



THE UNIVERSITY  
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Strangeness and  
Charge Symmetry Violation  
in Nucleon Structure

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# Abstract

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The role of strange quarks in generating the structure of the nucleon provides a key testing-ground for our understanding of Quantum Chromodynamics (QCD). Because the nucleon has zero net strangeness, strange observables give tremendous insight into the nature of the vacuum; they can only arise through quantum fluctuations in which strange-antistrange quark pairs are generated. Strange observables are also relevant to searches for physics beyond the Standard Model; the role of the strange quark in generating the nucleon mass—encoded in the strange sigma term—is essential information for the interpretation of dark matter direct-detection experiments. For these reasons, strangeness in the nucleon is currently a particular focus of the nuclear physics community.

We use the numerical lattice gauge theory approach to QCD, and the chiral perturbation theory formalism, to build a clear picture of the role of strange quarks in various nucleon-structure observables. A detailed analysis of the octet baryon masses provides precise new values of the nucleon sigma terms. By combining experimental and lattice input, we deduce the strange electromagnetic form factors of the nucleon over a far larger range of momentum-scales than is accessible experimentally. Our calculation of the strange magnetic moment is an order of magnitude more precise than the closest experimental result.

Until now, the dominant uncertainty in experimental determinations of the strange proton form factors has come from a lack of knowledge about the size of charge symmetry violation (CSV) in these quantities. CSV effects quantify the breaking of the approximate  $SU(2)$ -flavour symmetry of the up and down quarks. As well as their relevance to experimental determinations of nucleon strangeness, the precise knowledge of CSV observables has, with increasing experimental precision, become essential to the interpretation of many searches for physics beyond the Standard Model. We develop a formalism for the calculation of CSV observables from isospin-averaged  $2 + 1$ -flavour lattice QCD simulations.

Applying this formalism to a comprehensive lattice-based study of the electric and magnetic Sachs form factors of the baryon octet reveals that the CSV form factors are an order of magnitude smaller than suggested by previous work. This calculation opens the door for new, precise, experimental measurements of the strange nucleon form factors. We also investigate the proton-neutron mass difference and quantify the long-neglected CSV effects in the low Mellin moments of the spin-dependent and spin-independent parton distribution functions. This analysis improves the interpretation of neutrino-nucleus deep inelastic scattering experiments.



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Phiala E. Shanahan



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