

Track-Before-Detect for Active Sonar

Han Xuan Vu

*Thesis submitted for the degree of
Doctor of Philosophy
in
Electrical and Electronic Engineering
at
The University of Adelaide*

School of Electrical and Electronic Engineering



THE UNIVERSITY
of ADELAIDE

February 3, 2015

Abstract

The detection and tracking of underwater targets with active sonar is a challenging problem because of high acoustic clutter, fluctuating target returns and a relatively low measurement update rate. In this thesis, a Bayesian framework for the detection and tracking of underwater targets using active sonar is formulated. In general, Bayesian tracking algorithms are built on two statistical models: the target dynamics model and the measurement model. The target dynamics model describes the evolution of the target state with time and is almost always assumed to be a Markov process. The typical measurement model approximates the sensor image with a collection of discrete points at each frame and allows point measurement tracking to be performed. This thesis investigates alternative target and measurement models and considers their application to active sonar tracking.

The Markov process commonly used for target modelling assumes that the state evolves without knowledge of its future destination. Random realisations of a Markov process can also display a large amount of variability and do not, in general, resemble realistic target trajectories. An alternative is the reciprocal process, which assumes conditioning on a known destination state. The first key contribution is the derivation and implementation of a Maximum Likelihood Sequence Estimator (MLSE) for a Hidden Reciprocal Process (HRP). The performance of the proposed algorithm is demonstrated in simulated scenarios and shown to give improved state estimation performance over Markov processes for scenarios featuring reciprocal targets.

In point measurement tracking, reducing the sensor data to point detections results in the loss of valuable information. This method is generally sufficient for tracking high Signal-to-Noise Ratio (SNR) targets but can fail in the case of low SNR targets. The alternative to point measurement tracking is to provide the sensor intensity map, an image, as an input into the tracker. This paradigm is referred to as Track-Before-Detect (TkBD). This thesis will focus on a particular TkBD algorithm based on Expectation-Maximisation (EM) data association called the Histogram-Probabilistic Multi-Hypothesis Tracker (H-PMHT) as it handles multiple targets with low complexity. In the second key contribution, we demonstrate a Viterbi implementation of the H-PMHT algorithm, and show that it outperforms the Kalman Filter in the linear non-

Gaussian case.

A problem with H-PMHT is that it fails to model fluctuating target amplitude, which can degrade performance in realistic sensing conditions. The third key contribution addresses this by replacing the multinomial measurement model with a Poisson mixture process. The new Poisson mixture is shown to be consistent with the original H-PMHT modelling assumptions but it now allows for a randomly evolving mean target amplitude state with instantaneous fluctuations. This new TkBD algorithm is referred to as the Poisson H-PMHT. The Bayesian prior on the target state is also modified to ensure more robust performance.

The fourth contribution is a novel TkBD algorithm based on the application of EM data association to a new measurement model that directly describes continuous valued intensity maps and avoids using an intermediate quantisation stage like the H-PMHT. This model is referred to as the Interpolated Poisson measurement model and is integrated into the Probabilistic Multi-Hypothesis Tracker (PMHT) framework to derive a TkBD algorithm for continuous data called the Interpolated Poisson-PMHT (IP-PMHT). The performance of the Poisson H-PMHT and IP-PMHT algorithms are verified through simulations and are shown to outperform the standard H-PMHT in terms of SNR estimation, particularly for scenarios featuring targets with highly fluctuating amplitude.

The final key contribution is the application of several TkBD algorithms based on EM data association to the active sonar problem through a comparative study using trial data from an active towed array sonar. The TkBD algorithms are modified to incorporate changes in target appearance with received array bearing, and are shown to give improved SNR and state estimation performance compared with a conventional point measurement tracking algorithm. The thesis concludes by discussing the limitations of the proposed algorithms and possible avenues for future work.

