bone in which a pin was inserted, a technical coordinate system (TCS) was created and aligned with the reflective marker locations. Consequently, we had $\mathbf{T}_{\text{Fore}T}^{GCT}$, $\mathbf{T}_{\text{Hand}T}^{GCT}$, $\mathbf{T}_{\text{Scat}T}^{GCT}$, $\mathbf{T}_{\text{Lun}T}^{GCT}$, $\mathbf{T}_{\text{Tri}T}^{GCT}$, and $\mathbf{T}_{\text{Cap}T}^{GCT}$ to describe the poses of forearm TCS, hand TCS, scaphoid TCS, lunate TCS, triquetrum TCS, and capitate TCS respectively relative to the CT GCS. Additionally, the following anatomical landmarks (shown as red dots in Fig. 5.3B) were identified on the 3D reconstructions: medial and lateral humeral epicondyles, radial and ulnar styloids and the base and head of 2nd and 5th metacarpals. These landmarks were used to define the forearm ACS and hand ACS as described in Table 5.1. Then $\mathbf{T}_{\text{Hand}A}^{GCT}$ and $\mathbf{T}_{\text{Fore}A}^{GCT}$ were obtained to describe the poses of forearm ACS and hand ACS respectively relative to the CT GCS. The poses of the forearm TCS and hand TCS relative to their own ACS can be calculated as

$$\mathbf{T}_{\text{Fore}A}^{\text{Fore}T} = (\mathbf{T}_{\text{Fore}T}^{GCT})^{-1}\mathbf{T}_{\text{Fore}A}^{GCT}, \tag{5.1}$$

$$\mathbf{T}_{\text{Hand}A}^{\text{Hand}T} = (\mathbf{T}_{\text{Hand}T}^{GCT})^{-1}\mathbf{T}_{\text{Hand}A}^{GCT}. \tag{5.2}$$

Figure 5.3. A. Prepared cadaveric specimen with bone pins inserted and reflective markers coupled to the bone pins. B. Reconstructed CT scan including bone pins and reflective markers.
5.3.4 Wrist Kinematics Transformation

Reproducing motion on the cadaveric specimens required the computation of the pose of the hand ACS relative to the forearm ACS ($T_{\text{Hand}A}^{\text{Fore}A}$) from the input wrist angles. To achieve this, a hypothesis was established that the global wrist motion was a 2-DOF rotational motion (FE and RUD) with a fixed PS, the centre of rotation (COR) being the midpoint between the most distal tip of the lunate and the most proximal tip of the capitate (Youm et al., 1978), here called Wrist COR. This point was identified on the CT reconstructions for each specimen and was assumed as a fixed COR for each specimen. Consequently, knowing the time history of the wrist FE and RUD angles, the value of the (fixed) wrist PS angle, and the location of the wrist COR, the pose history of the hand ACS relative to the forearm ACS can be obtained as

$$\mathbf{t}_{\text{Hand}A}(t) = (\mathbf{t}_{\text{CT}^{\text{Fore}A}})^{-1} \mathbf{t}_{\text{COR}^{\text{CT}}}(t) \mathbf{T}_{\text{Hand}A}^{\text{COR}}$$

where $\mathbf{t}_{\text{CT}^{\text{Fore}A}}$ is a constant matrix as the forearm pose is fixed relative to the GCS, $\mathbf{t}_{\text{COR}^{\text{CT}}}(t)$ is the pose of the wrist COR relative to the GCS and can be easily calculated from the input wrist angles, and $\mathbf{T}_{\text{Hand}A}^{\text{COR}}$ is a constant matrix describing the pose of the hand ACS relative to the wrist COR. Note that the hypothesis of a fixed wrist COR was made only to reproduce the desired motions. When reconstructing motion from captured in-vitro data the hand was considered to have 6-DOF relative to the forearm.

Three wrist motions were considered for reproduction: 1) Flexion-extension, for this motion the RUD angle was kept at 0° while the FE angle followed a sine wave form, starting at 0°, moving to 15° flexion, then 15° extension, and back to 0°. 2) radio-ulnar deviation, which followed the same principle, with the FE angle kept constant at 0° and the RUD angle following a sine wave form from 20° ulnar deviation to 20° radial deviation. 3) The hammering motion obtained from the in-vivo experiment (Fig. 5.2A).

For all the above three motions, the wrist PS angle was kept constant at the value obtained from the CT scans.

Knowing the input wrist angles and the position of the wrist COR, $\mathbf{T}_{\text{Hand}A}^{\text{Fore}A}(t)$ was computed via Eq. (5.3). Subsequently, using the constant transformation matrices $\mathbf{T}_{\text{Fore}A}^{\text{Fore}E}$ and $\mathbf{T}_{\text{Hand}A}^{\text{Hand}E}$ calculated from Eq. (5.1) and Eq. (5.2) respectively, the pose history of the hand TCS relative to the forearm TCS $\mathbf{T}_{\text{Hand}A}^{\text{Fore}E}(t)$ can be computed for each of the eight specimens and each of the three wrist motions as
5.3. Methods

\[ T_{\text{Hand}}^{\text{Fore}} = T_{\text{Fore}}^{\text{Fore}} T_{\text{Hand}}^A (I) (T_{\text{Hand}}^A)^{-1}. \]  

(5.4)

5.3.5 In-vitro Experiment

Each specimen was thawed 12 to 16 hours prior to the in-vitro experiment. During preparation, the phalanges were disarticulated and the radius and ulna were transected at their distal third, so that the distance between proximal and distal ends of the specimen was 280mm. Skin and soft tissues were removed from the proximal 50mm of the forearm, and on the distal half of the metacarpals.

The specimen was mounted in a custom-built alignment device to ensure repeatable mounting. The radius and ulna were positioned inside an aluminium pot (diameter 75mm, depth 70mm) and fixed in Poly Methyl Methacrylate (PMMA). The specimen was then turned upside-down and the metacarpals were placed in another aluminium pot (diameter 120mm, depth 45mm) and fixed in PMMA (Fig. 5.4A). The specimen was then mounted in the Stewart platform-based testing system with rigid fixings (Fig. 5.4B). Twelve retroreflective markers were placed on the robot, six on the base and six on the end-effector plate (specimen fixation plate), defining the robot base TCS and the robot end-effector TCS which were coincident with the robot global

Figure 5.4. A. Preparation and fixation of the wrist prior to mounting in the robotic testing system. B. Specimen mounted in the Stewart platform-based testing system ready for in-vitro testing.
coordinate system and the robot end-effector coordinate system respectively. A two-second static capture trial was recorded, from which the pose matrices $T_{\text{Base}}^{\text{ForeT}}$ and $T_{\text{End}}^{\text{HandT}}$ were computed as

$$T_{\text{Base}}^{\text{ForeT}} = \left( T_{\text{ForeT}}^{G_{\text{CAM}}} \right)^{-1} T_{\text{Base}}^{G_{\text{CAM}}}, \quad (5.5)$$

$$T_{\text{End}}^{\text{HandT}} = \left( T_{\text{HandT}}^{G_{\text{CAM}}} \right)^{-1} T_{\text{End}}^{G_{\text{CAM}}}, \quad (5.6)$$

where $T_{\text{ForeT}}^{G_{\text{CAM}}}, T_{\text{Base}}^{G_{\text{CAM}}}, T_{\text{HandT}}^{G_{\text{CAM}}},$ and $T_{\text{End}}^{G_{\text{CAM}}}$ represent the poses of the defined TCS in the camera GCS. $T_{\text{Base}}^{\text{ForeT}}$ and $T_{\text{End}}^{\text{HandT}}$ represent the poses of the robot base and end-effector TCS relative to the forearm and hand TCS respectively, and were constant for each specimen. Finally, the pose history matrix $T_{\text{HandT}}^{\text{ForeT}}(t)$ calculated from Eq. (5.4) was used to obtain the pose of the robot end-effector relative to its GCS

$$T_{\text{End}}^{\text{HandT}}(t) = \left( T_{\text{Base}}^{\text{ForeT}} \right)^{-1} T_{\text{HandT}}^{\text{ForeT}}(t) T_{\text{End}}^{\text{HandT}}. \quad (5.7)$$

The time-varying pose matrix $T_{\text{End}}^{\text{BaseT}}(t)$ was used to drive the robot for reproducing the wrist motion.

All movements were performed over a 10-second duration (for the hammering motion this corresponds to 1/10th of the in-vivo speed) and each movement was repeated for three cycles (Fig. 5.2B). During in-vitro testing, marker trajectories were recorded using a 12-camera Vicon MX-F20 motion capture system (Vicon, Oxford, UK) at 100Hz. From the marker trajectories, the in-vitro kinematics of the wrist and its carpal bones were captured and consequently the following angles were obtained: 3rd metacarpal relative to radius (global wrist angles); scaphoid, lunate and triquetrum relative to radius (radiocarpal angles); capitate relative to scaphoid, lunate and triquetrum (midcarpal angles); and 3rd metacarpal relative to capitate (carpometacarpal angles). The means and standard deviations (SD) of the angles over repetitions and specimens were calculated. The RMS error between the input and measured global wrist angles over time were also computed. The ratios of carpal angles to global wrist angles (FE and RUD) were calculated for the following wrist displacements: from 0° to 15° of flexion and 0° to 15° of extension for the flexion-extension motion; from 0° to 20° of ulnar deviation and 0° to 20° of radial deviation for the radio-ulnar deviation motion. For the hammering motions the ratios were computed from 0° to maximal wrist flexion (7.1°), extension (11.1°), ulnar deviation (13.6°) and radial deviation (4.8°), respectively. In order to investigate whether the
ratio of carpal angle ($\theta_c$) to wrist angle ($\theta_w$) was constant over the tested range of motion, a linear curve was fit to the experimental data $\theta_c = f(\theta_w)$, and the R-square (regression coefficient) of this fit was computed.

