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An Enhanced Physics-based Model to Estimate the Displacement of Piezoelectric Actuators

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ABSTRACT: Piezoelectric actuators are the foremost actuators in the area of nanopositioning. However, the sensors, employed to measure the actuator displacement, are expensive and difficult if not impossible to be used. Mathematical models can map the easy-to-measure electrical signals to the displacements of these actuators as the displacement sensors are replaced with the models. In addition, these models can be used in model-based control system design. Two main groups of mathematical models are used for this purpose: black box models and physics-based models. As an advantage, the latter models have a much less number of parameters reducing computational demand. However, physics-based models suffer from (1) the relatively low accuracy of the models and, (2) non-standard and ad-hoc parameter identification methods. In this research, to improve the model accuracy, mathematical structure of the model is enhanced by adding two complementary terms inspired by the Preisach model. Then, a standard method based on the evolutionary algorithms is proposed to identify the model’s parameters. To this purpose, a well-known physics-based model, the Voigt model, is to be improved. The proposed ideas are substantiated to increase the applicability and accuracy of the model and they are easily extendable to other physics-based models of piezoelectric actuators. The newly proposed enhanced structure of the Voigt model doubles the estimation accuracy of the original model and results in accuracies comparable with black box models.

INTRODUCTION

Piezoelectric materials are a class of smart materials coupling the electric voltage and mechanical force. If an electric voltage is applied to a piezoelectric material, the material starts to vibrate and deform. This can actuate the adjacent structures; thus, the piezoelectric material plays the role of an actuator (Kim et al., 2005). Nanopositioning, manipulating materials at nano/micro metre scale, is the main application area for piezoelectric actuators. (Karam, 1999). Nanopositioning plays a critical role in areas such as Atomic Force Microscopy (AFM) (Minne et al., 1995, Last et al., 2010) and highly precise manufacturing (Park and Moon, 2010).

In nanopositioning, the displacement of piezoelectric actuators should be accurately controlled. To this purpose, feedback control systems are employed needing highly accurate displacement sensors. However, the sensors are expensive and/or their application is limited by practical constraints such as non-accessible installation and difficult calibration (Boukari, 2010). Accordingly, these sensors can be replaced with mathematical models. The models estimate the displacement of piezoelectric actuators using the voltage across the actuators.

Different types of mathematical models have been employed for the modelling purpose such as black box models designed through system-identification techniques (Mohammadzaheri et al., 2012a, Mohammadzaheri et al., 2012c). Moreover, a few physics-based models, inspired by physical phenomena, have been introduced to estimate the displacement of piezoelectric actuators. Physics-based models are superior to black box models in terms of offering physically interpretable and small numbers of parameters. Small numbers of
parameter reduces the computation requirement for running the models and increases the potential use for real-time feedback control. Physics-based models are an analogy of either magnetic systems, e.g. the Preisach model (Farsangi and Saidi, 2012), or an analogy of mechanical systems including masses, springs and dampers, e.g. the Kelvin-Voigt model (Yeh et al., 2008). One of the main challenges for physics-based models of piezoelectric actuators is the relatively low accuracy of the model, partially due to insufficient mathematical structure of the models. Accordingly, complementary terms/inputs are presented and justified to be added to a physics-base model constructing an enhanced structure for the model.

The method for identifying the parameters of physics-based models is another primary challenge for these models. In general, the parameters of these models (which are usually nonlinear) are identified by ad-hoc methods.

Hi,
In this research, the Voigt model is enhanced by adding terms containing the extremum values of the displacement and voltage. This new model has nine independent parameters; five parameters of the Voigt model and four parameters of the extremum terms.

Two reduced versions of the introduced model, only using the extremum terms, are also presented in this research:

\[
y(k + 1) = y(k) + \left( \text{sgn}(V(k)) \left( \left\lfloor \frac{V(k)}{b} \right\rfloor \right) - \frac{y(k)}{d(V(k))} \right) t_s. \tag{10}
\]

In short, the Voigt model is enhanced by adding terms containing the extremum values of the piezoelectric voltage or displacement. This new model has nine independent parameters; five parameters of the Voigt model and four parameters of the extremum terms.

