Extensional and Surface-Tension-Driven Fluid Flows in Microstructured Optical Fibre Fabrication

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For my mother, Sharon

And I looked at that painting *Sunflowers*. And for a bogan from Hamilton like myself I could actually see beauty in that frustration.

Phil Walsh

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Abstract

Microstructured optical fibres (MOFs) are a design of optical fibre comprising a series of longitudinal air channels within a thread of material that form a waveguide for light. The flexibility of this design allows optical fibres to be created with adaptable and previously unrealised optical properties. A MOF is typically constructed by first creating a macroscopic version of the design, known as a preform, with a centimetre-scale diameter that is later drawn into a fibre with a micrometer-scale diameter.

There are several methods for constructing a preform. In the extrusion method molten material is forced through a die containing an array of blocking elements that match the required pattern of channels. Preforms may also be constructed by stacking tubes and fusing them together with heat. In both processes the fluid flow that arises can deform the air channels, rendering the fibre useless. At present there is only a limited understanding of the relative importance of the various physical parameters in determining the final preform geometry, which means that the development of new MOF technology requires time-consuming and costly experimentation.

This thesis develops mathematical models of the fluid flows that occur during the extrusion and stacking methods of MOF preform fabrication. These models are used to determine which physical mechanisms are important during the manufacturing process so as to inform the fabrication of MOF preforms.

A model is constructed of a fixed slender fluid cylinder with internal structure stretching under gravity and with surface-tension-driven deformation. The molten material is modelled as a Newtonian fluid with a temperature-dependent viscosity, which is assumed known. The variables are expanded as series in powers of a slenderness parameter so that, after dropping higher-order terms, the resulting equations partially decouple into a one-dimensional model for the axial flow and a two-dimensional model for the transverse flow. Under a suitable transformation of variables the transverse equations are precisely the Stokes equations with unit surface tension. After reviewing the use of complex variables to represent the transverse problem, three numerical solution methods are considered: two based upon spectral methods and one using the method of fundamental solutions (MFS). These methods are compared for their efficiency and accuracy.

Several example solutions for stretching cylinders are presented and the role of surface tension is investigated using approximate solutions derived for zero and small surface tension. The model is validated against experimental data and found to be in good agreement. The stretching model is extended to the case of an extruded fluid cylinder, neglecting extrudate-swell effects, where again the fluid flow decouples in axial and transverse models. The results are compared with experimental observations and the model used to analyse the formation of distortions during preform extrusion and how these may be controlled. Two problems related to preform fabrication are considered that feature cross sections with non-circular initial outer boundaries. A technique is developed for deriving initial conformal maps describing such domains, which are used in the stretching and extrusion models to analyse the proposed problems.

Signed Statement

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List of Variables

Bi	$\frac{h\mathcal{L}}{k}$	Biot number
Ca	$\frac{\mu_0\mathcal{U}}{\gamma}$	Capillary number
De	$\frac{\mathcal{T}_c}{\mathcal{T}}$	Deborah number
Fr	$\frac{\mathcal{U}}{\sqrt{g\mathcal{L}}}$	Froude number
Pe	$\frac{\alpha}{\mathcal{L}\mathcal{U}}$	Péclet number
Re	$\frac{\rho\mathcal{U}\mathcal{L}}{\mu_0}$	Reynolds number
g^*	$\frac{\rho g\mathcal{L}^2}{\mu_0\mathcal{U}}$	Dimensionless gravity
γ^*	$\frac{1}{\epsilon \mathrm{Ca}}$	Dimensionless surface tension

$\mathcal{A}$	Airy stress function	r	radial co-ordinate
c	axial tension	R	radius
c_p	specific heat	s	arc length
C_n	boundary n	S	cross-sectional area
d	hole spacing/diameter	$\mathcal{S}$	area scale
e	rate of strain	t	time
E	strain	t_a	cooling time
f	Goursat function	t_c	critical time
F	composed Goursat function	t_e	effective cooling time
g	gravitational acceleration/ Goursat function	t_f	final time
		T	temperature
G	composed Goursat function	T_a	atmospheric temperature
h	heat transfer coefficient	$\mathcal{T}$	temperature scale
H	integrating factor	u	x velocity
i	imaginary unit	$\boldsymbol{u}$	velocity vector
$\boldsymbol{i}$	x unit vector	$\mathcal{U}$	velocity scale
$\boldsymbol{j}$	y unit vector	v	y velocity
k	thermal conductivity	V	volume
$\bar{k}$	Eötvös constant	V_m	molar volume
$\boldsymbol{k}$	z unit vector	w	z velocity
L	cylinder length	$\mathcal{W}$	bianalytic function
$\mathcal{L}$	axial length scale	x	Cartesian co-ordinate
m	viscosity harmonic mean	$\boldsymbol{x}$	Cartesian co-ordinate vector
p	pressure	y	Cartesian co-ordinate
$\mathcal{P}$	analytic function	z	Cartesian co-ordinate/ complex number
Q	flux		
$\mathcal{Q}$	flux scale		

α	thermal diffusivity
β	coefficient of thermal expansion
γ	surface tension
Γ	boundary length
ϵ	slenderness ratio
ζ	complex variable
η	Lagrangian co-ordinate
θ	angle
κ	curvature
Λ	hole spacing
μ	viscosity
ξ	Lagrangian co-ordinate
ρ	density
σ	stress tensor
σ_E	tensile stress
τ	reduced time
ϕ	annular aspect ratio
χ	square root of cross-sectional area
ψ	stream function
ω	vorticity
Ω	domain

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