Resulting forces and moments were recorded at 200Hz using the robot load-cell, low-pass filtered at 20Hz, transformed to the loads at the wrist COR and expressed in the forearm ACS. The means and SD of the loads over repetitions and specimens were calculated as well as the RMS variation of the loads over time.

5.4 Results

5.4.1 Accuracy of the Reproduced Motion

Both the input and in-vitro measured wrist angles for all three motions are shown respectively in Fig. 5.5-5.7 where the red solid line represents the input value (in-vivo mean value for hammering motion), the blue solid line represents the in-vitro mean value, and the black dashed line represents the in-vitro mean value ± standard deviation. The PS angles for all three motions were not computed as their influences were minimal on the carpal kinematics. The RUD angles under flexion-extension motion and the FE angles under radio-ulnar deviation motion were computed but not shown in the figures. As we can see from the plots, the SD of the in-vitro angles were not trivial, particularly at the maximum angle positions, however, the mean values of the in-vitro angles were very close to the input values. This proved the fidelity of the method, which reproduced the averaged in-vivo motions on multiple specimens to minimize the errors corresponding to individual cases. The RMS errors between the input and measured wrist angles are presented in Table. 5.2 where the RMS errors for the FE angles were within 2.6° and the RMS errors for the RUD angles were within 1.3° for all three motions. These errors have two main sources: the accuracy of the motion capture system and that of the robotic testing system. The displacement accuracies of the robotic testing system are orders of magnitude smaller than the RMS errors (see Chapters 2 & 6). For this reason, it is safe to assume that the vast majority of the error comes from the accuracy of the motion capture process. As a result, it was considered that all measured angles with a total range of variation of less than 2° were below measurement error and therefore not meaningful, referred to as negligible.
Table 5.2. RMS errors between commands and measured wrist angles over time, repetitions and subjects. FE: flexion and extension, RUD: radio-ulnar deviation, HAM: hammering motion.

<table>
<thead>
<tr>
<th></th>
<th>FE</th>
<th>RUD</th>
<th>HAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{RMS}_{\text{FE}}(°)$</td>
<td>1.8</td>
<td>2.6</td>
<td>2.0</td>
</tr>
<tr>
<td>$\text{RMS}_{\text{RUD}}(°)$</td>
<td>1.0</td>
<td>1.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 5.5. Wrist FE angle during flexion-extension motion: Input value (red solid), in-vitro mean value (blue solid), and in-vitro mean value ± SD (black dash).

Figure 5.6. Wrist RUD angle during radio-ulnar deviation motion: Input value (red solid), in-vitro mean value (blue solid), and in-vitro mean value ± SD (black dash).
5.4. Results

Figure 5.7. Wrist FE and RUD angles during hammering motion: Input/In-vivo mean value (red solid), in-vitro mean value (blue solid), and in-vitro mean value ± SD (black dash).

5.4.2 Load Responses at the Wrist COR

The 6-DOF load responses at the wrist COR during hammering motion are shown in Fig. 5.8 where each of the six subplots respectively shows the mean value as the thicker line and the mean value ± standard deviation as the thinner lines along each of the 6-DOF. As can be seen from the plots, the SD of the loads were minimal and the magnitude of the mean values of the loads were below 15N on translational axes and 1.3Nm on rotational axes. Consequently, we can conclude that the load responses
Chapter 5. A Scientific Method for Reproducing the In-vivo Measured Kinematics on Human Cadaver Joints Using a Stewart Platform-based Manipulator

Figure 5.8. 6-DOF load responses at the wrist COR during hammering motion. Each of the six subplots shows the mean value (thicker line) and the mean value ± SD (thinner lines) along each of the 6-DOF at the normalised time (0% to 100% percentage of the hammering motion).

were physiologically possible for the human wrist motion. This proved the assumption of a fixed wrist COR was realistic which further ensured the fidelity of the proposed reproducing method.

5.4.3 Carpal Kinematics

In wrist flexion (Fig. 5.9A) and extension (Fig. 5.9B), the majority of carpal motion happened at the radiocarpal level. Relative to the radius, the scaphoid, lunate and triquetrum flexed by 88% (SD 12%), 50% (SD 21%) and 68% (SD 7%) of global wrist flexion and extended by 84% (11%), 49% (6%) and 69% (8%) of global wrist extension, respectively. At the midcarpal level, the capitate flexed by 32% (22%) and 15% (8%) of global wrist flexion relative to the lunate and triquetrum respectively, while the flexion relative to the scaphoid was negligible (e.g. <2°); and extended by 32% (29%) of global wrist extension relative to the lunate, while its extension relative to the scaphoid and triquetrum was negligible. Carpal flexion varied linearly with global wrist flexion, with a minimum value of R-square of 0.823, and all correlations were found significant (p<0.05).
In ulnar deviation (Fig. 5.9C) the radiocarpal joints accounted for the majority of the motion again, although the effect was less pronounced than for flexion-extension. In radial deviation (Fig. 5.9D) the opposite effect was observed with the majority of the motion occurring at the midcarpal level. Relative to the radius, the scaphoid, lunate and triquetrum deviated ulnarily by 63% (9%), 41% (8%) and 71% (3%), while the capitate deviated ulnarily by 30% (9%), 52% (8%) and 22% (3%) of global wrist ulnar deviation relative to these bones. In radial deviation, the scaphoid and triquetrum deviated radially by 23% (7%) and 49% (5%) of global wrist radial deviation relatively to the radius, while the lunate RUD was negligible. In the midcarpal row the capitate deviated radially by 58% (14%), 86% (14%) and 34% (14%) of global wrist radial deviation relative to the scaphoid, lunate and triquetrum, respectively. Again, carpal rotations were proportional to wrist rotations (minimum R-square=0.854, p<0.05). The 3rd metacarpal flexed and extended during wrist flexion and extensions, respectively; but showed negligible amounts of RUD during wrist RUD.

Regarding out-of-plane rotations, the only rotations found non-negligible were scaphoid/radius radial deviation during wrist extension (18% of wrist extension, SD=9%), and scaphoid/radius and lunate/radius flexion during wrist radial deviation (28±16% and 28±17% of wrist radial deviation respectively).

During the hammering motion, the majority of the scaphoid and triquetrum flexion-extension motion occurred at the radiocarpal level (Fig. 5.10A and B). Relative to the radius, the scaphoid and triquetrum flexed by 64% (11%) and 81% (15%) of wrist flexion, and extended by 59% (13%) and 56% (11%) of wrist extension, respectively. At the midcarpal level, the capitate flexed by 37% (26%) of total wrist flexion and extended by 25% (13%) of total wrist extension with respect to the scaphoid. Capitate/triquetrum flexion was negligible while capitate/triquetrum extension was 28% (17%) of total wrist extension. The lunate exhibited an opposite behaviour: in wrist flexion lunate/radius flexion was negligible, while capitate/lunate flexion was 95% (39%) of total wrist flexion. In wrist extension, lunate/radius extension was 40% (8%) and capitate/lunate was 44% (18%) of total wrist extension. In ulnar deviation (Fig. 5.10C), scaphoid, lunate and triquetrum ulnar deviations relative to the radius were 56% (14%), 38% (16%) and 75% (7%) of total wrist ulnar deviation. The capitate deviated ulnarily by 32% (17%) and 49% (16%) relatively to the scaphoid and lunate, while its RUD relative to the triquetrum was negligible. In radial deviation
(Fig. 5.10D) only the triquetrum exhibited non-negligible motion relatively to the radius, with a radial deviation of 66% (27%) of total wrist radial deviation. At the midcarpal level the capitate deviated radially by 70% (24%) and 45% (60%) relatively to the scaphoid and lunate, while its RUD relative to the triquetrum was negligible.

5.5 Discussion and Conclusion

5.5.1 Evaluation of the Motion Reproduction Method

An assumption was made that the global motion of the wrist occurred about a fixed COR. This assumption was necessary to allow reproduction of the same joint angles on specimens of different dimensions. If this assumption was too coarse, or the estimation too inaccurate, it would have resulted in high loads at the wrist during motion reproduction, possibly damaging the specimens. However, as shown in Fig. 5.8, the loads at the wrist COR remained small and physiologically possible. Data on passive wrist moments are scarce in the literature; one study (Sinkjaer and Hyashi, 1989) reports wrist angular stiffness of 0.21 Nm/degree over the whole flexion-extension range of motion, and another one (Leger and Milner, 2000) reports passive stiffness of 0.03 Nm/degree for a small amount (3°) of wrist flexion. For rotations of 10° in flexion and extension, those values would yield passive moments of 2.1 Nm and 0.3 Nm, respectively. The wrist flexion-extension moments recorded in the present study (e.g. 0.5 Nm as shown in Fig. 5.8) are in the order of magnitude of these reported passive moments. This further strengthens our initial hypothesis of the approximate location of the wrist COR.

