Shihua Li, Haibo Du, Peng Shi

Distributed attitude control for multiple spacecraft with communication delays

IEEE Transactions on Aerospace and Electronic Systems, 2014; 50(3):1765-1773

Distributed attitude control for multiple spacecraft with communication delays

Shihua Li, *Senior Member, IEEE,* Haibo Du, *Member, IEEE,* Peng Shi, *Senior Member, IEEE*

***Abstract*—This paper considers the attitude synchronization problem for a group of spacecraft in the presence of communication delays. Based on the backstepping control and finite-time control techniques, a novel non-smooth distributed cooperative attitude control algorithm is proposed for multiple spacecraft with attitude described by quaternion. Rigorous proof shows that attitude synchronization can be achieved asymptotically under the proposed control law if the communication topology graph among the spacecraft is strongly connected. Finally, a simulation example is given to demonstrate the efficiency of the proposed method.**

***Index Terms*—Attitude synchronization, Multiple spacecraft, Communication delays, Finite-time control.**

I. Introduction

In recent years, the consensus problem of multi-agents systems has attracted a great deal of interest, see for example [8], [9], [10]. As an important application of consensus, the attitude coordination control of multiple spacecraft has been a hot topic in this area. The interest is motivated by its many different types of applications, such as formation flying [6], [7], space-based interferometry, etc. However, it is well known that the attitude dynamics of the spacecraft are coupled and highly non-linear, which is the main obstruction to design a high precision attitude control law.

Usually, the attitude coordination control is cataloged as centralized coordination control and decentralized coordination control. Compared with the centralized coordination control, the decentralized coordination control often achieves more benefits, such as greater efficiency, higher robustness, and less communication requirement [9], [10]. Considering these benefits, many distributed cooperative control algorithms have been developed for linear multi-agent systems [8], [11], [12] and multiple spacecraft attitude systems. In [13], two distributed control strategies were proposed to guarantee attitude synchronization under a ring communication graph. In [14], [15], the results of [13] were extended to a more general communication graph. In [16], the attitude synchronization algorithm by using only relative attitude and relative angular velocity information was developed. In the case that the angular velocity is unmeasurable, the attitude synchronization problem was also considered in [13], [17], [18]. When there exist a single leader or multiple leaders, the cooperative attitude tracking control problem was discussed in [18], [19], respectively. Recently, in order to enhance the convergence speed and robustness to uncertainties and disturbances, the finite-time control technique [20], [21] has been employed to solve the attitude control problem for spacecraft and the consensus problem for multi-agent systems. For example, in [22], [23], the finite-time consensus problem for linear multi-agent systems were discussed. In [24], [3], [4], [5] the finite-time attitude stabilization problem for a single spacecraft was investigated. In [3], [25], [2], the finite-time cooperative attitude tracking control problem for a group of spacecraft with a single leader or multiple leaders was solved, respectively.

Note that all the preceding listed literature on attitude coordination control does not consider the effect of communication delays. Usually, for the multi-agent network, the communication delays between agents are unavoidable. These delays may deteriorate the system performance and may even cause instability. For multi-agent systems with linear dynamics, this issue has been extensively studied and a number of interesting results have been reported in [8], [26], [27], [28], [29], [30], [1], to name just a few. However, it is not straightforward to extend these consensus algorithms from linear model to spacecraft model. The difficulty lies in the nonlinear characteristics of rigid spacecraft. In literature, there are very few results on attitude synchronization problem in the presence of communication delays. In [31], using a variable structure control method, a robust cooperative attitude tracking control law with communication delays was presented. Later, in [32], a continuous robust attitude tracking control algorithm was proposed. Recently, the authors of [33] also considered the attitude cooperative tracking control problems for multiple spacecraft in the presence of communication delays. It should be pointed out that all the aforementioned results [31], [32], [33] only discussed the cooperative attitude tracking control problem. A common assumption in [31], [32], [33] is that each follower needs to keep communication with the leader. When there is no external reference state or leader, i.e., leaderless multiple spacecraft systems, these attitude tracking algorithms can not be applied again. To this end, in [34], a delayed attitude synchronization was proposed for leaderless multiple spacecraft. However, in [34], only kinematic model of attitude system is considered and the control law is local rather than global.

Shihua Li and Haibo Du are with School of Automation, Southeast University, Nanjing 210096, China. Haibo Du is also with the School of Electrical Engineering and Automation, Hefei University of Technology, Hefei 230009, China. Peng Shi is with School of Engineering and Science, Victoria University, Melbourne, VIC 8001, Australia, and he is also with School of Electrical and Electronic Engineering, The University of Adelaide, SA 5005, Australia

This work was supported by Natural Science Foundation of China (61074013,61304007), Program for New Century Excellent Talents in University (NCET-10-0328), Science Foundation for Distinguished Young Scholars of Jiangsu Province (BK20130018), and Natural Science Foundation of Anhui Province (1308085QF106).

Corresponding author. E-mail: lsh@seu.edu.cn

In this paper, we concentrate on the design of global attitude synchronization algorithm for a group spacecraft with dynamic models when there exist communication delays. Based on the backstepping control and finite-time control techniques, the global controller design is divided into two steps. Specifically, for the kinematic subsystem, a virtual angular velocity is first designed such that the attitude synchronization can be achieved in the presence of communication delays. Then, for the dynamic subsystem, a finite-time control law is designed for the control torque such that the virtual angular velocity can be tracked in a finite time.

The main contribution of this paper is that the proposed attitude synchronization algorithm can be applied to the leaderless multiple spacecraft systems in the presence of communication delays. Rigorous global stability analysis shows that the attitude synchronization can be achieved asymptotically. So far, to the best of authors' knowledge, there is no global attitude synchronization result for leaderless multiple spacecraft systems with communication delays.