Publications

- H. X. Vu, S. J. Davey, S. Arulampalam, F. K. Fletcher, and C.C. Lim. A new state prior for the Histogram-PMHT. *IEEE Signal Processing Letters*, in preparation
- H. X. Vu, S. J. Davey, S. Arulampalam, F. K. Fletcher, and C.C. Lim. Histogram-PMHT with an evolving Poisson prior. In *2015 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, April 2015
- H. X. Vu, S. J. Davey, F. K. Fletcher, S. Arulampalam, R. Ellem, and C.-C. Lim. Track-before-detect for an active towed array sonar. In *Proceedings of Acoustics 2013*, November 2013
- H. X. Vu, S. J. Davey, S. Arulampalam, F. K. Fletcher, and C.-C. Lim. H-PMHT with a Poisson measurement model. In *Proceedings of the 2013 International Conference on Radar (Radar)*, pages 446–451, September 2013
- H. X. Vu and S. J. Davey. Track-before-detect using Histogram PMHT and dynamic programming. In *Proceedings of the 2012 International Conference on Digital Image Computing Techniques and Applications (DICTA)*, pages 1–8, December 2012
- S. J. Davey, H. X. Vu, S. Arulampalam, F. Fletcher, and C.C. Lim. Clutter mapping for Histogram PMHT. In *2014 IEEE Workshop on Statistical Signal Processing (SSP)*, pages 153–156, Gold Coast, Australia, June-July 2014
- S. J. Davey, M. Wieneke, and H. Vu. Histogram-PMHT unfettered. *IEEE Journal of Selected Topics in Signal Processing*, 7(3):435–447, June 2013
- L. B. White and H. X. Vu. Maximum likelihood sequence estimation for hidden reciprocal processes. *IEEE Transactions on Automatic Control*, 58(10):2670–2674, October 2013

Declaration

I, Han Xuan Vu, certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree. I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

The author acknowledges that copyright of published works contained within this thesis resides with the copyright holder(s) of those works.

I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

Signature..... *Date*.....

Acknowledgements

I would like to acknowledge my phenomenal team of supervisors, Sam Davey, Sanjeev Arulampalam, Fiona Fletcher and Cheng-Chew Lim for their ongoing support and encouragement throughout my candidature. I would also like to express my gratitude for their generosity and willingness to share their valuable time and expertise. Their mentorship, guidance and knowledge have been invaluable. Thank you also to Richard Ellem for the provision of trial data used in this research.

I would like to thank my host institutions the School of Electrical and Electronic Engineering at the University of Adelaide and the Maritime Division of the Defence Science and Technology Organisation for their continued support throughout my candidature.

Finally, a sincere thank you to all my friends and family for their unwavering support and patience through the ups and downs of my candidature. A special mention to my partner Chris for taking the brunt of it, without (too much) complaint.

Contents

List of Figures	xv
List of Tables	xx
List of Acronyms	xxi
List of Principal Symbols	xxiv
1 Introduction	1
1.1 Motivation	2
1.2 Active Sonar Tracking Problem	2
1.3 Thesis Scope and Overview	4
2 Background	9
2.1 The General Tracking Problem	10
2.1.1 Model Order	10
2.1.2 Data Association	11
2.1.3 Filtering	12
2.2 Active Sonar Tracking Problem	15
2.2.1 Conventional Point Measurement Problem	16
2.2.2 Track-Before-Detect Problem	17
2.2.3 Multistatics	20
2.3 Conventional Point Measurement Filtering based on Markov Processes	21
2.3.1 Kalman Filter	21