The average RMS differences between the input and in-vitro recorded wrist angles were 2.1° and 1.1° on average for flexion-extension and radio-ulnar deviation, respectively. RMS errors in the order of 1° are consistent with the values of 2.4° to 4.1° reported by Chiari et al., (2005) for the estimation of lower limb joint angles during gait. The smaller errors obtained in the present study may be a result of the smaller motion range reproduced. Overall, the method employed in this study allows reproducing the in-vivo measured kinematics on human cadaveric specimens with acceptable accuracy (less than 2.1° RMS error).
Figure 5.9. Ratios of carpal FE to wrist FE angle, for A. 0 to 15° of flexion and for B. 0 to 15° of extension. Ratios of carpal RUD to wrist RUD angle, for C. 0 to 20° of ulnar deviation and for D. 0 to 20° of radial deviation. Average (bar) and SD (line) are shown for all specimens. The numbers on the bars are the R-square between the experimental curve and the linear fit to it. No value indicates that the amplitude of bone rotation was negligible (e.g. <2°). SCA: scaphoid, LUN: lunate, TRI: triquetrum, CAP: capitate, MET: 3rd metacarpal.
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Figure 5.10. Ratios of carpal FE to wrist FE angle during hammering for A. 0 to 7.1° of flexion and for B. 0 to 11.1° of extension. Ratios of carpal RUD to wrist RUD angle during hammering, for C. 0 to 13.6° of ulnar deviation and for D. 0 to 4.8° of radial deviation. Average (bar) and SD (line) are shown for all specimens. The numbers on the bars are the R-square between the experimental curve and the linear fit to it. No value indicates that the amplitude of bone rotation was negligible (e.g. <2°). SCA: scaphoid, LUN: lunate, TRI: triquetrum, CAP: capitate, MET: 3rd metacarpal.
5.5.2 Kinematics of the Carpal Bones

During wrist flexion-extension, the majority of the carpal motion occurs at the radiocarpal level. The flexion-extension of the central column is distributed between radiocarpal (radiolunate) and midcarpal (lunocapitate) joints, whereas the vast majority of scaphoid and triquetrum flexion-extension occurs at the radiocarpal joint. This is in agreement with the results of previous studies (Berger et al., 1982).

Conversely, carpal motion in radio-ulnar deviation is more equally shared between radiocarpal and midcarpal joints. In ulnar deviation, there is more motion of the scaphoid and triquetrum at the radiocarpal than at the midcarpal level, whereas for the lunate, motion at the midcarpal (lunocapitate) level dominates. In radial deviation, most of the motion occurs at the midcarpal level, but once again, the lunate shows the least rotation relative to the radius compared to the scaphoid and triquetrum, and the radiolunate rotation was negligible, which is in agreement with the findings of Kobayashi et al. (1997a) reported similar findings but in radial deviation only. Additionally, the scaphoid and lunate flexed during wrist radial deviation. The amount of flexion and radial deviation of the scaphoid were comparable in magnitude, whereas for the lunate, flexion was of greater magnitude than radial deviation (which was negligible). However those bones did not show a significant flexion-extension motion during ulnar deviation which was observed by Berger et al. (1982).

Finally, the rotations of the carpal bones were proportional to the global rotation of the wrist over the ranges of motion studies. This was also observed by Kobayashi et al. (1997a) over large ranges of motion (60° of flexion and extension, and 30° of ulnar deviation). Therefore it seems justified to infer carpal kinematics from recordings of static postures of the wrist, as in Crisco et al. (2005).

The method presented here is not without limitations. The range of motion was limited by the capacities of the robotic testing systems. This was not a major concern in radio-ulnar deviation, where the rotations attained were very close to the total physiological range of motion. It did however limit the amount of flexion-extension to ±15°, whereas the physiological range of motion is typically ±60°. For the same reason the hammering motion had to be scaled down to 50% of the actual in vivo values, potentially preventing the observation of the changes in kinematics occurring at the extremes of motion. Additionally, the method used did not allow controlling the
force applied to the wrist. It has been shown that axial loading modifies carpal
kinematics (Kobayashi et al., 1997b), therefore controlling the amount of axial load
on the wrist during motion reproduction may lead to kinematics closer to
physiological reality.
Chapter 6

Robot Control System Design and Typical Applications of the Developed Robotic Testing System in Biomechanics

6.1 Introduction

The robot control system was designed and consistently updated throughout this project. The whole robotic testing system was fully functional and launched in September 2011. Since then, the robotic testing system has been used for several biomechanical testing projects including the wrist kinematics reproduction project as described in Chapter 5. In this chapter, the technical aspects of the robot control system including control hardware and software design, functionality and GUI design, and robot specifications are initially presented followed by some typical applications of the robotic testing system in biomechanics technically assisted by the author.

6.2 Robot Control System Design

6.2.1 Control Hardware and Software Design

The robot control system employs a host-target structure as shown in Fig. 6.1. The host computer runs Windows and LabVIEW 2009 (National Instruments) for programming, and for displaying the graphical user interface (GUI). The user is able to configure the robot, communicate with the servo amplifiers via Ethernet, control
Chapter 6. Robot Control System Design and Typical Applications of the Developed Robotic Testing System in Biomechanics

Real Time Controller
- Time-critical Loop (1kHz)
  - Inverse kinematics
  - Trajectory generation
- State-flow Loop (100Hz)
  - Safety-critical control
  - Direct kinematics
  - Force control
- Data Sampling Loop (100Hz)
  - Sample and store data
- Load-cell Data Loop (200Hz)
  - Input load-cell signals

FPGA (16bit fixed point)
- Dual Loop PID Control (10kHz)
  - 6x analog output torque commands
- I/O Interface Loop (10kHz)
  - 6x analog load cell signals
  - 12x limit switch digital inputs
  - 1x estop digital input
  - 1x servo-amp disable digital output
- Encoder Counter Loop (40MHz)
  - 6x linear encoder quadrature inputs
  - 6x rotary encoder quadrature inputs

Soloist Controller
- Current Loop (20kHz)
  - 6x 3phase current for motors
- Fault Diagnosis Loop (20kHz)
  - Ballscrew overtravel
  - Motor current overlimits
  - Hall-effect sensor errors
  - Rotary/linear encoder errors
  - Motor overheating/overspeed
  - Load-cell overload
  - Estop
  - Many other limits and faults

Legend
- Analog
- Digital
- Ethernet
- DMA
- RS232

Figure 6.1. Schematic showing the control system architecture (for six legs only). It shows the four platforms on which control loops are running (including rates).
the motion of the robot, and view and collect data on the GUI. A real-time controller (PXI-8106, National Instruments) connects with the host computer via Ethernet, and is running a real-time operating system programmed using LabVIEW Real-Time. It is used to generate the deterministic trajectory, perform inverse/direct kinematics, implement safety-critical control, run the force control algorithm, and sample and store data. FORTRAN code employing the DUNLSF function from the International Mathematics and Statistics Library (IMSL) was used to solve the direct kinematics, and was complied to a Dynamic-Link Library (DLL) file which can be called by LabVIEW on the real-time controller. The real-time controller connects with two field programmable gate array (FPGA) boards (PXI-7852R, National Instruments) through direct memory access (DMA). The FPGA boards run six dual-loop PID controllers, send analogue torque commands to the servo amplifiers, read analog load-cell signals, read the digital E-stop and limit switch signals, output the servo-amp disable flag, and count the six rotary incremental encoders on the servos as well as the six linear incremental encoders. By using the FPGAs, the control system is able to achieve hardware-level (nanosecond) determinism.

Six servo amplifiers (Soloist CP20 Controller, Aerotech) receive the analog torque commands from the FPGA boards and run 20 kHz PI current loops to regulate the current sent to the servo motors, which in effect results in torque being regulated. The Soloist Controllers are used to diagnose many defined faults (such as ballscrew over-travel, motor current over-limits, Hall-effect sensor errors, rotary/linear encoder errors, motor overheating/overspeed, E-stop, etc) and disable the motor currents as soon as the faults are detected. Some other faults are detected by safety-critical control on the PXI real-time controller, such as robot tracking error over-limits and load-cell overload. If any of these faults occur, the FPGA will send a servo-amp disable flag to the servo amplifiers to disable the motors. An emergency stop (E-stop) button is used to disable the servo amplifiers immediately at any time when any emergency occurs.

The analog load-cell signals were initially collected through the FPGAs. However, these analog signals were found to contain very high levels of broad spectrum electrical noise generated by the servo amplifiers and propagated through the mains. Hence, the load-cell signals were digitized to a RS232 protocol and fed through a custom isolation device (built based on Analog Devices ADM3251E isolated RS-232 Driver/Receiver) before input into the PXI real-time controller. The isolation device
Figure 6.2. Comparison of the load-cell noise energy level in dB (Fz channel only) in power spectrum. Sample frequency: 50Hz, FFT points: 512, FFT method: Welch, Window type: Hann, Overlap: 0.75.