PARAMETER IDENTIFICATION

In this paper, available recorded experimental input-output (piezoelectric voltage-piezoelectric displacement) data are used to identify the model parameters (Mohammadzaheri et al., 2012b). The model error is defined as the average of absolute discrepancy for the model output and the real output:

\[
\text{model error} = \frac{\sum_{k=1}^{m} |y_{\text{model}}(k) - y_{\text{output}}(k)|}{m}. \tag{13}
\]

where \(m\) is the number of estimations. The small value of the model error is a sign of the closeness of the model parameters to their correct values. As a result, a ‘cost function’ or model error exists requiring minimisation through fine-tuning the values of the model parameters. This process is a classical optimisation algorithm. In other words, the parameter identification problem is converted into an optimisation problem.

In system identification problems, in order to optimise the cost function, several numerical algorithms have been developed. From the range of algorithms, the Genetic Algorithm (GA) is employed because the GA is a global optimisation algorithm determining the global minimum of the cost function (Haupt and Haupt, 1998).

Modelling Error Approaches

There are two approaches to estimate the model error. In one approach, all the inputs to the model are assumed to be known at a given instant, then the model output at the next instant (step) is estimated, and the output is compared with the real output of the system. The resultant discrepancy is called ‘One Step Prediction’ (OSP) error (Mohammadzaheri et al., 2012a). In the second approach to calculate the model error, after the first estimation, previously estimated value(s) of the system output(s) are used as model input(s) (instead of the measured values). The model error in this approach is called ‘Simulation’ error (Mohammadzaheri et al., 2012a).

In dynamic systems, the current value of the system output(s) depends on its (their) previous value(s). However, the previous values are not measured/available to be fed into the model during long-term simulations. Therefore, for dynamic systems, previously model outputs are used as the model inputs to estimate the output at each instant; this approach is the Simulation approach.

Let us consider a first order discrete model that represents a single-input-single-output (SISO) system:

\[
y_{\text{model}}(k + 1) = f(V(k), y(k)). \tag{14}
\]

After the initial steps, in order to use the OSP approach, the model output to be used in (15) is calculated as:

\[
y_{\text{model}}(k + 1) = f(V(k), y_{\text{output}}(k)), \tag{15}
\]

and for the Simulation approach:

\[
y_{\text{model}}(k + 1) = f(V(k), y_{\text{model}}(k)). \tag{16}
\]

Figure 1 illustrates the difference between the OSP and Simulation approaches. The variables with \(\hat{\text{hat}}\) are estimated values which are different in two approaches as shown in Equations (16) and (17). The value of the model error is found by Equation (13).

Both approaches, presented by Equations (16) and (17), together with Equation (13), are used to find the model error. The model error is to be used both for the purpose of modelling, e.g. parameter identification, and model validation. In modelling, the parameters need to be identified so as to minimise the model error while in model validation, the model error shows the accuracy/validity of the model.

The OSP model error is usually much lower than the Simulation model error, due to the error accumulation phenomenon in the Simulation approach. Nevertheless, the OSP approach is not applicable to validate the models of dynamic systems (Mohammadzaheri et al., 2012a). Therefore, in this research, to validate a model, the simulation approach is employed.

![Figure 1. Estimation of the model error through (a) One Step Prediction (OSP) and, (b) Simulation approaches.](image-url)
**Parameter Identification to Optimisation**

In this research, a Genetic Algorithm (GA) is to be used for parameter identification through minimising the model error, because it is a global optimisation method. (Haupt and Haupt, 1998). Moreover, the original binary GA is employed in which a binary number is assigned to each parameter so as to minimise the model error as presented in Equation (13). This binary number is called a gene. First, a number of bits with the value of 0 or 1 are assigned to each gene. The whole set of genes forms a chromosome, which is a full set of parameters and a probable solution to the optimisation problem. A number of chromosomes are made and tuned to solve the problem. The whole set of chromosomes is called the population and the mechanism for fine-tuning the genes and chromosomes is called evolution (Popov, 2005). The algorithm starts from assigning random values to the bits to form the initial population. Subsequently, the model error for each chromosome (set of parameters) is evaluated, then the chromosomes are evolved; that is, they go through a selection, crossover and mutation process* (Chipperfield and Fleming, 1995, Homaifar et al., 1994), and a new population is generated. This sequence is repeated several times until it is demonstrated that there will be no further improvement through evolution; this happens when the ‘cost function’ does not decrease through more evolutions. Each time that the evolution is repeated it is called iteration. After several iterations, the best chromosome amongst the population (with the lowest model error) is chosen as the solution.