Moreover, note that we employ finite-time control techniques to get a faster convergence rate for the tracking of angular velocity to virtual angular velocity. The reason of using finite-time control is of interest because the systems with finite-time convergence demonstrate some nice features such as faster convergence as well as better robustness and disturbance rejection properties [20], [24], [35]. By regulating the additional parameter, i.e., the fractional power, the disturbance rejection performance of closed-loop system can be enhanced. It is known that conventional disturbance analysis results usually shows that to reduce the bounds of steady output errors, one needs to increase the control gains to be sufficient larger. However, high gain feedback control system often exhibits instability in the actual operation. The proposed method of this paper can reduce the bounds of steady output errors without increasing the control gains.

II. Preliminaries and problem formulation

In this paper, let $P > 0$ denote a symmetric positive definite matrix P. Let $\lambda_{max}(P)$ and $\lambda_{min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. Define $\text{sig}^{\alpha}(x) = \text{sign}(x)|x|^{\alpha}$, where $\alpha > 0$, $x \in R$ and $\text{sign}(\cdot)$ is the standard signum function. If $x = [x_1, x_2, \cdots, x_n]^T$ is a vector, then $\text{sig}^{\alpha}(x) = [\text{sig}^{\alpha}(x_1), \text{sig}^{\alpha}(x_2), \cdots, \text{sig}^{\alpha}(x_n)]^T$.

Next, let us review some concepts about graph theory.

A. Graph theory

Assume that the information exchange of n spacecraft is modelled by a directed graph $G(A) = \{V, E, A\}$. $V = \{v_i, i = 1, \cdots, n\}$ is the set of nodes, $E \subseteq V \times V$ is the set of edges and $A = [a_{ij}] \in R^{n\times n}$ is the weighted adjacency matrix of the graph $G(A)$ with non-negative adjacency elements a_{ij}. The node indexes belong to a finite index set $\Gamma = \{1, \cdots, n\}$. Assume that the adjacency elements associated with the edges of the digraph are positive, i.e., $a_{ij} > 0 \Leftrightarrow (v_j, v_i) \in E$. Moreover, we assume that $a_{ii} = 0$ for all $i \in \Gamma$. The set of neighbors of agent i is denoted by $N_i = \{j : (v_j, v_i) \in E\}$.

If there is an edge from agent i to agent j, i.e., $(v_i, v_j) \in E$, then there exists an available information channel from agent i to agent j. A path in directed graph $G(A)$ from v_{i_1} to v_{i_k} is a sequence of $v_{i_1}, v_{i_2}, \cdots, v_{i_k}$ of finite nodes starting with v_{i_1} and ending with v_{i_k} such that $(v_{i_l}, v_{i_{l+1}}) \in E$ for $l = 1, 2, \cdots, k-1$. If $(v_i, v_j) \in E$, then v_i is called the parent node of v_j and v_j is called the child node of v_i. The directed graph G is strongly connected if there exists a path between any two distinct vertices. The out-degree of node v_i is defined as $\deg_{out}(v_i) = d_i = \sum_{j=1}^{n} a_{ij} = \sum_{j\in N_i} a_{ij}$. Then the degree matrix of digraph G is $D = \text{diag}\{d_1, \cdots, d_n\}$ and the Laplacian matrix of digraph G is $L = D - A$.

B. Spacecraft attitude kinematics and dynamics

The spacecraft attitude can be described by two sets of equations, namely, the kinematic equation which relates the time derivatives of the angular coordinates to the angular velocity vector, and the dynamic equation which describes the evolution of the velocity vector. Note that quaternion uses the least possible number of parameters (four parameters) to represent orientation globally. Therefore, we use quaternion to describe spacecraft attitude in this paper. Without loss of generality, assume that there exist n spacecraft in this paper. The communication topology among these n spacecraft is described by the directed graph $G(A)$. Let $\Gamma = \{1, \cdots, n\}$.

As in [36], [37], the dynamic equation of i-th spacecraft can be described by

$$J_i\dot{\omega}_i = s(\omega_i)J_i\omega_i + \tau_i, \quad i \in \Gamma, \tag{1}$$

where $J_i = J_i^T$ is the positive definite inertia matrix, $\omega_i = [\omega_{i,1}, \omega_{i,2}, \omega_{i,3}]^T$ is the angular velocity vector, $\tau_i = [\tau_{i,1}, \tau_{i,2}, \tau_{i,3}]^T$ is the control torque vector, and $s(\omega_i)$ is the following matrix $s(\omega_i) = \begin{bmatrix} 0 & \omega_{i,3} & -\omega_{i,2} \\ -\omega_{i,3} & 0 & \omega_{i,1} \\ \omega_{i,2} & -\omega_{i,1} & 0 \end{bmatrix}$.

The kinematic equation of i-th spacecraft can be described as follows

$$\dot{q}_i = \frac{1}{2}E(q_i)\omega_i, \quad i \in \Gamma, \tag{2}$$

where $q_i = [q_{i,0}, q_{i,1}, q_{i,2}, q_{i,3}]^T = [q_{i,0}, q_{i,v}^T]^T$ is unit quaternion, and

$$E(q_i) = \begin{pmatrix} -q_{i,v}^T \\ -s(q_{i,v}) + q_{i,0}I_3 \end{pmatrix},$$

where I_3 denotes the 3×3 identity matrix.

Actually, for $i \in \Gamma$, let Φ_i denote the principal angle and $e_i = [e_{i,1}, e_{i,2}, e_{i,3}]^T$ denote the principal axis associated with Euler's Theorem with $e_i^T e_i = 1$. Then the quaternion can be defined as

$$q_{i,0} = \cos\frac{\Phi_i}{2}, \quad q_{i,v} = e_i \sin\frac{\Phi_i}{2}. \tag{3}$$

From (3), we obtain

$$q_{i,0}^2 + q_{i,v}^T q_{i,v} = 1. \tag{4}$$

Furthermore, we have $E^T(q_i)E(q_i) = I_3$.