(a) Analog signal noise level when the Soloist amplifiers are off

(b) Analog signal noise level when the Soloist amplifiers are on

(c) RS232 signal noise level when the Soloist amplifiers are on
was used to isolate the RS232 link between the load cell amplifier and the PXI system and consequently prevent the noise from propagating into the input signal. Figure 6.2 shows a comparison of the load-cell noise energy level (Fz channel only) in power spectrum where subplot (a) shows the analog signal noise level when the Soloist amplifiers are off, subplot (b) shows the analog signal noise level when the Soloist amplifiers are on, and subplot (c) shows the RS232 signal noise level when the Soloist amplifiers are on. The analog signal noise level is significantly increased by approximately 30dB once the Soloist amplifiers are turned on. However, by inputting the load-cell signal via RS232 with appropriate isolation, the noise level during the operation of the Soloist amplifiers is only 3dB higher than the case when the Soloist amplifiers are off.

The high complexity of the control system required extensive wiring work and coding work. Six custom printed circuit boards (PCB) were built to replace the numerous cables initially employed in early prototyping. The electrical drawings of the PCBs and isolation device and the wiring diagrams of the control system can be found in Appendix B. The developed LabVIEW codes involve more than 100 VIs which are attached as a zip file as part of this thesis submission.

### 6.2.2 Functionality and GUI Design

Figure 6.3 shows the custom graphical user interface (GUI) built for operating the robotic testing system and observing data. Overall, the GUI has five modes—home, jogging, joint-space motion, task-space motion, and load control—for the user to command the robot to implement various tasks. The home mode is used to automatically return all six linear incremental encoders to their ‘home’ index (near the fully extended position of the leg) and reset their positions in sequence for initializing the lengths of the leg. Entering the jogging mode, the user can slowly jog the robot along each of its 6-DOF or jog each of the six legs. The jogging mode is normally used when mounting the specimen on the robot or recovering the robot from an operational fault. In joint-space motion mode, the trajectories of the robot can be planned and commanded in joint-space. In another words, the robot can be moved from current pose to another predefined pose through the shortest travel distance (straight line between the two poses). This mode is usually used to quickly position
Chapter 6. Robot Control System Design and Typical Applications of the Developed Robotic Testing System in Biomechanics

Figure 6.3. Screenshot showing the main body of the custom GUI
the robot before mounting the specimen. Entering the task-space motion mode, the user can plan and command the 6-DOF displacement trajectories of the specimen to be tested in task-space. This is the most often used mode during biomechanical testing, where the displacement trajectories can be planned on the GUI in various forms (e.g. ramp, impact, sine or haver-sine waveform) or input from files generated by other motion planning software (e.g. Matlab, C-motion). The load control mode is used to control the 6-DOF loads rather than the displacements on specimen. In this mode, the user can implement fully unconstrained or partially unconstrained testing on the specimen. The loading trajectories can also be planned on the GUI or input from other sources. Besides the above mentioned five main modes, the user can also modify the settings of the system (e.g. fault thresholds, control gains), and monitor and clear the faults of the system (e.g. limit switch fault, load-cell overload fault) using the GUI. In addition, the user can monitor and record the data flow on the GUI such as the forces and moments on the specimen, the displacements of the specimen, the pose of the robot, and many more states at up to 100Hz sample rate. The recorded data is then output to the host computer for further processing and analysis.

### 6.2.3 Robot Specifications

The technical specifications of the robot are shown in Table 6.1, approximated at the nominal pose of the robot. The dynamic accuracy of the robot was measured by the linear encoders when the robot tracked a pure sinusoidal motion with $\pm 1$ mm or $\pm 1$ degree amplitude along each of the 6-DOF at three different cycle rates (0.1Hz, 1Hz and 2Hz). The robot has a significant load capacity to fail any biological joint, a proper workspace which allows testing specimen with various stiffness and sizes, a fast speed to simulate physical joint motion, and an acceptable dynamic accuracy to track motion commands.

The absolute accuracy of the robot was verified by Duncan Tool & Gauge Pty Ltd. accredited by National Association of Testing Authorities (NATA) based on ISO 10360, geometrical product specification (GPS) acceptance test and verification test for coordinate measuring machines (CMM) and is shown in Table 6.2. The linear accuracy of the three axes was measured using a 150mm stepped calliper check piece. The robot was placed in the middle position of travel of the axes not being checked,
Chapter 6. Robot Control System Design and Typical Applications of the Developed Robotic Testing System in Biomechanics

Table 6.1. Specifications of the robotic testing system (6 legs). Displacement accuracy measured when the robot tracks a pure sinusoidal motion waveform with ±1mm or ±1 degree amplitude.

<table>
<thead>
<tr>
<th>Load Capacity</th>
<th>Stroke</th>
<th>Disp. Resolution</th>
<th>Max. Speed</th>
<th>Disp. Accuracy (RMS)</th>
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<td></td>
<td></td>
<td></td>
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<td>An/Posterior Shear</td>
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<td>±0.3µm</td>
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<tr>
<td>Lateral Shear</td>
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<td>±0.3µm</td>
<td>540mm/s</td>
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<tr>
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<td>±0.25µm</td>
<td>210mm/s</td>
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<tr>
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<td>±0.001°</td>
<td>60°/s</td>
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<tr>
<td>Flex/Extension</td>
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<td>63°/s</td>
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<td>Axial Rotation</td>
<td>2000Nm</td>
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<td>±0.0005°</td>
<td>135°/s</td>
</tr>
</tbody>
</table>

Table 6.2. Results from the verification test on the robotic testing system implemented by a NATA accredited test facility

| FEATURE ACTUAL |
|------------|-------------|
| 1. Linear Accuracy (over 150mm) |
| (a) X axis | 0.160mm |
| (b) Y axis | -0.006mm |
| (c) Z axis | -0.075mm |
| 2. Straightness of Travel of the Central 60mm of Each Axis |
| (a) X axis | No measurable error |
| (b) Y axis | No measurable error |
| (c) Z axis | 0.008mm |
| 3. Squareness of Axes |
| (a) XZ | 0.001mm over 32mm |
| (b) YZ | 0.004mm over 32mm |
| (c) XY | 0.010mm over 32mm |

while the end effector was moved over the centre 150mm travel of the total axis travel. The straightness of travel was measured by comparison with a calibrated straight edge (60mm length). The squareness of axes was measured by comparison with a precision cube (32mm×32mm×32mm). An electronic lever probe indicator was used to measure the errors. From Table 6.2, we can see that the robotic testing system has ultra-high accuracy and precision within its workspace. Within the testing range of human joints, the accuracy of the robot is approximately 10µm.
6.3 Typical Applications of the Robotic Testing System in Biomechanics

This section briefly introduces three biomechanics applications of the robotic testing system in which the author was involved. The main responsibilities of the author were to design the testing protocols, program and operate the robot, and collect data. A project summary and primary results are presented for each of the three applications.

6.3.1 Assessment of Impaction Bone Grafting

6.3.1.1 Project Summary

Bone allograft is used to facilitate the healing of fracture non-union with segmental defects. However initial construct stability is poor until bone healing is achieved, which may take between 3-12 months and even longer in complicated cases, accompanied with a significant cost burden to society. The aim of this study was to compare initial stability of impaction bone grafting in a segmental defect to intramedullary nailing of a transverse mid-diaphyseal fracture.

Seven sheep tibia underwent biomechanical testing in compression (1000N), bending and torsion (6Nm) on the custom robotic testing system. Each tibia underwent the same tests across three sequential groups: control (Group 1 – Intact), mid-diaphyseal transverse fracture stabilized by intramedullary nailing (Group 2 – Fracture), segmental defect stabilized with a nail and impacted bone graft in a compliant mesh (Group 3 – Defect). Stiffness was calculated for all tests in each group and normalised as a fraction of the intact tibia. Repeated measures analysis of variance (ANOVA) were conducted for each loading direction with significant differences accepted when p<0.05.

6.3.1.2 Results

Statistical analyses for normalised stiffness between each group revealed that the overall effect of group was significant for all directions of loading (Fig. 6.4, p<0.028). For compression, pairwise compressions revealed that Group 1 was significantly different to Group 3 (p=0.044), no significant differences were present between Groups 1 and 2 (p=0.225, power=0.62) or between Groups 2 and 3 (p=0.205,
Figure 6.4. Mean (95% confidence interval) stiffness expressed as a fraction of the intact tibia (normalised) for each direction of loading. Significant differences compared to the intact group are denoted by *.

power=0.09). The difference between groups that could have been detected, to have a power of 0.8 (e.g. 80%) with the present sample size and assuming the standard deviations did not change was 24%. Pairwise comparisons between groups for bending and torsion (both left and right rotation) revealed significant differences between Groups 1 and 2, and Groups 1 and 3 (p<0.010), with no significant differences existing between Groups 2 and 3 (p=1, power ranged from 0.02-0.06 across the three test directions). The difference between groups that could have been detected, to have a power of 80% with the present sample size and assuming the standard deviations did not change, ranged from 11-23% across the three test directions.

These results are encouraging in translating the technique of impaction bone grafting into the treatment of posttraumatic human long bone segmental defects that are known not only to be difficult to treat, but also require lengthy treatment time and significant patient compliance.
6.3.2 Understanding of 3D Lumbar Intervertebral Disc Internal Strains during Repetitive Loading

6.3.2.1 Project Summary

Chronic low back pain (LBP) is a crippling and insidious drain on one’s quality of life and is a significant burden to both the health care system and the workforce. The mechanisms of LBP are largely poorly understood but it is well known that loss of intervertebral disc (disc) height due to degeneration is a common cause of chronic low back and referred pain. Gross disc injury such as herniation can be caused cumulatively or by sudden overload and is both a cause of acute LBP and an accelerant of disc degeneration. This study analyses a direction and style of motion hypothesised to place the disc at greatest risk of posterolateral herniation.

