

# **Staggered and Non-Staggered Time-Domain Meshless Radial Point Interpolation Method in Electromagnetics**

by

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*To my parents  
and my husband, Ali  
with all my love.*



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# Abstract

Meshless methods have gained attention recently as a new class of numerical methods for the solution of partial differential equations in various disciplines of computational engineering. This class of methods offers several promising features compared to mesh-based approaches. The principle of domain discretization with arbitrary node distributions allows accurate modeling of complex geometries with fine details. Moreover, an elaborate and time-consuming re-meshing in the grid-based methods can be replaced in meshless counterparts by an adaptive node refinement during the simulation. This can be exploited to enhance solution accuracy or in optimization procedures.

In this thesis, the meshless Radial Point Interpolation Method (RPIM) is investigated for application in time-domain computational electromagnetics. The numerical algorithm is based on a combination of locally defined radial and polynomial basis functions and yields a highly accurate local interpolation of field values and associated derivatives based on the values at close neighboring positions. These interpolated partial derivatives are used to solve the partial differential equations.

The thesis is firstly focused on the staggered meshless RPIM. The classical implementation of the staggered meshless RPIM in electromagnetics using the first-order Maxwell's curl equations is described and the update equations for the staggered electric and magnetic fields are shown. To enhance the capability of the algorithm, a novel implementation of the Uniaxial Perfectly Matched Layer (UPML) is introduced. It is shown however that UPML has intrinsically a long-time instability. Therefore, to avoid this instability two loss terms are introduced, which are added to the update equations in the UPML region after almost all the energy from the computational domain is absorbed. Various capabilities of the meshless method are then validated through different numerical examples using staggered node arrangements in the staggered meshless RPIM. However, the generation of a dual node distribution can be computationally costly and restricts the freedom of node positions, which might reduce the potential advantages of the scheme.

To overcome this challenge, the thesis next proposes a novel non-staggered algorithm for the meshless RPIM based on a magnetic vector potential technique. In this method instead of solving Maxwell's curl equations for the electric and magnetic fields, the

wave equation for the magnetic vector potential is solved. Therefore, a single set of nodes can be used to discretize the computational domain. Importantly in the proposed implementation, solving the second-order vector potential wave equation intrinsically enforces the divergence-free property of the electric and magnetic fields and the computational effort associated with the generation of a dual node distribution is avoided. In this part of the thesis, a hybrid algorithm is further proposed to implement staggered perfectly matched layers in the non-staggered RPIM framework. The properties of the proposed non-staggered RPIM are evaluated through several numerical examples both in 2D and 3D implementations.

In the last part of the thesis, the staggered and non-staggered implementations of meshless RPIM are directly compared in terms of efficiency and accuracy. It is shown that the non-staggered meshless RPIM not only bypasses the requirement of the dual node distribution, but also suppresses the spurious solutions observed in the staggered implementation.

The results of this research show the capability of meshless RPIM for being used efficiently in time-domain computational electromagnetics.

# Statement of Originality

I certify that this work contains no material, which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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# Thesis Conventions

The following conventions have been adopted in this Thesis:

## Typesetting

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This document was compiled using L<sup>A</sup>T<sub>E</sub>X2<sub>ε</sub>. Texmaker and TeXstudio were used as text editor interfaced to L<sup>A</sup>T<sub>E</sub>X2<sub>ε</sub>. Inkscape was used to produce schematic diagrams and other drawings.

## Referencing

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Referencing and citation style in this thesis are based on the Institute of Electrical and Electronics Engineers (IEEE) Transaction style.

## System of units

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The units comply with the international system of units recommended in an Australian Standard: AS ISO 1000–1998 (Standards Australia Committee ME/71, Quantities, Units and Conversions 1998).

## Spelling

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American English spelling is adopted in this thesis.



# Publications

## Book Chapter

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1. C. Fumeaux, T. Kaufmann, Z. Shaterian, D. Baumann, and M. Klemm, *Conformal and Multi-Scale Time-Domain Methods: From Unstructured Meshes to Meshless Discretisations*. Chapter 6 in *Computational Electromagnetics Retrospective and Outlook: In Honor of Wolfgang J. R. Hoefer*, Springer, 2015.\*

## Journal Articles

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1. Z. Shaterian, T. Kaufmann, and C. Fumeaux, "Time-domain vector potential technique for the meshless radial point interpolation method," *International Journal for Numerical Methods in Engineering*, 2015 (in print).\*
2. A. K. Horestani, J. Naqui, Z. Shaterian, D. Abbott, C. Fumeaux, and F. Martín, "Two-dimensional alignment and displacement sensor based on movable broadside-coupled split ring resonators," *Sensors and Actuators A: Physical*, vol. 210, pp. 18–24, 2014.