C. Control objectives

The goal of this paper is to deign a distributed attitude control law for a group of spacecraft in the presence of communication delays. The communication topology among spacecraft is modelled by a directed graph. The distributed law is based on the local state information from itself and its neighbors. Rigorous stability proof will show that under the proposed distributed attitude control law, the attitude synchronization can be achieved.

D. Some lemmas

Lemma 1. [20] Consider the following system

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in R^n, \tag{5}$$

where $f(\cdot) : R^n \to R^n$ is a continuous function. Suppose there exist a positive definite continuous function $V(x) : U \to R$, real numbers $c > 0$ and $\alpha \in (0,1)$, and an open neighborhood $U_0 \subset U$ of the origin such that $\dot{V}(x) + c(V(x))^\alpha \leq 0, x \in U_0 \backslash \{0\}$. Then $V(x)$ approaches 0 in finite time. In addition, the finite settling time T satisfies that $T \leq \frac{V(x(0))^{1-\alpha}}{c(1-\alpha)}$.

Lemma 2. [38] For $x_i \in R, i = 1, \cdots, n$, $0 < p \leq 1$, then $(|x_1| + \cdots + |x_n|)^p \leq |x_1|^p + \cdots + |x_n|^p$.

Lemma 3. [39] Let $c, d > 0$. For any $\gamma > 0$, the following inequality holds for $\forall x, y \in R$: $|x|^c|y|^d \leq \frac{c}{c+d}\gamma|x|^{c+d} + \frac{d}{c+d}\gamma^{-c/d}|y|^{c+d}$.

Lemma 4. [22] If a directed graph G is strongly connected, then there exists a positive column vector $\gamma = [\gamma_1, \cdots, \gamma_n]^T \in R^n$ such that $\gamma^T L = 0$, where L the corresponding Laplacian matrix L of graph G.

III. Main results

In this section, the attitude synchronization problem in the presence of communication delays is investigated. Let $T_{ij} > 0$ represent the communication delay from agent j to agent i. The controller design method can be regarded as a integration with the backstepping control and finite-time control methods. Specifically speaking, the design procedure is divided into two steps:

- For kinematic subsystem (2), using the backstepping control idea and considering ω_i as the virtual input, a virtual angular velocity ω_i^* is designed such that the attitudes of kinematic subsystem achieve consensus.
- For dynamic subsystem (1), using the finite-time control technique, a control law is designed such that the virtual velocity ω_i^* can be tracked by the real angular velocity ω_i in a finite time.

A. Virtual angular velocity design

Proposition 1. Consider the kinematic subsystem (2). If the directed graph $G(A)$ is strongly connected and the virtual angular velocity is designed as

$$\omega_i^*(t) = -k_2 \left(\sum_{j \in N_i} a_{ij}[q_{i,v}(t) - q_{j,v}(t - T_{ij})] \right), i \in \Gamma, \tag{6}$$

where $k_2 > 0$, then the attitude synchronization can be achieved asymptotically.

Proof. Substituting control law (6) into subsystems (2) yields

$$\dot{q}_i(t) = -\frac{k_2}{2} E(q_i) \left(\sum_{j \in N_i} a_{ij}[q_{i,v}(t) - q_{j,v}(t - T_{ij})] \right), \quad i \in \Gamma. \tag{7}$$

According to Lemma 4, if the directed graph $G(A)$ is strongly connected, there exists a positive column vector $\gamma = [\gamma_1, \cdots, \gamma_n]^T \in R^n$ such that $\gamma^T L = 0$. Consider the following candidate Lyapunov function

$$V(t) = \sum_{i=1}^n \gamma_i [2 - 2q_{i,0}(t)] + \frac{k_2}{2} \sum_{i=1}^n \gamma_i \sum_{j \in N_i} a_{ij} \int_{t-T_{ij}}^t q_{j,v}^T(\tau) q_{j,v}(\tau) d\tau. \tag{8}$$

According to (2) and the definition of $E(q_i)$, the derivative of $2 - 2q_{i,0}(t)$ with respect to t is

$$\begin{aligned} \frac{\mathrm{d}[2 - 2q_{i,0}(t)]}{\mathrm{d}t} &= q_{i,v}^T(t)\omega(t) \\ &= -k_2 q_{i,v}^T(t) \left(\sum_{j \in N_i} a_{ij}[q_{i,v}(t) - q_{j,v}(t - T_{ij})] \right). \end{aligned} \tag{9}$$

Then, taking the derivative of $V(t)$ along (7), yields

$$\begin{aligned} \dot{V}(t) =& -k_2 \sum_{i=1}^n \sum_{j \in N_i} \gamma_i a_{ij} \Big(q_{i,v}^T(t) q_{i,v}(t) - q_{i,v}^T(t) q_{j,v}(t - T_{ij}) \\ & - \frac{1}{2} q_{j,v}^T(t) q_{j,v}(t) + \frac{1}{2} q_{j,v}^T(t - T_{ij}) q_{j,v}(t - T_{ij}) \Big) \\ =& -\frac{k_2}{2} \sum_{i=1}^n \gamma_i \sum_{j \in N_i} a_{ij} (q_{i,v}^T q_{i,v} - q_{j,v}^T q_{j,v}) \\ & - \frac{k_2}{2} \sum_{i=1}^n \gamma_i \sum_{j \in N_i} a_{ij} [q_{i,v}(t) - q_{j,v}(t - T_{ij})]^T \\ & \times [q_{i,v}(t) - q_{j,v}(t - T_{ij})]. \end{aligned} \tag{10}$$

Define $\xi = [q_{1,v}^T q_{1,v}, \cdots, q_{n,v}^T q_{n,v}]^T$. By the definition of L, we obtain $\sum_{j \in N_i} a_{ij}(q_{i,v}^T q_{i,v} - q_{j,v}^T q_{j,v}) = (L\xi)_i$, where $(L\xi)_i$ denotes the i-th element of vector $L\xi$. Since $\gamma^T L = 0$, then

$$\sum_{i=1}^n \gamma_i \sum_{j \in N_i} a_{ij} (q_{i,v}^T q_{i,v} - q_{j,v}^T q_{j,v}) = \gamma^T L \xi = 0. \tag{11}$$