2.3.2	Extended Kalman Filter	22
2.3.3	Unscented Kalman Filter	23
2.3.4	Particle Filter	23
2.3.5	Random Finite Sets	24
2.4	Hidden Reciprocal Processes	25
2.5	Track-Before-Detect	26
2.5.1	Forward-Backward Algorithm	27
2.5.2	Viterbi Algorithm	28
2.5.3	Likelihood Ratio Detection and Tracking	29
2.5.4	Particle Filter	29
2.5.5	Random Finite Sets	30
2.5.6	Histogram-Probabilistic Multi-Hypothesis Tracker	31
3	Maximum Likelihood Sequence Estimation for Hidden Reciprocal Processes	33
3.1	Introduction	34
3.2	Hidden Reciprocal Processes	35
3.3	Optimal Smoothing	37
3.3.1	Optimal smoothing for Hidden Markov Models: Forward-Backward Algorithm	37
3.3.2	Optimal Smoothing for Hidden Reciprocal Chains	39
3.4	Maximum Likelihood Sequence Estimation	43
3.4.1	MLSE for Hidden Markov Models: Viterbi Algorithm	43
3.4.2	MLSE for Hidden Reciprocal Chains	46
3.5	Simulations	48
3.6	Summary	60
4	Histogram-Probabilistic Multi-Hypothesis Tracker (H-PMHT)	61
4.1	Introduction	62
4.2	Derivation	63
4.2.1	Prior Density	66

4.2.2	Expectation-Maximisation	67
4.2.3	E-Step	67
4.2.4	Taking the Limit of the Quantisation	76
4.2.5	M-Step	78
4.3	Implementations	79
4.3.1	Kalman Filter Implementation	80
4.3.2	Particle Filter Implementation	83
4.3.3	Viterbi Algorithm Implementation	86
4.4	Simulations	88
4.4.1	Linear Gaussian Scenario	90
4.4.2	Linear Non-Gaussian Scenario	93
4.5	Limitations	94
4.5.1	Unsmoothed Mixing Proportion Estimate	94
4.5.2	Quantisation issues	97
4.6	Summary	98
5	H-PMHT with a Poisson Measurement Model	101
5.1	Introduction	102
5.2	Relationship between Multinomial and Poisson Distributions	103
5.3	Derivation	105
5.3.1	Prior Density	107
5.3.2	Expectation-Maximisation	108
5.3.3	E-Step	109
5.3.4	Taking the Limit of the Quantisation	116
5.3.5	M-Step	118
5.4	Implementation	122
5.5	Simulations	122
5.6	Summary	133
6	An Interpolated Poisson Measurement Model for Track-Before-Detect	137

6.1	Introduction	138
6.2	Interpolated Poisson Distribution	139
6.3	Derivation	141
6.3.1	Expectation-Maximisation	143
6.3.2	E-Step	143
6.4	Kalman Filter Implementation	148
6.5	Simulations	150
6.6	Summary	154
7	Comparative Study using Trial Data from an Active Towed Array Sonar	165
7.1	Introduction	166
7.2	Active Sonar Problem	167
7.3	Tracking Algorithms	169
7.3.1	Conventional Tracking using Integrated Probabilistic Data Association	169
7.3.2	Track-Before-Detect using Expectation-Maximisation Data Association	171
7.4	Comparative Study using Sonar Trial Data	173
7.4.1	Bearing Dependent Point Spread Function	177
7.4.2	Conventional Tracking versus Track-Before-Detect	181
7.5	Summary	186
8	Summary	189
8.1	Conclusions	190
8.1.1	Maximum Likelihood Sequence Estimation for Track-Before-Detect	190
8.1.2	Viterbi Implementation for H-PMHT	190
8.1.3	Poisson Measurement Model for H-PMHT	190
8.1.4	Interpolated Poisson Measurement Model for Track-Before-Detect	191
8.1.5	Comparative Study of Track-Before-Detect and Conventional Point Measurement Tracking using Sonar Trial Data	191
8.2	Future Work	192
8.2.1	Application of Track-Before-Detect to the Active Sonar Problem	192

8.2.2	Extensions to Sonar Trial Data Application	192
8.2.3	Extension of Track-Before-Detect to the Multi-target Active Sonar Tracking Problem	193
A	Interpolated Poisson Distribution	195
A.1	Superposition	195
A.2	Proof of Integral (6.23) in the Derivation of the Interpolated Poisson-PMHT . . .	196