Ten human lumbar Functional Spinal Units (FSUs) had a grid of tantalum wires inserted into the disc and were subjected to 20,000 cycles of repetitive loading in combined compression, flexion and right axial rotation on the robotic testing system. Stereoradiographs were taken at cyclic intervals (1, 500, 1000, 5000, 10000, 15000 and 20000 cycles) from which 3D internal principal strains and maximum shear strains (MSS) in the disc were calculated and partitioned into nine disc anatomical regions. After testing the discs were sectioned and macroscopically assessed to correlate tissue damage with regions of highest internal disc strain. An ANOVA was used to examine the effects of cycle number and anatomical region on MSS.

6.3.2.2 Results

The displacements and maximum shear strains on the disc after various cycles of receptive loading are shown in Figs. 6.5 and 6.6 respectively. No visible evidence of disc herniation occurred after 20,000 cycles, however an annular tear was present in all cases. There was a significant effect of both number of cycles and disc region on maximum shear strain magnitude (p<0.001). There was an increase in MSS with increasing cycle number in the anterior, left lateral, left/right anterolateral, left posterolateral regions and nucleus. An overall decrease in MSS was seen in the right lateral and right posterolateral regions. The largest increases were observed in the left anterolateral and left posterolateral regions after 20,000 cycles. An increase in MSS
Chapter 6. Robot Control System Design and Typical Applications of the Developed Robotic Testing System in Biomechanics

Figure 6.5. Displacements on the disc after various cycles of repetitive loading
6.3. Typical Applications of the Robotic Testing System in Biomechanics

Figure 6.6. Maximum shear strains (%) on the disc after various cycles of repetitive loading
was observed across most regions in the disc, especially in the left posterolateral region, suggesting internal disc tissue disorganisation that may indicate a progression towards annular tears and eventual herniation.

6.3.3 Assessment of Primary Stability of Cementless Tibial Tray

6.3.3.1 Project Summary

Cementless tibial fixation has been used for over 30 years in total knee arthroplasty. There are several potential advantages including preservation of bone stock and ease of revision. More importantly, for young active patients there is the potential for increased longevity of fixation. However, the clinical results have been variable, with reports of extensive radiolucent lines, rapid early migration and aseptic loosening. Problems appear to stem from a failure to become sufficiently osseointegrated, which in turn suggests a lack of primary stability. In order to achieve boney ingrowth, interface micromotions should be less than 50 microns, whereas fibrous tissue formation is known to occur if micromotions are in excess of 150 microns. The degree of micromotion at the bone-implant interface are dependent on the kinematics and kinetics of the replaced joint. The tibial tray experiences complex six degree of freedom (6-DOF) loading as a consequence of activities of daily living (ADL) comprising a combination of axial, anterior-posterior and medial-lateral loads as well as flexion-extension, varus-valgus and internal-external moments. Until recently our knowledge of the magnitude and temporal variation of these forces and moments has been limited, but recent data from telemetric implants (Kutzner et al., 2010, Mundermann et al., 2008) have now given detailed information that will prove invaluable for pre-clinical testing. In-vitro studies investigating primary stability have used simplified loading conditions incorporating a static axial load in combination with internal-external moments (Sala et al., 1999, Walker et al., 1990), AP shear forces (Walker et al., 1990, Chong et al., 2011) and ML forces (Chong et al., 2011). Typically, these studies have used a force equivalent to the peak load which occurs during the stance phase of gait. To date only computational studies have been able to simulate the 6-DOF loading conditions for a variety of activities (Taylor et al., 2012). This study aims to develop an in-vitro testing protocol capable of applying 6-DOF loads for ADL’s in order to assess the primary stability of cementless tibial trays.
A cadaveric tibia, with no signs of disease or gross defects, was retrieved and stored at -20 degrees Celsius. Prior to testing, the tibia was defrosted and cleaned of all soft tissue. The tibia was implanted with an idealized stainless steel, cementless tibial tray, which had a single conical keel (based on the geometry of the LCS, DePuy Inc). There were no coatings applied to the tray and cone, which only had a course grit blasted finish. The tibia was resected approx. 100mm below the distal tip of the prostheses and potted into a fixture using liquid metal. The mechanical tests were performed on the custom developed 6-DOF Hexapod Robot (Stewart platform), which has been designed to achieve high load (~20 kN/1500 Nm), high precision (~10 microns) performance and can precisely reproduce the 6-DOF loads histories of activities of daily living. The distal tibia was rigidly attached to the base of the machine and the tibial tray rigidly attached to the actuator. The 6-DOF forces (anterior-posterior, medial-lateral and inferior-superior forces and flexion-extension, abduction-adduction and internal-external moments) associated with the stance phase of level gait were applied directly to the tibial tray, based on data derived from Orthoload.com (subject K2) (Kutzner et al., 2010). Due to the limits of the control system, the forces were applied quasi-statically, with loading frequency of 0.001Hz. All forces and moments were scaled to a peak axial load of 1500N. Previous studies have shown the micromotion to be dominated by lift-off the tibial tray. Therefore, the vertical micromotion of the tray relative to the bone on the medial, lateral and anterior aspects of the implant was recorded at a frequency of 2Hz using linear variable displacement transducers, with a measurement resolution of 5 microns.

6.3.3.2 Results

The Hexapod robot successfully achieved the desired 6-DOF loading profile. For example there was an RMS error of 29N for the axial load. The 6DOF forces resulted in anterior liftoff of the tray (Fig. 6.7), with a peak of 332 micron which occurred at approx. 15% of the gait cycle, just after heel strike. The micromotion appeared to be dominated by the applied axial load and the flexion moment, with the peak micromotion occurring due to a combination of a high axial load (1400N) and the peak flexion moment (approx. 12Nm). In terms of lift-off, the other loading components (anterior-posterior and medial-lateral shear forces and abduction-adduction and internal-external moments) appeared to have less of an influence on
axial micromotion. The effects of the anterior lift was also evident in the medial and lateral micromotion data, with axial micromotions of 106 and 88 microns respectively, suggesting that the anterior lift off resulted in a wedge shaped gap that extended beyond the AP midpoint of the tray.

In order to fully evaluate the behaviour of cementless tibial trays, there is a need for designs to be subjected to all of the forces that occur in vivo for a wide range of activities, including level gait, stair ascent/descent, chair rise and a deep squat. This can be achieved using finite element analysis (Taylor et al., 2012), but to date, reproducing the 6-DOF loading in-vitro has been difficult due to the limitations of conventional materials testing machines. Using a 6-DOF Hexapod and data derived from telemetric implants, this study has successfully assessed the micromotions of a tibial tray subjected to the complex loads experienced during the stance phase of gait. The applied loads resulted in an anterior gap being generated as a result of the combination of the high axial loads and flexion moments. There are a number of limitations with this study. Only axial micromotions were assessed and shear micromotions were not measured. The other loading components, particularly internal-external moments and anterior-posterior forces may have a greater influence on shear micromotions. Also, the geometry and surface finish of the tibial tray may not be representative of cementless tibial trays used clinically. However, this pilot study has demonstrated that this approach has potential to explore a wide range of activities and provide a method for validating finite element models of the implanted proximal tibia subjected to complex loads.

Figure 6.7. Anterior lift off of the tibial tray during the stance phase of gait (0-60%)
Chapter 7

Conclusion and Future Work

7.1 Conclusion

A Stewart platform-based manipulator was developed as part of this study and was fully functional for use in applications of biomechanical testing. Four main features of the proposed manipulator were investigated in this thesis. The first two features—non-collocated actuator-sensor mechanism and active preload control using actuation redundancy—can be used in general robotic-based applications requiring ultra-high stiffness, improved bandwidth and accuracy from the robot. The other two features—adaptive velocity-based load control and reproducing the \textit{in-vivo} measured kinematics on human cadaveric joints—were specifically designed for use in biomechanical testing.

7.1.1 Stiffness and Control of the Non-collocated Actuator-Sensor Mechanism

A Stewart platform-based manipulator with decoupled sensor-actuator locations was proposed for applications in biomechanical testing. Studies were performed on both the stiffness and control aspects of the manipulator. Results show that the unique non-collocated sensor-actuator mechanism was able to increase the stiffness of a general Stewart platform testing system by a factor of 15 over a common collocated design, and therefore significantly improved the static accuracy of the manipulator when subjected to large reaction forces and moments. Results also show that using the proposed decoupled control algorithm on the manipulator improved the dynamic accuracy of the manipulator by 25% on average.
7.1.2 Active Preload Control Using Actuation Redundancy for Backlash Elimination

This thesis studied the use of actuation redundancy to eliminate backlash inaccuracy for a general 6-DOF Stewart platform. A novel redundancy arrangement with a refined active preload control method was proposed for real-time control applications. Simulation results demonstrated preload efficacy by the redundant arrangement within the workspace of the robot. Simulation results also showed that the proposed method can effectively achieve backlash-free positioning of the manipulator under large 6-DOF external loads. Mainly because of the hardware limitations, the experiment was implemented at speeds slower than the robot could deliver. The experimental results further demonstrated that the proposed method can eliminate backlash instabilities from control and consequently higher bandwidth control can be achieved by the robot with improved accuracy.