Substituting (11) into (10), we obtain

$$\begin{aligned} \dot{V}(t) =& -\frac{k_2}{2} \sum_{i=1}^n \gamma_i \sum_{j \in N_i} a_{ij} [q_{i,v}(t) - q_{j,v}(t - T_{ij})]^T \\ & \times [q_{i,v}(t) - q_{j,v}(t - T_{ij})] \leq 0. \end{aligned} \tag{12}$$

Hence $V(t)$ is monotonously non-increasing. With this in mind, by noticing that $V(t)$ is bounded from below since

$V(t) \geq 0$, it can be concluded that $\lim_{t\to\infty}\int_0^t \dot{V}(\tau)d\tau = \lim_{t\to\infty}(V(t)-V(0))$ exists and is finite. Meanwhile,

$$\begin{aligned}\ddot{V}(t) =& -k_2\sum_{i=1}^{n}\gamma_i\sum_{j\in N_i}a_{ij}[q_{i,v}(t)-q_{j,v}(t-T_{ij})]^T \\ &\times[\dot{q}_{i,v}(t)-\dot{q}_{j,v}(t-T_{ij})] \\ =& -k_2\sum_{i=1}^{n}\gamma_i\sum_{j\in N_i}a_{ij}[q_{i,v}(t)-q_{j,v}(t-T_{ij})]^T \\ &\times\frac{1}{2}\left([-s(q_{i,v}(t))+q_{i,0}(t)I_3]\omega_i^*(t)\right. \\ &\left.-[-s(q_{j,v}(t-T_{ij}))+q_{j,0}(t-T_{ij})I_3]\omega_j^*(t-T_{ij})\right). \end{aligned} \tag{13}$$

By (4), we know that $q_{i,v}, q_{i,0} (\forall i \in \Gamma)$ are always bounded. As a result, $\ddot{V}(t)$ is bounded, which means that $\dot{V}(t)$ is uniformly continuous. By Barbalat's Lemma, it can be concluded that $\dot{V}(t) \to 0$ as $t\to\infty$. Since $\gamma_i > 0$ and $a_{ij} > 0$ if $j \in N_i$, then we have for all $i\in\Gamma$, $q_{i,v}(t)-q_{j,v}(t-T_{ij}) \to 0, \forall j\in N_i$, as $t\to\infty$. With this in mind and by (7), it can be concluded that $\dot{q}_j(t)\to 0$, $\forall j\in\Gamma$ as $t\to\infty$, which means that $q_{j,v}(t-T_{ij})-q_{j,v}(t)\to 0, \forall j\in N_i$, as $t\to\infty$. Hence, for all $i\in\Gamma$, $q_{i,v}(t)-q_{j,v}(t)\to 0, \forall j\in N_i$, as $t\to\infty$. Since the graph $G(A)$ is strong connected, then there exists a path between any two distinct agents. As a matter of fact, $q_{i,v}(t)-q_{j,v}(t)\to 0$ as $t\to\infty$ for all $i,j\in\Gamma$. In addition, by noticing the constraint condition (4), $q_{i,v}=q_{j,v}$ implies that $q_{i,0}=q_{j,0}$ or $q_{i,0}=-q_{j,0}$. Since quaternions $(q_{i,0}, q_{i,v}^T)^T$ and $(-q_{i,0}, q_{i,v}^T)^T$ represent the same rotation in the physical space [37], the attitude synchronization is achieved asymptotically. □

Remark 1. Using the idea of backstepping method, the angular velocity is taken as a virtual input to design an attitude synchronization algorithm for the kinematic subsystem. Next, we should design a control law for the dynamic subsystem such that the virtual angular velocity can be tracked by the real angular velocity. Different from the conventional backstepping control method, here, the finite-time control technique is used to design the tracking control law. The reason for using this technique is that it can increase the convergence rate and improve the disturbance rejection performance by regulating the additional parameter, i.e., the fractional power.

B. Finite-time control law design

In this section, we first design a finite-time control law such that the angular velocity ω_i can track the virtual angular velocity ω_i^* in a finite time. Then we give a rigorous global stability analysis.

Define

$$\varepsilon_i(t) = (\varepsilon_{i,1}(t), \varepsilon_{i,2}(t), \varepsilon_{i,3}(t))^T = \omega_i - \omega_i^*, i\in\Gamma. \tag{14}$$

Combining (14) with (1) and (2), we have the following error model:

$$\dot{q}_i = \frac{1}{2}E(q_i)\omega_i = \frac{1}{2}E(q_i)\omega_i^* + \frac{1}{2}E(q_i)\varepsilon_i, \tag{15}$$

$$J_i\dot{\varepsilon}_i = s(\omega_i)J_i\omega_i - J_i\dot{\omega}_i^* + \tau_i, \qquad i\in\Gamma. \tag{16}$$

Obviously, systems (15) and (16) can be regarded as a cascaded system. The interconnection term is $\frac{1}{2}E(q_i)\varepsilon_i$. Now, we present the main result.

Theorem 1. Consider the multiple spacecraft system (1)-(2). If the directed graph $G(A)$ is strongly connected and the control torque τ_i is chosen as

$$\begin{aligned}\tau_i(t) =& -s(\omega_i(t))J_i\omega_i(t) \\ &- k_1\mathrm{sig}^\alpha\left(\omega_i(t)+k_2\sum_{j\in N_i}a_{ij}(q_{i,v}(t)-q_{j,v}(t-T_{ij}))\right) \\ &- k_2J_i\sum_{j\in N_i}a_{ij}(\dot{q}_{i,v}(t)-\dot{q}_{j,v}(t-T_{ij})), i\in\Gamma, \end{aligned}\tag{17}$$

where $k_1 > 0$, $k_2 > 0$, $0 < \alpha < 1$, then the attitude synchronization can be achieved asymptotically.