As a subtle point, a gene with m bits may have a value between 0 and \( \sum_{i=0}^{m-1} 2^i \) (equal to \( 2^m - 1 \)), where m is the number of bits in a gene. Initially, an arbitrary binary population is created, and then the population is converted to decimal values. These values may fall obviously outside the constraints of some parameters. In order to address this issue, before evaluation of the chromosomes, the real value of each parameter (presented by a gene) is found by mapping the constraint presented by its associated gene onto its real constraint. For example, the value of a gene \( p_b \) should go through the following mapping to produce its real value in the constraint of \( [p_{min}, p_{max}] \):

\[
p = \frac{p_{max} - p_{min}}{\sum_{i=0}^{m-1} 2^i} \times p_b + p_{min}.
\]

According to Equation (18), as the minimum absolute change of \( p_b \) is 1, the minimum absolute change (MAC) of each parameter is:

\[
MAC = \frac{p_{max} - p_{min}}{\sum_{i=0}^{m-1} 2^i}.
\]

Therefore, as the number of bits increases, the minimum absolute change for the parameters decreases, and consequently the global minima are less likely to be missed out by the algorithm. However, the number of bits is problem dependent initially generated by a random value (Cao and Wu, 1999).

**SOLVING A REAL PROBLEM**

**Experimental Setup and Data Gathering**

The experimental setup consists of a Nek AE0505D44H40 stack piezoelectric actuator. A PHILITECH D20 optical sensor is employed to measure the displacement of the piezoelectric actuator. Both the sensor and actuator are connected to a PC via a dSpace (a DS1104 R&D Controller Board) and a voltage amplifier amplifies the output voltage of the dSpace before being applied to the actuator and after being received by the sensors. The excitation voltage, in different experiments, generates sinusoidal and triangular functions of time. For both aforementioned functions, three frequencies of 1, 10 and 100 Hz were used and the voltage amplitude was within the range of \( \pm 20V \). Each experiment lasted 2 seconds. The data gathered through the excitation by the triangular voltage function at a frequency of 10Hz was used for the model validation. The rest of the data was employed for modelling.

**Modelling: Sampling Time and Parameter Identification**

The sampling time \( t_s \) appearing in the models, presented by Equations (9), (10), (11) and (12), should be defined before identifying the model parameters. The sampling time has a crucial role in models: too large and too short sampling times make the model unable to capture the system dynamics. In this case study, the natural/resonant frequency of the actuator is 34 kHz and driving frequencies up to about \( 1/3 \times 34 \) kHz is possible (Micromechatronics, 2013). Therefore, a dSpace (DS1104 R&D Controller Board) on a PC-compatible computer with a sampling rate of 10 kHz was used to generate the digital driving voltage for the actuator; i.e., an experimental sampling rate every \( 10^{-4} \) s (Mhamadzaheri et al., 2012c). In the literature, the range of sampling time for the models of piezoelectric actuator is between \( 3.3 \times 10^{-3} \) s and \( 10^{-5} \) s (Mhamadzaheri et al., 2012a). Therefore, to find the optimum sampling time for the current model, the modelling was performed for a few sampling times of larger than \( 10^{-4} \) s and the model error was identified for each of them. Table 1 summarises MAEs of the model validation for different sampling times and Figure 2 shows these results. Based on these outcomes, a sampling time of \( 5 \times 10^{-4} \) s was selected for the model. (Miri et al., 2013b).

As explained in Section Parameter Identification to Optimisation, a constraint should be defined for each parameter. Considering the literature (Boukari et al., 2011),