List of Figures

3.1	Example of Markov transition probabilities from state 5 (blue) and from state 1. Assumes number of states $N_v = 10$	51
3.2	Comparison of mean square state estimation error for HRC and HMC MLSEs and optimal smoothers for the uniform endpoints scenario. Assumes number of states $N_v = 10$	52
3.3	Comparison of mean square state estimation error for HRC and HMC MLSEs and optimal smoothers for the informative endpoints scenario. Assumes number of states $N_v = 10$	54
3.4	Mean square state estimation error vs. sequence length extended to $T = 100$ for the informative endpoints scenario. Assumes number of states $N_v = 10$, measurement noise variance $\sigma^2 = 1$	55
3.5	Target trajectories representative of Markovian and reciprocal behaviour to varying degrees.	57
3.6	Goodness of fit F for each scenario using MLSE for varying noise variance σ^2 . Assumes number of states $N_v = 20$ and sequence length $T = 12$	58
3.7	Goodness of fit F vs. “reciprocalness” of a target using optimal smoothing for varying noise variance σ^2 . Assumes number of states $N = 20$ and sequence length $T = 12$	58
3.8	Goodness of fit F for each scenario using MLSE for varying sequence length T . Assumes number of states $N_v = 20$ and measurement noise variance $\sigma^2 = 1$. . .	59
3.9	Goodness of fit F for each scenario using optimal smoothing for varying sequence length T . Assumes number of states $N_v = 20$ and measurement noise variance $\sigma^2 = 1$	59
4.1	Linear Gaussian scenario	91

4.2	Localisation accuracy, linear Gaussian scenario	92
4.3	Non-Gaussian point spread function	94
4.4	Localisation accuracy, linear non-Gaussian scenario	95
4.5	Localisation error in X component, linear non-Gaussian scenario	95
4.6	Localisation error in Y component, linear non-Gaussian scenario	96
5.1	RMS error averaged over 100 Monte Carlo runs comparing the standard H-PMHT with the Poisson H-PMHT under various target amplitude models for $\eta = 10$	126
5.2	SNR averaged over 100 Monte Carlo runs comparing the standard H-PMHT with the Poisson H-PMHT under various target amplitude models for $\eta = 10$	127
5.3	Constant Amplitude scenario: Comparison of the average target SNR for a single run for the standard H-PMHT and Poisson H-PMHT for varying forgetting factor η	129
5.4	Slowly Varying Amplitude Scenario: Comparison of the average target SNR for a single run for the standard H-PMHT and Poisson H-PMHT for varying forgetting factor η	130
5.5	Swerling I Scenario: Comparison of the average target SNR for a single run for the standard H-PMHT and Poisson H-PMHT for varying forgetting factor η	131
5.6	Step Function with Gaussian noise Scenario: Comparison of the average target SNR for a single run for the standard H-PMHT and Poisson H-PMHT for varying forgetting factor η	132
5.7	Track SNR variance versus forgetting factor η for the standard H-PMHT and Poisson H-PMHT under various target amplitude models.	134
5.8	Comparison of the average track SNR variance for the standard H-PMHT versus the Poisson H-PMHT for varying forgetting factor η for the Swerling I Scenario.	135
6.1	Integral of the Interpolated Poisson function for varying rate parameter λ	140
6.2	ϵ_Q versus time averaged over 100 Monte Carlo and 50 iterations assuming a forgetting factor of $\eta = 10$	153
6.3	Comparison of RMS error versus time (averaged over 100 Monte Carlo runs) for the standard H-PMHT, Poisson H-PMHT and IP-PMHT for various target amplitude models assuming $\eta = 10$	155