Proof. The proof procedure can be divided into two steps. Firstly, we prove that the virtual angular velocity can be tracked by the real angular velocity in a finite time under control law (17). Then, we show that the attitude synchronization can be achieved under control law (17).

Step 1. In this step, we will show that the virtual angular velocity ω_i^* can be tracked by the real angular velocity $\omega_i, i\in\Gamma$ in a finite time under control law (17). In other words, we should prove that the error states ε_i will converge to zero in a finite time under control law (17).

Substituting control law (17) into system (16) yields

$$J_i\dot{\varepsilon}_i = -k_1\mathrm{sig}^\alpha(\varepsilon_i), \qquad i\in\Gamma. \tag{18}$$

Without loss of generality, let us first consider the i-th ($i\in\Gamma$) agent. Consider the Lyapunov function

$$V_i(\varepsilon_i) = \frac{1}{2}\varepsilon_i^T J_i\varepsilon_i.$$

Taking the derivative of $V_i(\varepsilon)$ along (18) yields

$$\begin{aligned}\dot{V}_i(\varepsilon_i) &= \varepsilon_i^T J_i\dot{\varepsilon}_i = -k_1\varepsilon_i^T\mathrm{sig}^\alpha(\varepsilon_i) \\ &= -k_1(|\varepsilon_{i,1}|^{1+\alpha}+|\varepsilon_{i,2}|^{1+\alpha}+|\varepsilon_{i,3}|^{1+\alpha}). \end{aligned}\tag{19}$$

By Lemma 2, $|\varepsilon_{i,1}|^{1+\alpha}+|\varepsilon_{i,2}|^{1+\alpha}+|\varepsilon_{i,3}|^{1+\alpha} \geq (\varepsilon_{i,1}^2+\varepsilon_{i,2}^2+\varepsilon_{i,3}^2)^{(1+\alpha)/2}$. Note that $V_i(\varepsilon_i) = \frac{1}{2}\varepsilon_i^T J_i\varepsilon_i \leq \frac{1}{2}J_{i,max}\varepsilon_i^T\varepsilon_i$, where $J_{i,max} = \lambda_{max}(J_i)$. Hence, we have

$$\begin{aligned}\dot{V}_i(\varepsilon_i) \leq -k_1(\varepsilon_i^T\varepsilon_i)^{\frac{1+\alpha}{2}} &\leq -k_1(2/J_{i,max})^{\frac{1+\alpha}{2}}(V_i(\varepsilon_i))^{\frac{1+\alpha}{2}} \\ &= -k_1c(V_i(\varepsilon_i))^{\frac{1+\alpha}{2}}, \end{aligned}\tag{20}$$

where $c = (2/J_{i,max})^{\frac{1+\alpha}{2}}$. By Lemma 1, $V_i(\varepsilon_i)$ reaches zero in finite time, which implies that there exists a time $T_i = \frac{V_i(\varepsilon_i(0))^{(1-\alpha)/2}}{k_1c(1-\alpha)/2}$, such that $V_i(\varepsilon_i)(t)=0$, i.e., $\varepsilon_i(t)=0$, $\forall t\geq T_i$. Define $T^* = \max\{T_1,\cdots,T_n\}$. Then, we have $\varepsilon_i(t)=0$, when $t\geq T^*$, $\forall i\in\Gamma$. In other words, after the time T^*, $\omega_i=\omega_i^*$, for all $i\in\Gamma$.

Step 2. In this step, we will prove that the attitude synchronization can be achieved asymptotically under control law (17).

By Step 1, we know that under control law (17), when $t\geq T^*$, $\omega_i=\omega_i^*$, $\forall i\in\Gamma$. Combining the results of Proposition 1, once $\omega_i\equiv\omega_i^*$, then attitude synchronization can be achieved asymptotically. Note that we do not discuss the state trajectories in the time interval $[0, T^*]$. In what follows, we will prove that the states $(q_i(t), \omega_i(t))(\forall i \in \Gamma)$ are bounded in the interval $[0, T^*]$.

First, the state $q_i(\forall i \in \Gamma)$ is always bounded due to the constraint condition (4), which implies that $\omega_i^* = -k_2\left(\sum_{j\in N_i} a_{ij}[q_{i,v}(t) - q_{j,v}(t-T_{ij})]\right)(\forall i \in \Gamma)$ is always bounded.

Second, from (20), we have $\dot{V}_i(\varepsilon_i(t)) \le 0$ for any t, $\forall i \in \Gamma$. It implies that

$$\frac{1}{2}J_{min,i}\varepsilon_i^T(t)\varepsilon_i(t) \le \frac{1}{2}\varepsilon_i^T(t)J_i\varepsilon_i(t) = V_i(\varepsilon_i(t)) \le V_i(\varepsilon_i(0)) = \frac{1}{2}\varepsilon_i^T(0)J_i\varepsilon_i(0), \forall t \ge 0, \forall i \in \Gamma, \tag{21}$$

where $J_{min,i} = \lambda_{min}(J_i)$. That is to say that the error state $\varepsilon_i(\forall i \in \Gamma)$ is always bounded. By noticing that $\omega_i = \varepsilon_i + \omega_i^*$ and ω_i^* is bounded, then $\omega_i(\forall i \in \Gamma)$ is always bounded. Hence, the states $(q_i(t), \omega_i(t))(\forall i \in \Gamma)$ are always bounded.