\[
MAC = \frac{p_{max} - p_{min}}{\sum_{i=0}^{m-1} 2^i}.
\]
after trial and error, the parameter constraints were defined as seen in Table 2. The full constraints and minimum absolute changes for each parameter are listed in this table. The OSP model error, explained in Section Modelling Error Approaches, and the experimental data, explained in Section Experimental Setup and Data Gathering, were used during the optimisation/identification process.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Min</th>
<th>Max</th>
<th>Range</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha(\mu)$</td>
<td>100</td>
<td>2500</td>
<td>2400</td>
<td>2.35</td>
</tr>
<tr>
<td>$\beta$</td>
<td>200</td>
<td>1000</td>
<td>800</td>
<td>0.78</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.001</td>
<td>0.01</td>
<td>0.009</td>
<td>8.79</td>
</tr>
<tr>
<td>$b$</td>
<td>5000</td>
<td>200000</td>
<td>195000</td>
<td>190.61</td>
</tr>
<tr>
<td>$p$</td>
<td>0.1</td>
<td>1.2</td>
<td>1.1</td>
<td>0.0011</td>
</tr>
<tr>
<td>$g(\mu)$</td>
<td>5</td>
<td>150</td>
<td>145</td>
<td>0.142</td>
</tr>
<tr>
<td>$h$</td>
<td>0.02</td>
<td>1.6</td>
<td>1.58</td>
<td>0.0015</td>
</tr>
<tr>
<td>$e$</td>
<td>500</td>
<td>1000</td>
<td>500</td>
<td>0.489</td>
</tr>
<tr>
<td>$f$</td>
<td>0.1</td>
<td>1.3</td>
<td>1.2</td>
<td>0.0012</td>
</tr>
</tbody>
</table>
### Table 3: Identified parameters of the models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation (10)</th>
<th>Equation (11)</th>
<th>Equation (12)</th>
<th>Equation (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSP</td>
<td>( \alpha = 672.434 \mu m )</td>
<td>( \alpha = 2414135 \mu m )</td>
<td>( \alpha = 2260.405 \mu m )</td>
<td>( \alpha = 1770.343 \mu m )</td>
</tr>
<tr>
<td></td>
<td>( \beta = 729.423 V )</td>
<td>( \beta = 676810 V )</td>
<td>( \beta = 863.929 V )</td>
<td>( \beta = 563.636 V )</td>
</tr>
<tr>
<td></td>
<td>( \lambda = 0.00725 V^{-1} )</td>
<td>( \lambda = 0.00805 V^{-1} )</td>
<td>( \lambda = 0.00622 V^{-1} )</td>
<td>( \lambda = 0.00950 V^{-1} )</td>
</tr>
<tr>
<td></td>
<td>( b = 95923754 \Omega )</td>
<td>( b = 31272727 \Omega )</td>
<td>( b = 68909.09 \Omega )</td>
<td>( b = 109511.73 \Omega )</td>
</tr>
<tr>
<td></td>
<td>( P = 0.742 )</td>
<td>( P = 0.863 )</td>
<td>( P = 0.8445 )</td>
<td>( P = 0.872 )</td>
</tr>
<tr>
<td></td>
<td>( g = 1421 \mu V^{-1} s^{-1} )</td>
<td>( e = 250.825 m^{-1} s^{-1} )</td>
<td>( g = 60.428 \mu V^{-1} s^{-1} )</td>
<td>( e = 736.694 m^{-1} s^{-1} )</td>
</tr>
<tr>
<td></td>
<td>( h = 0.628 )</td>
<td>( f = 1.252 )</td>
<td>( h = 0.582 )</td>
<td>( f = 1.222 )</td>
</tr>
</tbody>
</table>

### Table 4: Validation errors of the models made through the OSP approach.

<table>
<thead>
<tr>
<th>Model Structure</th>
<th>Equation (10)</th>
<th>Equation (11)</th>
<th>Equation (12)</th>
<th>Equation (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Indicator</td>
<td>MAE</td>
<td>MAX</td>
<td>MAE</td>
<td>MAX</td>
</tr>
<tr>
<td>Error value (( \mu m ))</td>
<td>0.3324</td>
<td>0.9937</td>
<td>0.299</td>
<td>1.02</td>
</tr>
</tbody>
</table>

### Table 5: Validation errors of the models made through the Simulation approach.

<table>
<thead>
<tr>
<th>Model Structure</th>
<th>Equation (10)</th>
<th>Equation (11)</th>
<th>Equation (12)</th>
<th>Equation (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Indicator</td>
<td>MAE</td>
<td>MAX</td>
<td>MAE</td>
<td>MAX</td>
</tr>
<tr>
<td>Error value (( \mu m ))</td>
<td>0.3357</td>
<td>0.9313</td>
<td>0.294</td>
<td>0.973</td>
</tr>
</tbody>
</table>

### Simulation Results and Analysis

The parameters of the model Equations (9), (10), (11) and (12) were identified based on minimising the model error obtained through two aforementioned approaches in Section Modelling Error Approaches: the OSP and Simulation. These parameters are shown in Table 3.

