

Markov Random Fields with Unknown Heterogeneous Graphs



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For my parents and my girlfriend.

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Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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- Zhenhua Wang, Qinfeng Shi, Chunhua Shen, Pawan Kumar, Anton van den Hengel. Linear and Quadratic Relaxations for Inferring MRF Labels and Graphs Simultaneously. Submitted to some conference. Under review.
- Zhenhua Wang, Zhiyi Zhang, Nan Geng, A Message Passing Algorithm for MRF Inference with Unknown Graphs and Its Applications. *Asian Conference on Computer Vision (ACCV)*, accepted as oral presentation, Singapore, 2014.

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- Qinfeng Shi, Mark Reid, Tiberio Caetano, Anton van den Hengel, and Zhenhua Wang. A Hybrid Loss for Multiclass and Structured Prediction. In *IEEE Trans. on Pattern Analysis and Machine Intelligence (TPAMI)*, to appear, 2014.

Abstract

Markov Random Fields have been widely used in computer vision problems, for example image denoising, segmentation and human action recognition. The structure of the graph can be determined using human heuristics or domain knowledge, or can be learned from data when assuming graphs are homogeneous in topology. However, there are many applications for heterogeneous graphs. This research concentrates on estimating heterogeneous graphs and labels simultaneously from the observation. The joint estimation of graphs and labels is formulated into maximising a joint likelihood inference. Unfortunately, these inference problems are generally NP-complete, and we thus develop novel algorithms for effectively and efficiently finding approximate solutions. We also demonstrate how to learn Markov random field parameters from data with our inference techniques.

Our contributions are as follows. First, we show that estimating graphs and performing maximum a posteriori inference can be achieved simultaneously by solving a bilinear programming problem, for which we provide a branch and bound solver. Second, we relax the inference problem into a tighter non-convex quadratic programming problem, and propose a convex-concave procedure algorithm to solve the non-convex quadratic programming. Third, we derive the partial-dual of a mixed integer programming relaxation of the inference problem, which admits a scalable message passing-style algorithm. Lastly, we show how to learn the parameters of Markov random fields with the structured max-margin training and the proposed inference algorithms.

We evaluate the proposed algorithms on both synthetic data and real applications including human action/activity recognition and semantic image segmentation. Our inference algorithms usually outperform the state-of-the-art. Within our proposed algorithms, the quadratic programming algorithm often performs better than the bilinear programming and the message passing algorithms, while the message passing algorithm is the most efficient.

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Nomenclature

Mathematical Notations

a activity variable

\mathcal{E} the set of all possible edges

$\boldsymbol{\theta}$ vector representation of all unary and binary potentials

\mathcal{O} the set of constraints of the non-convex QP formulation

χ the variable of the QP formulation

\mathbf{Q} the matrix of the quadratic term in convex QP formulation

$\text{deg}(i)$ the degree of node i

$\text{diag}(\mathbf{d})$ a diagonal matrix formed by the vector \mathbf{d}

$\text{dis}(y_i, y_j)$ the distance function of y_i, y_j used to generate random potentials

$F(\mathbf{x}, \mathbf{y}, G; \mathbf{w})$ discriminant function

E Edge set of G

\emptyset empty set

G Graph

$\mathbb{1}(\cdot)$ indicator vector

$\mathbf{e}_c(y_i)$ the indicator vectors of label

$\mathbf{e}_o(o_i)$ the indicator vectors of body pose

$\mathbf{e}_p(r_{ij})$ the indicator vectors of relative body position

Φ_2	the binary joint feature map
Φ_1	the unary joint feature map
\otimes	Kronecker tensor
$\Delta(\mathbf{y}, \hat{\mathbf{y}})$	the cost of predicting \mathbf{y} as $\hat{\mathbf{y}}$
$\delta(y, \hat{y})$	the cost of predicting y as \hat{y}
$\mathcal{L}(\mathcal{Q})$	the length of \mathcal{Q}
Φ_{lb}	the lower bound function
$\mu_{ij}(y_i, y_j)$	the joint distribution of random variables y_i, y_j
\mathcal{M}	set contains all constraints of the mixed integer bilinear programming problem
$\mu_i(y_i)$	the distribution of random variable y_i
$Z(\boldsymbol{\theta})$	partition function
\mathbf{w}	the parameter vector of potential function
\mathbf{w}_1	the parameter vector of unary potential
\mathbf{w}_2	the parameter vector of pairwise potential
o_i	the body pose of person i
\mathcal{Q}	rectangle domain
$\mathcal{Q}_{\text{init}}$	the initial rectangular domain
r_{ij}	the relative body position of person i to person j
\preceq	negative semi-definite matrix notation
h	graph sparsity parameter
$\theta_{st}(y_s, y_t)$	pairwise (binary) potential function of y_s, y_t
$\theta_i(y_i)$	unary potential function of y_i
$\hat{\Theta}$	the matrix of the quadratic term in the non-convex QP formulation

Θ	block diagonal matrix defined in Section 3.1
\mathcal{U}	uniform distribution
Φ_{ub}	the upper bound function
V	Node set of G
\mathbf{x}	input (observation)
\mathcal{X}	input space
y	a random variable
\mathbf{y}	vector contains all y variables
\mathcal{Y}	output (labelling) space
z_{ij}	the variable that represents the existence of edge (i, j)
\mathbf{z}	vector contains all z_{ij} variables
Z-mst	MST

Abbreviations

BLP	bilinear programming
BP	belief propagation
CCCP	convex-concave procedure
CRF	conditional random field
HAR	human action recognition
HoG/HoF	histogram of gradients/optical flow
LP	linear programming
MAP	maximum a posteriori
MCSVM	multi-class support vector machine
MRF	Markov Random Field

PCA principle component analysis

PDMP partial-dual based message passing

QP quadratic programming

SSVM structured support vector machine

SVM support vector machine