Therefore, by the results of Steps 1-2, we can conclude that under control law (17), $q_i \to q_j$ as $t \to \infty$. Moreover, by the definition of ω_i^*, $q_i = q_j, \forall i, j \in \Gamma$, implies that $\omega_i^* = 0$. Thus, $\omega_i = \omega_i^* \to 0$ as $t \to \infty$. □

Remark 2. Note that here the finite-time control technique is employed to design the tracking control law such that the virtual angular velocity can be tracked in a finite time. From the proof procedure, we know that the explicit expression for the finite settling time is bounded by $T_i = \frac{V_i(\varepsilon_i(0))^{(1-\alpha)/2}}{k_1 c(1-\alpha)/2}$. Clearly, increasing gain $k_1(k_1 > 0)$ will enhance the convergence speed. Moreover, decreasing fractional power $\alpha(0 < \alpha < 1)$ will enhance the convergence speed of closed-loop system as well. Besides faster convergence rate, the closed-loop systems under finite-time control usually demonstrate better disturbance rejection properties. Since the finite-time control technique is only employed for designing tracking control law for the dynamic subsystems, a rigorous theoretical analysis of disturbance rejection performance will be given for this subsystems. Assume there exist bounded external disturbances $d_i(t) \in R^3(\|d_i\| \le l < +\infty)$ in the same channel as the control torque. Then it follows from (16) and (19) that $\dot{V}_i(\varepsilon_i) = -k_1\sum_{j=1}^3 |\varepsilon_{i,j}|^{1+\alpha} + \varepsilon_i^T d_i$. By Lemma 3, we obtain $\varepsilon_i^T d_i \le \sum_{j=1}^3 |\varepsilon_{i,j}|(l^{1/\alpha})^\alpha \le \frac{k_1}{1+\alpha}\sum_{j=1}^3 |\varepsilon_{i,j}|^{1+\alpha} + \frac{3\alpha}{(1+\alpha)k_1^{1/\alpha}}l^{(1+\alpha)/\alpha}$, which leads to

$$\dot{V}_i(\varepsilon_i) = -\frac{k_1\alpha}{1+\alpha}\Big(\sum_{j=1}^3 |\varepsilon_{i,j}|^{1+\alpha} - 3(l/k)^{\frac{1+\alpha}{\alpha}}\Big). \tag{22}$$

According to the proof of (20), we have

$$\dot{V}_i(\varepsilon_i) \le -\frac{k_1\alpha}{1+\alpha}\Big(cV_i(\varepsilon_i)^{(1+\alpha)/2} - 3(l/k)^{\frac{1+\alpha}{\alpha}}\Big). \tag{23}$$

Following a similar analysis as that in [35], $V_i(\varepsilon_i)$ will enter the region $V_i(\varepsilon_i) \le \frac{3}{c}\left(\frac{l+\triangle}{k_1}\right)^{2/\alpha}$ and stay there for ever, where $\triangle$ is an arbitrarily small positive constant. Since $V_i(\varepsilon_i) \ge \frac{1}{2}J_{i,min}\varepsilon_i^T\varepsilon_i$, where $J_{i,min} = \lambda_{min}(J_i)$, then

$$|\varepsilon_{i,j}| \le (\varepsilon_i^T\varepsilon_i)^{1/2} \le \frac{6}{cJ_{i,min}}\left(\frac{l+\triangle}{k_1}\right)^{1/\alpha}. \tag{24}$$

Usually, conventional disturbance analysis results shows that to reduce the bounds of steady output errors, one needs to increase the control gains to be sufficiently large. However, high gain feedback control system often exhibits instability in the actual operation. So under the considerations of stability as well as control saturation constraint, the proposed non-smooth control method can reduce the bounds of steady output errors only by adjusting the fractional power α. For example, according to (24) and selecting k_1 such that $k_1 > l + \triangle$, then we can select α to approximate to 0 such that the steady tracking error $|\varepsilon_{i,j}|$ can be made as small as desired. Based on this analysis and noticing $\|E(q_i)\| = 1$, it follows from (15) that the steady output errors for attitude can be rendered as small as possible as well. The numerical simulations in Section 4 will illustrate this statement.

IV. Numerical examples and simulations

Consider a team with four spacecraft described by (1)-(2). The information exchange topology among spacecraft is shown in Fig. 1. The weights of the directed edges are: $a_{13} = a_{21} = a_{31} = a_{34} = a_{42} = 0.5, a_{41} = 1$. The communication delays are $T_{13} = 0.2s, T_{21} = T_{31} = T_{34} = T_{42} = 0.4s, T_{41} = 0.8s$. The inertia matrices of the spacecraft are given as in [19]: $J_1 =$diag$(18, 12, 10)$, $J_2 =$diag$(22, 16, 12)$, $J_3 =$diag$(17, 14, 12)$, $J_4 =$diag$(15, 13, 8)$.

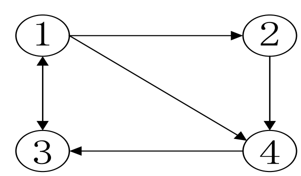

Fig. 1. The information exchange among four spacecraft.

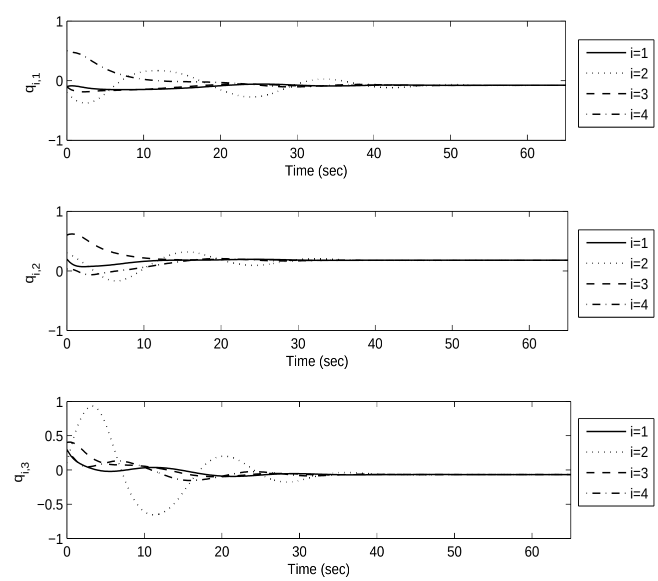

Fig. 2. Attitudes of all spacecraft.

