A Variational Approach to Hadron Structure in Lattice QCD

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Abstract

In order to understand how hadrons acquire their physical properties from their constituents, we must resort to the underlying theory of the strong interaction, Quantum Chromodynamics (QCD). However, the non-perturbative nature of this theory in the relevant energy scales renders the standard perturbative methods ineffective. The formulation of QCD on a discrete space-time lattice allows for a first principles, non-perturbative approach to studying the strong interaction, in a manner well suited to numerical computation.

Over the past decade, the use of variational techniques has provided an effective framework for spectroscopic studies of the full hadron spectrum. Herein we generalise the use of the variational approach to hadron form factor calculations and examine its use in a number of different hadronic systems. As such an approach allows for the isolation of terms relevant to a single eigenstate or eigenstate transition, we show that this method is both an effective way to remove excited state contamination from the study of ground state systems and an effective framework through which one can study the structure of hadronic excitations.

We begin with an evaluation of the nucleon axial charge, $g_A$, to investigate the improvement offered through this method and consider the role that excited states play in the discrepancy observed between lattice determinations and experiment. This is followed by a determination of the $\rho$-meson electromagnetic form factors $G_C$, $G_M$ and $G_Q$, and the corresponding radiative transition form factor $G_{M1}$ using near physical masses. We then turn our attention to the electromagnetic form factors of the two lowest-lying negative parity nucleons, where such techniques are required to disentangle the contributions of these two near degenerate states. Here we present the first evaluation of the elastic form factors $G_E$ and $G_M$ for both these low-lying states.

Finally, through careful consideration of the $N\gamma \rightarrow N^*$ vertex, we develop an innovative formalism that allows one to evaluate radiative transition form factors for all spin-$1/2$ nucleon excitations. This novel formalism is implemented to provide the world’s first examination of the odd-parity transitions of the nucleon in lattice QCD.
Statement of Originality

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Benjamin James Owen
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