In order to make the proposed active preload control method fully applicable in industry, more design and research work is required. Firstly, the design of the redundant leg assembly is critical. The type of shock absorber employed is not ideal not only because of its single-direction load capacity but also due to the instabilities in its transversal directions. Therefore, a more sophisticated mass-damper-spring system needs to be designed to allow a single DOF compliant motion along its longitudinal axis. As the redundant leg controls the preloads on all six position-controlled legs, the load capacity of the redundant leg is required to be 4 to 6 times higher than the position-controlled legs to ensure the controllability of the system. Secondly, the location of the redundant leg (e.g. spherical joint locations) needs further investigation to reach the optimal preload distribution efficiency of the redundant configuration. It is recommended to place the redundant leg into the robot inner space. This configuration avoids a perpendicular relation occurring between the legs, and consequently ensure the overall preload distribution efficiency of the structure. Thirdly, the control of the redundantly actuated manipulator requires further improvements. More intelligent control algorithms are necessary to improve the efficiency, speed and ease of robot control by taking more control indices into consideration (e.g. backlash dynamics, manipulator acceleration and tracking errors).
7.1.3 Adaptive Velocity-based Load Control of Human Joint for Unconstrained Testing

An adaptive velocity-based load control method was proposed for use on the robotic system to implement unconstrained testing on human joints. The control method was not only self-adaptive to the unknown behaviour of the joint but also computationally inexpensive. The efficiency of the method was proven using experiments and can be further improved if the following limitations in the system can be overcome: measurement noise in the load feedback was the main limitation which prevented the adaptive controller from making the appropriate decision. A possible solution is to use a more advanced load-cell amplifier having better noise rejection ability. In addition, the accuracy and stability of the higher level force control was highly dependent on the accuracy and stability of the lower level (inner loop) position control. As a result, any nonlinearity (e.g. backlash) degrading the position control performance can directly degrade the performance of the force control loop. The current solution was to ease off the position control gain to ensure control stability, although this can sacrifice control accuracy. A potential solution was to integrate the redundant leg method into this experiment but this was out of the timeline of this project.

7.1.4 Reproducing the In-vivo Measured Kinematics on Human Cadaveric Joints

This thesis developed a method to scientifically reproduce the general in-vivo kinematics measured from a living human on human cadaver joints using the custom-built Stewart platform-based manipulator. A human wrist was used as a typical example to elaborate the theory of the method and to assess the fidelity of the method. Experimental results show that the accuracy of the reproduced motion on the cadaveric samples was of similar magnitude to the measurement error of the motion capture system. Although largely limited due to this reason, the accuracy of the method was still acceptable on average for joint research. Experimental results also show that load responses at the joint centre of rotation during reproduction were minimal, which proved that the assumption of fixed wrist joint centre of rotation was valid for motion reproduction. In addition, the wrist carpal kinematics measured from
the reproduced motion matched the results obtained from previous research. This further proved the validity and fidelity of the proposed method.

### 7.2 Future Work

Although the work presented in this thesis has successfully contributed to the knowledge of the application of robotic systems to biomechanical testing, there are still a number of unanswered questions and potential improvements that could be made given time and the necessary financial resources. This section discusses such future work.

#### 7.2.1 Non-collocated Actuator-Sensor Mechanism

There are several ways in which the control of the developed non-collocated actuator-sensor mechanism could be improved. Dynamics-based control and more intelligent control methods are required to enable the manipulator to track a dynamic trajectory with improved accuracy. A model of the non-collocated actuator-sensor mechanism is critical for its control and could be derived based on system identification techniques (e.g. prediction and simulation), where both linear and nonlinear approaches could be employed. Furthermore, considering the complexity of the whole system, model-based fault detection and diagnosis techniques can be applied to the robot, where a mathematical or non-analytical model of the robot contributes to detecting possible faults/failures that may occur in the sensors and actuators of the robot.

As developed based on a general Stewart platform concept, the non-collocated actuator-sensor mechanism has limited ranges of motion, particularly on its rotational axes, which is a major limitation for some applications. Recent research has proposed several 6-DOF parallel robot mechanisms with a larger workspace compared to the traditional Stewart platform, however at the expense of sacrificing the stiffness of the system (Coulombe and Bonev, 2013). A potential solution for this issue is to decouple the locations of the actuators and sensors in these designs as was undertaken on the general Stewart platform. Depending on the original robot structure design, this task can be very challenging and considerable research work is required to investigate the mechanical arrangement, stiffness improvement, and robot control.
7.2.2 Intelligent Active Preload Control of the Redundantly Actuated Manipulator

In the proposed active preload control method, the preload trajectory of the redundant leg was determined using a simple online optimization algorithm based on the dynamics model of the redundant manipulator, the backlash-free threshold, and the feedback from the sensors. This inherently caused a short delay in the preload control of the redundant leg. The current solution was to define a higher backlash-free threshold to compensate for such a delay. Experimental results proved the efficacy of this solution. However, in the case when the external loads vary quickly, a large preload difference can appear on the redundant leg due to this delay, which can consequently move the preloads on the position-controlled legs into the backlash region. To properly address this issue, a more intelligent control algorithm such as online learning predictive control is required to control the redundant leg. In addition, it will be more efficient to define the backlash-free condition using the backlash dynamics model rather than a single threshold in control. The position tracking performance of the manipulator can be used as feedback in the control algorithm to recognize if the backlash-free condition is achieved in real-time.

7.2.3 Study the Control of the Manipulator to Simulate More Physiological Joint Motion and Loading

Although the robotic testing system developed from this study was fully functional for biomechanical testing, further studies are required to be conducted on the control of the manipulator to simulate more extensive physiological joint motion and loading. The load control for unconstrained testing was limited by its testing speed, while the position control for constrained testing can cause large errors in the simulated motion when the assumption of a fixed joint centre of rotation failed. Therefore, a multiple-input-multiple-output (MIMO) 6-DOF load control algorithm with a higher bandwidth is needed and an online algorithm is required to estimate the trajectory of the joint centre of rotation based on the fixed helical axis theory and additional sensors for position control.
Chapter 7. Conclusion and Future Work
References


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Reference


Appendix A

Mechanical Drawings of the Additional Frame and Redundant Leg Assembly
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.1. Assembly of the additional framework for mounting the redundant leg which consists of three individual pieces.
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.2. Drawing showing the dimensions of the front frame piece
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.3. Drawing showing the dimensions of the back frame piece

200
Figure A.4. Drawing showing the dimensions of the top frame piece
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.5. Assembly of the redundant leg design
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.6. Drawing showing the thread on the ballcrew-actuator shock-absorber connector

Item #4
Material to be Mild Steel.
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.7. Drawing showing ballcrew-actuator shock-absorber connector

Item #5
Material to be Mild Steel.

General Tolerance: ± 0.1 mm and to suit mating parts or unless otherwise stated.
Figure A.8. Drawing showing the assembly of the ballscrew-actuator shock-absorber connector
Figure A.9. Drawing showing the limit bolt for fixing the shock absorber to the connector
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.10. Drawing showing the shock absorber body

- Item #7
  Material to be Aluminum.
  General Tolerance: ± 0.1 mm and to suit mating parts or unless otherwise stated.
  Existing part: no manufacturing required from the workshop.
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.11. Drawing showing the shock-absorber load-cell connector

Item #8
Material to be Mild Steel.
General Tolerance: ±0.1 mm and to suit mating parts or unless otherwise stated.
Figure A.12. Drawing showing the F214 load-cell body
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.13. Drawing showing the load-cell rod-eye connector
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.14. Drawing showing the spacer tube for the rod eye
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.15. Drawing showing the limit bolt acting as the shaft of the rod eye
Appendix A. Mechanical Drawings of the Additional Frame and Redundant Leg Assembly

Figure A.16. Drawing showing the should for supporting the rod eye shaft
Figure A.17. Drawing showing the plate which couples the redundant leg to the top of the robot
Appendix B

Electrical Drawings and Wiring

Diagrams of the Robot Control System
Figure B.1. The wiring diagram of the PCB boards (1)
Figure B.2. The wiring diagram of the PCB boards (2)
Appendix B. Electrical Drawings and Wiring Diagrams of the Robot Control System

Figure B.3. Wiring diagram on RIO0 (FPGA board 1)
Figure B.4. Wiring diagram on RIO1 (FPGA board 2)
Appendix B. Electrical Drawings and Wiring Diagrams of the Robot Control System

Figure B.5. Electrical drawing of the custom isolation device
Appendix C

Matlab Codes

C.1 Code for Calculating the Stiffness of the Non-Collocated Actuator-Sensor Mechanism

% Estimation of load frame and sensing frame stiffness in task space
% 12-11-2012
% Matlab v. 7
% Boyin DIng

% Conversion from inches to mm
%
conv = 25.4;
%
% Build Actuator End Position Matrices
% Note that 0 is base and 1 is top
%
xa0 = 46.584; % Dimension in inches
ya0 = 366.845; % Dimension in inches
Appendix C. Matlab Codes

% xa1 = 166.260;            % Dimension in inches
ya1 = 130.631;            % Dimension in inches