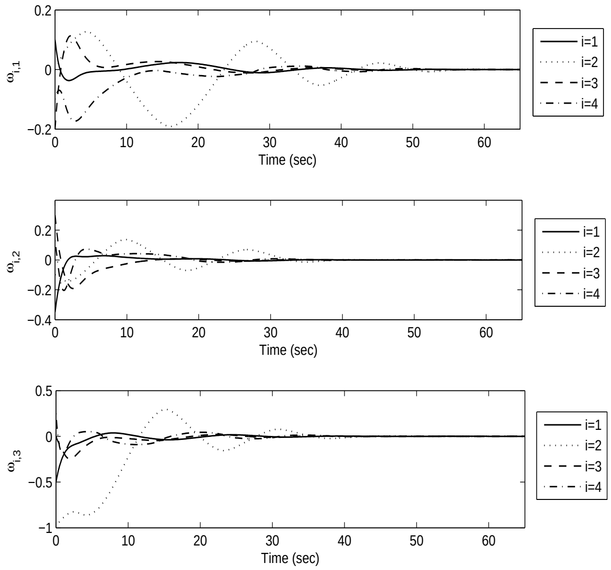

Fig. 3. Angular velocities of all spacecraft.

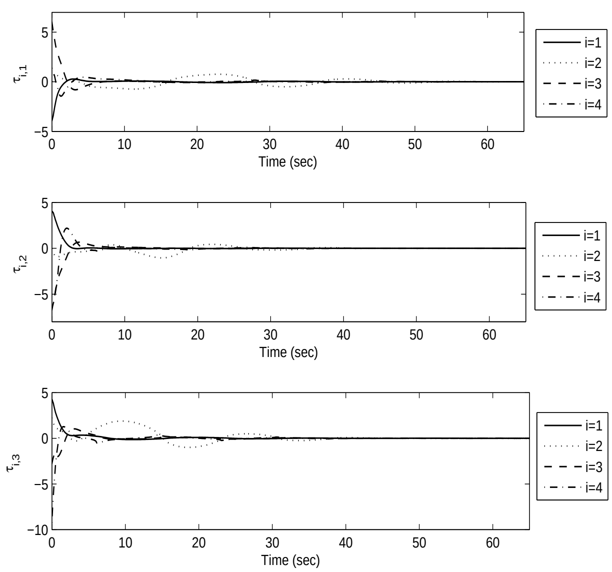

Fig. 4. Control torques of all spacecraft.

Let the control gains of attitude control law (17) be $k_1 = 2, k_2 = 2, \alpha = 2/3$. The initial conditions are randomly selected. Moreover, the control torques are limited not to exceed 10 $\mathrm{N.m}$.

In the absence of of external disturbances, under attitude control law (17), the response curves of the closed-loop system (1)-(2) with (17) are shown in Figs. 2-4. It can be found that the attitudes of each spacecraft converge to the same attitudes and the angular velocities of each spacecraft converge to zero.

Next, in the presence of external disturbances, the disturbance rejection property for the attitude control law (17) is also investigated. As in [31], the following external disturbances are added to each spacecraft system in the same channel as the control torque: $d_1(t) = 0.6\sin(t), d_2(t) = 0.3\cos(2t), d_3(t) = 0.5\sin(1.7t)$. Table 1 shows the steady-state errors for the closed-loop system under the different fractional power α. We can see that by regulating the additional parameter, i.e., the fractional power α, the bounds of steady output errors can be reduced without increasing the control gains k_1, k_2. That is to say that under the considerations of control saturation constraint (the control torques are limited not to exceed 10 $\mathrm{N.m}$), the disturbance rejection performance still can be enhanced.

Table 1. The steady-state errors under the different fractional power α

α	1	3/4	1/2	1/4
Steady-state errors: q_i	0.045	0.03	0.022	0.004
Steady-state errors: ω_i	0.06	0.06	0.045	0.011

V. Conclusion

In this paper, we have discussed the attitude synchronization problems for leaderless multiple spacecraft in the presence of communication delays. By using the backstepping control and finite-time control methods, a global continuous attitude synchronization algorithm has been presented. Future work includes extending the results in this paper to the cases when the communication delay is time-varying.

References

[1] Liu, X., Kumar, K. D.
Network-Based Tracking Control of Spacecraft Formation Flying with Communication Delays. *IEEE Transactions on Aerospace and Electronic Systems,* **48**, 3(2012), 2302-2314.

[2] Zou, A., Kumar, K. D.
Distributed Attitude Coordination Control for Spacecraft Formation Flying, *IEEE Transactions on Aerospace and Electronic Systems,* **48**, 2(2012), 1329-1346.

[3] Du, H., Li, S., Qian, C.
Finite-time attitude tracking control of spacecraft with application to attitude synchronization, *IEEE Transactions on Automatic Control,* **56**, 11(2011), 2711-2717.

[4] Du, H., Li, S.
Finite-time attitude stabilization for a spacecraft using homogeneous method, *Journal of Guidance, Control and Dynamics,* **35**, 3(2012), 740-749.

[5] Zou, A., Kumar, K. D., Hou, Z., Liu, X.
Finite-Time Attitude Tracking Control for Spacecraft Using Terminal Sliding Mode and Chebyshev Neural Network, *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics,* **41**, 4(2011), 950-963.

[6] Ren, W., Beard, R. W.
Decentralized scheme for spacecraft formation flying via the virtual structure approach, *Journal of Guidance, Control, and Dynamics,* **27**, 1(2004), 73-82.

[7] VanDyke, M. C., Hall, C. D.
Decentralized coordinated attitude control within a formation of spacecraft, *Journal of Guidance, Control, and Dynamics,* **29**, 5(2006), 1101-1109.