6.4	Comparison of track SNR versus time (averaged over 100 Monte Carlo runs) for the standard H-PMHT, Poisson H-PMHT and IP-PMHT for various target amplitude models assuming $\eta = 10$	156
6.5	Constant Amplitude scenario: Comparison of the average target SNR for the standard H-PMHT, Poisson H-PMHT and IP-PMHT for varying forgetting factor η	157
6.6	Slowly Varying Amplitude Scenario: Comparison of the average target SNR for the standard H-PMHT, Poisson H-PMHT and IP-PMHT for varying forgetting factor η	158
6.7	Swerling Model I Scenario: Comparison of the average target SNR for the standard H-PMHT, Poisson H-PMHT and IP-PMHT for varying forgetting factor η	159
6.8	Step Function with Gaussian noise Scenario: Comparison of the average target SNR for the standard H-PMHT, Poisson H-PMHT and IP-PMHT for varying forgetting factor η	160
6.9	Track SNR variance versus forgetting factor η for the standard H-PMHT, Poisson H-PMHT and IP-PMHT under various target amplitude models.	161
6.10	Comparison of the average track SNR variance for the standard H-PMHT, Poisson H-PMHT and IP-PMHT for varying forgetting factor η for the Swerling Model I Scenario.	162
7.1	Beampattern vs. bearing for transmissions at broadside, near aft and aft of the ship.	172
7.2	Variations in point spread function $h_\theta(\theta)$ in bearing space, for transmissions at broadside, near aft and aft of the ship.	173
7.3	Own-ship, ER and Observed ER Position for each dataset	175
7.4	TkBD measurement image for a target SNR return value of 24 dB.	178
7.5	TkBD measurement image for a target SNR return value of 13 dB.	178
7.6	Estimated target position: H-PMHT vs. the H-PMHT featuring a bearing dependent psf using SNR thresholding level of 11 dB.	179
7.7	Average track SNR estimates: H-PMHT vs the H-PMHT featuring a bearing dependent psf using SNR thresholding level of 11 dB.	180

7.8	Estimated target position: Various TkBD algorithms vs IPDA using a SNR thresholding level of 11 dB	182
7.9	Estimated target position: Various TkBD algorithms vs IPDA using a SNR thresholding level of 13 dB	183
7.10	Estimated target position: Various TkBD algorithms vs IPDA using a SNR thresholding level of 15 dB	184
7.11	Average track SNR estimates: Various TkBD algorithms using a SNR thresholding level of 15 dB	187

List of Tables

- 7.1 Number of false and divergent IPDA tracks at different SNR thresholding levels for the shallow dataset. 186
- 7.2 Number of false and divergent IPDA tracks at different SNR thresholding levels for the intermediate dataset. 186

List of Acronyms

EM	Expectation-Maximisation
EKF	Extended Kalman Filter
ER	Echo-Repeater
GNN	Global Nearest Neighbour
HMM	Hidden Markov Model
HMC	Hidden Markov Chain
HRC	Hidden Reciprocal Chain
HRP	Hidden Reciprocal Process
H-PMHT	Histogram-Probabilistic Multi-Hypothesis Tracker
iid	independent and identically distributed
IPDA	Integrated Probabilistic Data Association
IP-PMHT	Interpolated Poisson - Probabilistic Multi-Hypothesis Tracker
MAP	Maximum a Posteriori
MLSE	Maximum Likelihood Sequence Estimator
MMSE	Minimum Mean Square Error
ML	Maximum Likelihood
NN	Nearest Neighbour
KF	Kalman Filter
pdf	probability density function

psf	point spread function
PDA	Probabilistic Data Association
PF	Particle Filter
PMHT	Probabilistic Multi-Hypothesis Tracker
rhs	right hand side
RMS	Root Mean Square
RC	Reciprocal Chain
RP	Reciprocal Process
SMC	Sequential Monte Carlo
SNR	Signal-to-Noise Ratio
TkBD	Track-Before-Detect
UKF	Unscented Kalman Filter