## Conference Articles

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1. Z. Shaterian, T. Kaufmann, and C. Fumeaux, "On the choice of basis functions for the meshless radial point interpolation method with small local support domains," in *International Conference on Computational Electromagnetics (iCCEM)*, Hong Kong, 2-5 February 2015.\*
2. Z. Shaterian, A. K. Horestani, and C. Fumeaux, "Rotation sensing based on the symmetry properties of an open-ended microstrip line loaded with a split ring resonator," in *German Microwave Conference (GeMiC)*, Nuremberg, Germany, 16-18 March 2015.

3. Z. Shaterian, T. Kaufmann, and C. Fumeaux, "Hybrid staggered perfectly matched layers in non-staggered meshless time-domain vector potential technique," in *International Workshop on Antenna Technology (iWAT)*, Sydney, Australia, 4-6 March 2014, pp. 408–411.\* (Best Student Paper Award)
4. —, "First- and second-order meshless radial point interpolation methods in electromagnetics," in *1st Australian Microwave Symposium (AMS)*, Melbourne, Australia, 26-27 June 2014.\* (Best Student Paper Award)
5. —, "On the staggered and non-staggered time-domain meshless radial point interpolation method," in *17th International Symposium on ElectroMagnetic Compatibility (CEM)*, Clermont-Ferrand, France, 30 June- 3 July 2014.\* (Best Paper Award)
6. Z. Shaterian, A. K. Horestani, and C. Fumeaux, "Metamaterial-inspired displacement sensor with high dynamic range," in *International Conference on Metamaterials, Photonic Crystals and Plasmonics (META)*, United Arab Emirates, 2013.
7. Z. Shaterian, T. Kaufmann, and C. Fumeaux, "On the late-time instability of perfectly matched layers in the meshless radial point interpolation method," in *Asia-Pacific Microwave Conference (APMC) Proceedings*, Seoul, Korea, 5-8 November 2013, pp. 845–847.\*
8. —, "Impact of different node distributions on the meshless radial point interpolation method in time-domain electromagnetic simulations," in *Asia-Pacific Microwave Conference (APMC)*, Kaohsiung, Taiwan, 4-7 December 2012.\*
9. A. K. Horestani, Z. Shaterian, T. Kaufmann, and C. Fumeaux, "Single and dual band-notched ultra-wideband antenna based on dumbbell-shaped defects and complementary split ring resonators," in *German Microwave Conference (GeMiC)*, Nuremberg, Germany, 16-18 March 2015.
10. A. Horestani, Z. Shaterian, and C. Fumeaux, "Application of metamaterial-inspired resonators in compact microwave displacement sensors," in *Australian Microwave Symposium (AMS)*, Melbourne, Australia, 26-27 June 2014.
11. A. K. Horestani, Z. Shaterian, S. Al-Sarawi, D. Abbott, and C. Fumeaux, "Miniaturized bandpass filter with wide stopband using complementary spiral resonator," in *Proc. Asia-Pacific Microwave Conference (APMC)*, Kaohsiung, Taiwan, 4-7 December 2012, pp. 550–552.

12. A. Horestani, Z. Shaterian, S. Al-Sarawi, and D. Abbott, "High quality factor mm-wave coplanar strip resonator based on split ring resonators," in *36th International Conference on Infrared, Millimeter and Terahertz Waves (IRMMW-THz)*, Houston, 2-7 October 2011.
13. A. K. Horestani, Z. Shaterian, W. Withayachumnankul, C. Fumeaux, S. Al-Sarawi, and D. Abbott, "Compact wideband filter element-based on complementary splitting resonators," in *Smart Nano-Micro Materials and Devices*. International Society for Optics and Photonics, 2011.
14. Z. Shaterian and M. Ardebilipour, "Direct sequence and time hopping ultra wide-band over IEEE.802.15.3a channel model," in *16th International Conference on Software, Telecommunications and Computer Networks (SoftCOM)*, 2008, pp. 90–94.

Note: Articles with an asterisk (\*) are directly relevant to this thesis.



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