[8] Olfati-Saber, R., Murray, R.
Consensus problems in networks of agents with switching topology and time-delays, *IEEE Transactions on Automatic Control,* **49**, 9(2004), 1520-1533.

[9] Pettersen, K. Y., Gravdahl, J. T., Nijmeijer, H.
Group coordination and cooperative control, Berlin, Springer, 2006.

[10] Ren, W., Beard, R. W.
Distributed consensus in multivehicle cooperative control: Theory and applications, Berlin, Springer, 2007.

[11] Hong, Y., Hu, J., Gao, L.
Tracking control for multi-agent consensus with an active leader and variable topology, *Automatica,* **42**, 7(2006), 1177-1182.

[12] Hong, Y., Gao, L., Cheng, D., Hu, J.
Lyapunov-based approach to multiagent systems with switching jointly connected interconnection, *IEEE Transaction on Automatic Control,* **52**, 5(2007), 943-948.

[13] Lawton, J. R., Beard, R. W.
Synchronized multiple spacecraft rotations, *Automatica,* **38**, 8(2002), 1359-1364.

[14] Ren, W.
Distributed attitude alignment in spacecraft formation flying, *International Journal of Adaptive Control and Signal Processing,* **21**, 2-3(2007), 95-113.

[15] Sarlettea, A., Sepulchrea, R., Leonard, N. E.
Autonomous rigid body attitude synchronization, *Automatica,* **45**, 2(2009), 572-577.

[16] Bai, H., Arcak, M., Wen, J. T.-Y.
Rigid body attitude coordination without inertial frame information, *Automatica,* **44**, 12(2008), 3170-3175.

[17] Abdessameud, A., Tayebi, A.
Attitude synchronization of a group of spacecraft without velocity measurements, *IEEE Transactions on Automatic Control,* **54**, 11(2009), 2642-2648.

[18] Ren, W.
Distributed cooperative attitude synchronization and tracking for multiple rigid bodies, *IEEE Transactions on Control System Technology,* **18**, 2(2010), 383-392.

[19] Dimarogonas, D. V., Tsiotras, P., Kyriakopoulos, K. J. Leader-follower cooperative attitude control of multiple rigid bodies, *Systems and Control Letters*, **58**, 6(2009), 429-435.

[20] Bhat S. P., Bernstein, D. S.
Finite-time stability of continuous autonomous systems, *SIAM Journal on Control and Optimization,* **38**, 3(2000), 751-766.

[21] Hong, Y., Huang, J., Xu, Y.
On an output feedback finite-time stabilization problem, *IEEE Transaction on Automatic Control,* **46**, 2(2001), 305-309.

[22] Xiao, F., Wang, L., Chen, J., Gao, Y.
Finite-time formation control for multi-agent systems, *Automatica,* **45**, 11(2009), 2605-2611.

[23] Li, S., Du, H., Lin, X.
Finite-time consensus algorithm for multi-agent systems with double-interrator dynamics, *Automatica,* **47**, 8(2011), 1706-1712.

[24] Li, S., Ding S., Li, Q.
Global set stabilisation of the spacecraft attitude using finite-time control technique, *International Journal of Control,* **82**, 5(2009), 822-836.

[25] Meng, Z., Ren, W., You, Z.
Distributed finite-time attitude containment control for multiple rigid bodies, *Automatica,* **46**, 12(2010), 2092-2099.

[26] Wang, W., Slotine, J.J.E.
Contraction analysis of time-delayed communications and group cooperation, *IEEE Transactions on Automatic Control,* **51**, 4(2006), 712-717.

[27] Chopra, N., Spong, M.
Passivity-based control of multi-agent systems, *Advances in Robot Control,* 107-134, Springer, Berlin, Germany, 2006.

[28] Tian, Y., Liu, C.
Consensus of multi-agent systems with diverse input and communication delays, *IEEE Transactions on Automatic Control*, **53**, 9(2008), 2122-2128.

[29] Sun, Y., Wang, L.
Consensus of multi-agent systems in directed networks with nonuniform time-varying delays, *IEEE Transactions on Automatic Control,* **54**, 7(2009), 1607-1613.

[30] Yang, B., Fang, H.
Forced consensus in networks of double integrator systems with delayed input, *Automatica,* **46**, 3(2010), 629-632.

[31] Jin, E., Jiang, X., Sun, Z.
Robust decentralized attitude coordination control of spacecraft formation, *Systems and Control Letters,* **57**, 7(2008), 567-577.

[32] Jin, E., Sun, Z.
Robust attitude synchronisation controllers design for spacecraft formation, *IET Control Theory & Applications,* **3**, 3(2009), 325-339.

[33] Meng, Z., You, Z., Li, G., Fan, C.
Cooperative attitude control of multiple rigid bodies with multiple time-varying delays and dynamically changing topologies, *Mathematical Problems in Engineering*, 2010.

[34] Igarashi, Y., Hatanaka, T., Fujita, M., Spong, M. W.
Passivity-based attitude synchronization in SE(3), *IEEE Transaction on Control Systems Technology,* **17**, 5(2009), 1119-1134.

[35] Li, S., Ding, S., Li, Q.
Global set stabilization of the spacecraft attitude control problem based on quaternion, *International Journal of Robust and Nonlinear Control*, **20**, 1(2010), 84-105.

[36] Wen, J. T., Kreutz-Delgado, K.
The attitude control problem, *IEEE Transactions on Automatic Control,* **36**, 10(1991), 1148-1162.

[37] Shuster, M. D.
A survey of attitude representations, *Journal of the Astronautical Sciences,* **41**, 4(1993), 439-517.

[38] Hardy, G., Littlewood, J., Polya G.
Inequalities, Cambridge University Press, Cambridge, 1952.

[39] Qian, C., Lin, W.
A continuous feedback approach to global strong stabilization of nonlinear systems, *IEEE Transactions on Automatic Control,* **46**, 7(2001), 1061-1079.