List of Principal Symbols

$!$	Factorial operator
$[\cdot]^T$	Transpose operator
$[\cdot]'$	Indicates that the variable is dependent on estimates from the previous EM iteration
η	Forgetting factor
λ	Poisson mixing rate
λ_t^m	Poisson mixing rate for component m at time t
$\tilde{\lambda}$	Quantised Poisson mixing rate
$\tilde{\lambda}_t$	Quantised Poisson mixing rate at time t
$\tilde{\lambda}_t^m$	Quantised Poisson mixing rate for component m at time t
$\tilde{\lambda}_t^{im}$	Quantised Poisson mixing rate for component m in pixel i at time t
Λ	Collection of Poisson mixing rate terms for all times and components
Λ'	Collection of Poisson mixing rate terms at the previous EM iteration for all time and components
π	Multinomial mixing proportion term
π_t^m	Multinomial mixing proportion term for component m in pixel i at time t
Π	Collection of multinomial mixing proportion terms for all times and components
Π'	Collection of multinomial mixing proportion terms at the previous EM iteration for all times and components
μ_t^{im}	Proportion of power from component m in pixel i at time t
\mathfrak{Z}_t^{im}	Energy from component m in pixel i at time t
$B_{i,j}^k$	Markov bridge transitions

c^2	Quantisation level
f_t	H-PMHT probability density function at time t
f_t^i	H-PMHT per-pixel probability density at time t
\mathbb{F}_t	Poisson H-PMHT intensity function at time t
\mathbb{F}_t^i	Poisson H-PMHT per-pixel intensity at time t
F	Linear target transition matrix
F_t	Estimated total power in the image at time t as calculated by the H-PMHT
$F_t^{\mathcal{O}}$	Estimated total power in the observed image pixels at time t as calculated by the H-PMHT
\mathbb{F}_t	Estimated total power in the observed image pixels at time t as calculated by the Poisson H-PMHT
$G_0(\cdot)$	Clutter distribution
$h(\cdot)$	Point spread function
$h^i(\mathbf{x}_t^m)$	Probability of a shot due to a target \mathbf{x}_t^m falls in pixel i
$h^i(\emptyset)$	Probability of a shot due to clutter falls in pixel i
H	Linear measurement function
i	Measurement pixel index
I	Total number observed pixels
K_t^i	Set of components associated with shots in pixel i at time t
K_t^{ir}	Index of the component associated with the r^{th} shot in pixel i at time t
\mathbf{K}	Collection of the assignments of measurements to components for all times and components
\mathbf{L}	Collection of the precise locations of the shots inside its pixel for all time and components
m_t	Number of point detections at time t
m	Component index
M	Number of components
\mathbf{n}_t^i	Quantised measurements (counts) in pixel i at time t
\mathbf{n}_t^{im}	Quantised measurements (counts) from component m in pixel i at time t

\mathbf{n}_t^{ci}	Quantised unobserved measurements (counts) in pixel i at time t
\mathbf{N}	Collection of quantised observed measurements for all times and components
\mathbf{N}^c	Collection of quantised unobserved measurements for all times and components
$\ \mathbf{N}_t\ $	Total number of shots in the observed image at time t
$\ \mathbf{N}_t^c\ $	Total number of shots in the unobserved image at time t
$\ \mathbf{N}_t^{total}\ $	Total number of shots in image (from both observed and unobserved pixels) at time t
\mathbf{O}	Observer
\mathbf{Q}	Process noise covariance
$Q^{(H)}$	H-PMHT auxiliary function
$Q^{(P)}$	Poisson H-PMHT auxiliary function
$Q^{(IP)}$	IP-PMHT auxiliary function
Q_{ijl}	Reciprocal three point transition function
r	Measurement shot index
\mathbf{R}	Measurement covariance matrix
S	Total number of image pixels (unobserved and observed)
t	Time index
T	Number of time scans
\mathbf{x}_t^m	State of component m at time t
X_t	State in Markov Model
\mathbf{X}	Collection of components for all times and components
\mathbf{X}'	Collection of components at the previous EM iteration for all times and components
Y_t	Observations in the Markov model at time t
z_t	An observed measurement at time t
z_t^i	Observed energy in pixel i at time t
z_t^m	Observed energy from component m at time t
\mathbf{Z}	Collection of observed measurements for all times and components
\mathbf{Z}_t	Observed measurements at time t
Z_t	Collection of observed measurements up until time t

- \mathbf{Z}^c Collection of unobserved measurements for all times
- \mathbf{Z}_t^c Collection of unobserved measurements up to time t
- $\|\mathbf{Z}_t\|$ Total observed energy in image at time t