% A0 = geom(xa0,ya0);
A1 = geom(xa1,ya1);

% % Build Encoder End Position Matrices
%
xe0 = 6.855;           % Dimension in inches
ye0 = 316.867;           % Dimension in inches
%
xe1 = 125.834;           % Dimension in inches
ye1 = 82.044;           % Dimension in inches
%
E0 = geom(xe0,ye0);
E1 = geom(xe1,ye1);

% % Rotation Matrix
%
r = [20;0;0];               % Position of the end-effector
deg2rad = pi/180;       % Conversion from degrees to radians
M0 = rotxyz(r*deg2rad);

% % Offset Vector
%
C.1. Code for Calculating the Stiffness of the Non-collocated Actuator-sensor Mechanism

d = [0; 0; 500]; % Dimensions in mm
D0 = repmat(d,1,6);

% % Actuator Link Vector
% VA0 = M0*A1+D0-A0;
% % Encoder link Vector
% VE0 = M0*E1+D0-E0;
% % Lengths of Actuator Links
% LA0 = sqrt(sum(VA0.*VA0));
% % Lengths of Encoder Links
% LE0 = sqrt(sum(VE0.*VE0));
% % Jacobian matrix mapping end-effector force to leg force
% na=transpose([VA0(:,1)/LA0(1) VA0(:,2)/LA0(2) VA0(:,3)/LA0(3) VA0(:,4)/LA0(4)
VA0(:,5)/LA0(5) VA0(:,6)/LA0(6)]);
JA=[na cross(na, transpose(-M0*(A1./1000)))];
JF=transpose(inv(JA));
% Get force vector on legs for FEA analysis
%
Fe=[0 0 0 0 0 2000]';
Fl=JF*Fe
FA0(1,:)=VA0(1,:).*(Fl'/LA0);
FA0(2,:)=VA0(2,:).*(Fl'/LA0);
FA0(3,:)=VA0(3,:).*(Fl'/LA0);
FA0
FAE(:,1)=inv(M0)*(-FA0(:,1));
FAE(:,2)=inv(M0)*(-FA0(:,2));
FAE(:,3)=inv(M0)*(-FA0(:,3));
FAE(:,4)=inv(M0)*(-FA0(:,4));
FAE(:,5)=inv(M0)*(-FA0(:,5));
FAE(:,6)=inv(M0)*(-FA0(:,6));
FAE
%
% Calculate kinematics Jacobian of the load frame and sensing frame respectively
na=transpose([VA0(:,1)/LA0(1) VA0(:,2)/LA0(2) VA0(:,3)/LA0(3) VA0(:,4)/LA0(4)
VA0(:,5)/LA0(5) VA0(:,6)/LA0(6)]);
JA=[na cross(na, transpose(-M0*A1))];
%
ne=transpose([VE0(:,1)/LE0(1) VE0(:,2)/LE0(2) VE0(:,3)/LE0(3) VE0(:,4)/LE0(4)
VE0(:,5)/LE0(5) VE0(:,6)/LE0(6)]);
JE=[ne cross(ne, transpose(-M0*E1))];
%
% Transform the errors on the top spherical joints in ECS to GCS
%
C.1. Code for Calculating the Stiffness of the Non-collocated Actuator-sensor Mechanism

da11=M0*da1; %da1 is from FEA analysis
%
de11=M0*de1; %de1 is from FEA analysis
%
% Calculate stiffness of the ballscrew actuator
%
% Steel as material
%
E = 190000000000
%
% Piston length
%
lp = 7*25.4*0.001
%
% Piston cross-section area
%
Ap=pi*(1.13*25.4/2*0.001)^2
%
% Stiffness of piston
%
Kp=E*Ap/lp
%
% Cylinder length
%
lc = (4.84+7)*25.4*0.001
%
% Cylinder cross-section area
%
Ac=pi*(2.25*25.4/2*0.001)^2-pi*(1.13*25.4/2*0.001)^2
%
% Stiffness of cylinder
%
Kc=E*Ac/lc
%
% Overall stiffness
%
Kl=Kc*Kp/(Kc+Kp)
%
Kt=40000000
%
Ka=Kt*Kl/(Kt+Kl)
%
% Transform errors on subsystems to errors on the robot end-effector
%
dat=inv(JA)*(Fl./[Ka Ka Ka Ka Ka Ka])' + inv(JA)*([na(1,:)*da0(:,1);na(2,:)*da0(:,2);na(3,:)*da0(:,3);na(4,:)*da0(:,4);na(5,:)*da0(:,5);na(6,:)*da0(:,6)]) -
inv(JA)*([na(1,:)*da11(:,1);na(2,:)*da11(:,2);na(3,:)*da11(:,3);na(4,:)*da11(:,4);na(5,:)*da11(:,5);na(6,:)*da11(:,6)])
%
det=inv(JE)*([ne(1,:)*de0(:,1);ne(2,:)*de0(:,2);ne(3,:)*de0(:,3);ne(4,:)*de0(:,4);ne(5,:)*de0(:,5);ne(6,:)*de0(:,6)]) -
inv(JE)*([ne(1,:)*de11(:,1);ne(2,:)*de11(:,2);ne(3,:)*de11(:,3);ne(4,:)*de11(:,4);ne(5,:)*de11(:,5);ne(6,:)*de11(:,6)])
C.2 Code for Preload Control Simulation

 clear all
 close all

 %
 % Conversion from inches to mm
 %
 conv = 25.4;
 %
 % Build Actuator End Position Matrices
 % Note that 0 is base and 1 is top
 %
 xa0 = 46.584;       % Dimension in inches
 ya0 = 366.845;      % Dimension in inches
 %
 xa1 = 166.260;      % Dimension in inches
 ya1 = 130.631;      % Dimension in inches
 %
 A0 = geom(xa0,ya0);
 A1 = geom(xa1,ya1);
Appendix C. Matlab Codes

% Define some parameters

% 
Fnp = zeros(6,3000);    % forces on ED without preload control 
f = zeros(6,3000);    % forces on the legs without preload control 
Fwp = zeros(6,3000);    % forces on ED with preload control 
fp = zeros(6,3000);     % forces on the legs with preload control 
preload = zeros(1,3000);        % preload with time 
V7 = zeros(1,3000);              % 7th leg length with time 
SE=[0;0;100];          % offset between SCS and ECS
Sa=100;                  % parameter for positive direction 
Sb=100;                  % parameter for negative direction 
C7=4000;               % capacity of the 7th leg
%
% Start iteration

for i=0:1:2999

% 
rip = [0; 0; -100; 0; 0; 0];  % robot initial pose
%

r = rip(4:6);

deg2rad = pi/180;             % Conversion from degrees to radians
M0 = rotxyz(r*deg2rad);       % Calculate the rotation matrix of the robot initial pose
%

sip = rip(1:3)-M0*SE;  % specimen initial position
%

sd = [-((5/8)*sin(i/1000*2*pi)),0,-(2/5)*sin(i/1000*2*pi),0,(50/6)*sin(i/1000*2*pi)),0];  % specimen displacement
C.2. Code for Preload Control Simulation

snp = sip+sd(1:3);  % specimen new position
rn = r+sd(4:6);    % specimen and robot new rotation

M1 = rotxyz(rn*deg2rad);  % rotation matrix of the new robot and specimen pose
rnp = snp+M1*SE;   % New robot position

d = rnp+[0; 0; 590.738];  % end-effector position in GCS: Dimensions in mm
D0 = repmat(d,1,6);

% Actuator Link Vector
VA0 = M1*A1+D0-A0;

% Lengths of Actuator Links
LA0 = sqrt(sum(VA0.*VA0));

% Get Jacobian matrix maps ED pose to leg lengths
na=transpose([VA0(:,1)/LA0(1) VA0(:,2)/LA0(2) VA0(:,3)/LA0(3) VA0(:,4)/LA0(4) VA0(:,5)/LA0(5) VA0(:,6)/LA0(6)]);
JA=[na cross(na, transpose(-M0*(A1/1000)))];

% Get Transposed Jacobian matrix maps ED force to forces along leg

Appendix C. Matlab Codes

\[ JT = \text{transpose(inv(JA))}; \]

\%

\% Transform the force and moment at the specimen to the the force and
\% moment at end-effector
\%

\[ M = \text{diag([-80 -80 -500 -6 -6 -4])}; \]  \% Stiffness matrix

\[ Fs = M \times sd; \]  \% force and moment at specimen center of rotation

\[ Fe = [Fs(1:3); \text{cross}(M1 \times (-SE \times 0.001), Fs(1:3)) + Fs(4:6)]; \]  \% obtain the force acting on
\% the end-effector due to specimen deform

\%

\% Transform the top assembly gravity to the force and moment at end-effector
\%

\[ Mg = [0 0 -200 0 0 0]'; \]

\[ GE = [0 0 -60]'; \]

\[ Me = [Mg(1:3); \text{cross}(M1 \times (-GE \times 0.001), Mg(1:3)) + Mg(4:6)]; \]  \% obtain the force due
to top assembly gravity