Accuracy of the new model(s) was verified by validating the model for arbitrary inputs. The validation errors for these models are presented in Tables 4 and 5. These tables present two representatives of the model error: the mean of absolute error (MAE) and the maximum of absolute error (MAX).

As seen in Table 4, the model Equation (9) offers the best accuracy of the estimations while according to the model Equation (10), the Voigt model results in the minimum MAE of 0.3324 \( \mu m \). The model Equation (11) decreases the estimation error to 0.299 \( \mu m \) implying that introducing extremum values of the voltage, in the form of new terms, to the Voigt model boosts the model accuracy. However, the influence of adding the extremum terms of the displacement, as presented in Equation (12), does not meaningfully increase...
the accuracy, as shown in Tables 4 and 5, probably due to displacement measurement noises. Nonetheless, the estimation accuracy increases considerably by adding both aforementioned terms including the extrema; as presented in the last columns of Tables 4 and 5. The estimated displacement through the enhanced model of Equation (9) shows a significant improvement of the model accuracy compared with the Voigt model. This accuracy is comparable with similar studies using black box models (Mohammazaheri et al., 2012a).

Furthermore, the comparison between Table 4 and 5, i.e. Equations (11) and (12), shows that there is not a meaningful difference between two introduced modelling approaches: both OSP and Simulation approaches result in models with rather similar accuracies. However, according to Tables 4 and 5, the model Equation (9) made through the OSP approach demonstrates the highest accuracy among the presented models. As the OSP approach is also less computationally demanding, it is recommended for modelling in similar research work.

Real and estimated displacement values of piezoelectric actuators, generated through the Voigt and other three different models, are illustrated in Figures (3)-(6). In these figures, the measured and simulated displacements by the models are graphed using bold dashed and solid lines, respectively. The validation error for each model is shown by narrow dashed lines.

**Figure 3.** Real and estimated displacement values of the piezoelectric actuator made through Equation (10). (Parameter tuning by the GA with minimal model error achieved through the (a) OSP and, (b) Simulation approach.)

**Figure 4.** Real and estimated displacement values of the piezoelectric actuator made through Equation (11). (Parameter tuning by the GA with minimal model error achieved through the (a) OSP and, (b) Simulation approach.)
CONCLUSION

In this paper, improving the accuracy for a physics-based model of piezoelectric actuators was addressed. The model was enhanced through the modification of the model structure. The model structure was modified by adding two terms, including four new parameters, inspired by another physics-based model. The new structure has a total of nine parameters. Subsequently, the parameters were identified through the optimisation method using the genetic algorithm. The proposed parameter identification is not ad-hoc and may be used for other physics-based models.

The models’ errors were defined through two approaches: ‘One-Step-Prediction’ and ‘Simulation’. Both approaches and the new structure were assessed appropriately.

Improving black box models by adding new inputs had been proven to increase the model accuracy. However, physics-based models are popular due to physically interpretable and a small number of parameters in comparison with black box models. Accordingly, a physics-based model was enhanced in this research. The enhanced model, on a rather similar basis with black box models, doubled the estimation accuracy.

REFERENCES


*Note: Evolution encompasses three major stages: Selection, Crossover and Mutation: In the Selection stage, the chromosomes are selected to be mated. Roulette Wheels,
Tournament, Boltzmann, Ranking and Steady State are different strategies of Selection. In all methods, the Selection deals with the chromosomes according to their cost function. In this research, a Roulette Wheel is employed which is one of the most famous methods used to select the parents. In this method, each chromosome is represented by a slice proportional to the chromosome’s ‘cost function’. In this method, over a long period of time, the least cost functions are selected (Cao and Wu, 1999). Crossover is an important stage in which new generations are created. The crossover point is selected in the opted parents. This point is a random place in the string in which the values of two strings are replaced with each other. Different methods of crossover include single-point, two-point, multi point, uniform and matrix methods, while there is no difference among them (Chipperfield and Fleming, 1995). In this research, the single-point method has been adopted for Crossover. At the Mutation stage, each point in the string could mutate with a random probability but this probability is usually low. A mutation operator changes the bit code from ‘0’ to ‘1’ or vice-versa (Fraser, 1960).