\%

\% The forces acting on the ED arise from the 7th actuator
\%

\[ E7 = [0; 0; 136.5]; \]  \% offset between the ED and the seven leg lower pivot

\[ VA7 = d + M1 \times E7 - [0; 0; 1573.1]; \]  \% 7th actuator link vector

\[ V7(i+1) = \sqrt{\text{sum}(VA7.*VA7)}; \]  \% Record 7th leg length

\[ F = 0 \times VA7 / \sqrt{\text{sum}(VA7.*VA7)}; \]  \% no preload at all

\%

\% Calculate the force along six legs due to leg gravity

\[ Flg = \text{transpose}(VA0./\text{repmat(LA0,3,1))} \times [0; 0; -20]; \]

\%
% Calculate the forces along six legs
%
Fnp(:,i+1)=Me+Fe+[F;cross(M1*(E7*0.001), F)];       % Force acting on ED without preload control
f(:,i+1)=JT*(Fe+[F;cross(M1*(E7*0.001), F)]+Me)+Flg;     % Force on legs without preload control
%
Fcp = Sa*VA7/sqrt(sum(VA7.*VA7));                       % 7th leg force at the current position in GCS
S0=Sa;
Fa= Fcp;                                                % set Fa and Fb to the current preload value in GCS
Fb= Fcp;
fp1(:,i+1)=JT*(Fe+[Fcp;cross(M1*(E7*0.001), Fcp)]+Me)+Flg;    % preloads on six legs during upper search
fp2(:,i+1)=fp1(:,i+1);                                    % preloads on six legs during lower search
if min(abs(fp1(:,i+1)))<80
    while
        and(or(min(abs(fp1(:,i+1)))<80,and(sum(sign(fp1(:,i+1)))~=6,sum(sign(fp1(:,i+1)))~=-6)),or(min(abs(fp2(:,i+1)))<80,and(sum(sign(fp2(:,i+1)))~-6,sum(sign(fp2(:,i+1)))~=-6)))
            Sa = Sa+2;       % search along positive direction
            Sa = min(Sa, C7);     % make sure it's in range
            Sa = max(Sa, -C7);
            Sb = Sb-2;     % search along negative direction
            Sb = min(Sb, C7);     % make sure it's in range
            Sb = max(Sb, -C7);
    end
end
Appendix C. Matlab Codes

\[
Fa = Sa*VA7/sqrt(sum(VA7.*VA7)); \quad \% \text{upper search force in global coordinate system}
\]

\[
Fb = Sb*VA7/sqrt(sum(VA7.*VA7)); \quad \% \text{lower search force in global coordinate system}
\]

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C.2. Code for Preload Control Simulation

\[
\begin{align*}
fp1(:,i+1) &= JT*(Fe+[Fa;cross(M1*(E7*0.001), Fa)]+Me)+Flg;
\text{else} \\
\text{end}
\end{align*}
\]

Fwp(:,i+1)=Fe+[Fa;cross(M1*(E7*0.001), Fa)]+Me; \quad \% \text{Force acting on ED with preload control}

\[
\begin{align*}
fp(:,i+1) &= -fp1(:,i+1); \\
\text{preload}(i+1) &= Sa; \\
Sb &= Sa; \\
\text{else} \\
\text{if} \ (S0-Sb)>20 \\
\quad Sb &= S0-20; \\
\quad Fb &= Sb*VA7/sqrt(sum(VA7.*VA7)); \\
fp2(:,i+1) &= JT*(Fe+[Fb;cross(M1*(E7*0.001), Fb)]+Me)+Flg; \\
\text{else} \\
\text{end} \\
fp(:,i+1) &= -fp2(:,i+1); \\
\text{Fwp}(:,i+1) &= Fe+[Fb;cross(M1*(E7*0.001), Fb)]+Me; \\
\text{preload}(i+1) &= Sb; \\
\text{Sa} &= Sb; \\
\text{end} \\
\text{end} \\
\text{else} \\
FC &= S0*VA7/sqrt(sum(VA7.*VA7)); \quad \% \text{Current force in global coordinate system}
\]

\[
\begin{align*}
\text{LC} &= \text{transpose}([JT*(Fe+[FC;cross(M1*(E7*0.001), FC)]+Me);S0])*([JT*(Fe+[FC;cross(M1*(E7*0.001), FC)]+Me);S0]); \quad \% \text{Current control forces index}
\end{align*}
\]
% search along positive direction
Sa = Sa+2;

% make sure it's in range
Sa = min(Sa, C7);
Sa = max(Sa, -C7);

% search along negative direction
Sb = Sb-2;
Sb = min(Sb, C7);
Sb = max(Sb, -C7);

% upper search force in global coordinate system
Fa = Sa*VA7/sqrt(sum(VA7.*VA7));

% lower search force in global coordinate system
Fb = Sb*VA7/sqrt(sum(VA7.*VA7));

% Claulte the control forces index for upper search and lower search and current force
LU=transpose([JT*(Fe+[Fa;cross(M1*(E7*0.001), Fa)]+Me);Sa])*([JT*(Fe+[Fa;cross(M1*(E7*0.001), Fa)]+Me);Sa]);

LL=transpose([JT*(Fe+[Fb;cross(M1*(E7*0.001), Fb)]+Me);Sb])*([JT*(Fe+[Fb;cross(M1*(E7*0.001), Fb)]+Me);Sb]);

% Calculate the forces along six legs with the new force acting on the 7th leg

fp1(:,i+1)=JT*(Fe+[Fa;cross(M1*(E7*0.001), Fa)]+Me)+Flg;
fp2(:,i+1)=JT*(Fe+[Fb;cross(M1*(E7*0.001), Fb)]+Me)+Flg;

if and(and(min(abs(fp1(:,i+1)))>80,LU<LL),LU<LC)
    Fwp(:,i+1)=Fe+[Fa;cross(M1*(E7*0.001), Fa)]+Me; % Force acting on ED with preload control
    fp(:,i+1)=-fp1(:,i+1);
    preload(i+1)=Sa;
else
    FP(:,i+1)=Fe+[Fa;cross(M1*(E7*0.001), Fa)]+Me;
    fp(:,i+1)=-FP(:,i+1);
    preload(i+1)=Sa;
end
Sb=Sa;
else
    if and(min(abs(fp2(:,i+1)))>80,LL<LC)
        fp(:,i+1)=-fp2(:,i+1);
        Fwp(:,i+1)=Fe+[Fb;cross(M1*(E7*0.001), Fb)]+Me;
        preload(i+1)=Sb;
        Sa=Sb;
    else
        fp(:,i+1)=-JT*(Fe+[Fcp;cross(M1*(E7*0.001), Fcp)]+Me)-Flg;   % preload not changed
        Sa=S0;
        Sb=S0;
        Fwp(:,i+1)=Fe+[Fcp;cross(M1*(E7*0.001), Fcp)]+Me;
        preload(i+1)=S0;
    end
end
end

V7=V7-V7(1)*ones(1,3000);   % Get relative value

figure(1)
plot((1:3000)/100,Fnp(1,:),'r')
hold on
plot((1:3000)/100,Fnp(2,:),'c')
plot((1:3000)/100,Fnp(3,:),'g')
Appendix C. Matlab Codes

plot((1:3000)/100,Fnp(4,:),'b')
plot((1:3000)/100,Fnp(5,:),'y')
plot((1:3000)/100,Fnp(6,:),'k')
title('force acting on the ED wihtout preload control')
legend('x force','y force','z force','x moment','y moment','z moment')

figure(2)
plot((1:3000)/100,-f(1,:),'r','linewidth',2)
hold on
plot((1:3000)/100,-f(2,:),'c','linewidth',2)
plot((1:3000)/100,-f(3,:),'g','linewidth',2)
plot((1:3000)/100,-f(4,:),'b','linewidth',2)
plot((1:3000)/100,-f(5,:),'y','linewidth',2)
plot((1:3000)/100,-f(6,:),'k','linewidth',2)
legend('1st leg','2nd leg','3rd leg','4th leg','5th leg','6th leg')
ylabel('Actuator control force (N)')
xlabel('Time (seconds)')

figure(3)
plot((1:3000)/100,Fwp(1,:),'r')
hold on
plot((1:3000)/100,Fwp(2,:),'c')
plot((1:3000)/100,Fwp(3,:),'g')
plot((1:3000)/100,Fwp(4,:),'b')
plot((1:3000)/100,Fwp(5,:),'y')
plot((1:3000)/100,Fwp(6,:),'k')
title('force acting on the ED with preload control')
legend('x force','y force','z force','x moment','y moment','z moment')

figure(4)
plot((1:3000)/100,fp(1,:),r,'linewidth',2)
hold on
plot((1:3000)/100,fp(2,:),c,'linewidth',2)
plot((1:3000)/100,fp(3,:),g,'linewidth',2)
plot((1:3000)/100,fp(4,:),b,'linewidth',2)
plot((1:3000)/100,fp(5,:),y,'linewidth',2)
plot((1:3000)/100,fp(6,:),k,'linewidth',2)
plot((0:30),ones(31)*70,--m,'linewidth',2)
legend('1st leg','2nd leg','3rd leg','4th leg','5th leg','6th leg', 'BF threshold')
ylabel('Actuator control force (N)')
xlabel('Time (seconds)')

figure(5)
plot((1:3000)/100,preload,'linewidth',2)
legend('7th leg')
ylabel('Preload (N)')
xlabel('Time (seconds)')

figure(6)
plot((1:3000)/100,V7)
title('Interplated length of the 7th leg')
legend('length of the 7th leg')