Innovative Multi-level Methodology
Incorporating the Techniques of
Finite Element Modelling and Multimodal Optimization
for Concept Design
of Advanced Grid Stiffened Composite Panels
against Buckling

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B.Eng., M.Eng., Ph.D.

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of Doctor of Philosophy

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To my beloved parents

Tie-Jun Huang & Xiao-Lan Yuan
Preface

Since the Second World War fiber reinforced polymer (FRP) composites have become more attractive as a structural material in a variety of engineering practices, such as infrastructure construction, automobile industry and aerospace engineering, due to high specific strength and stiffness as well as flexibility in tailoring the structural performance. On the other hand, a stiffened panel always performs better in resisting loads compared to an unstiffened panel of same weight. Thus a combination of lightweight composite materials and stiffened structural forms can efficiently enhance the load resisting capability that can be buckling strength of a structure.

Stiffened composite panels are subjected to any combination of in-plane, out-of-plane and shear load conditions during service life. These types of thin-walled structures are vulnerable to lose global and local stability under compression loadings. Consequently, buckling-resistant design is one of the most critical issues of stiffened composite panels applied in real practices. Moreover, the buckling optimization design of composite panels is usually a typical multimodal optimization problem, in which there exist multiple global optimal solutions with identical or closely comparable optima of structural performance.

Recently, with the development of manufacturing techniques, advanced grid stiffened (AGS) composite panels have increasingly emerged and gained more attention as these grid-stiffening configurations help to enhance the structural efficiency in a more effective way in complex loading conditions compared with conventional unidirectionally-stiffened composite panels. These grid-stiffening configurations provide more available options to select outstanding concept designs of AGS composite panels against buckling for the final appropriate design development at the final construction stage.
In this PhD thesis, a novel multi-level optimization methodology for concept design of advanced grid stiffened composite panels against buckling has been developed. Furthermore, an efficient finite element (FE) modelling component for buckling analysis and a robust particle swarm optimization (PSO) algorithms for multimodal optimization have been presented, in order to further consolidate the performance of the proposed methodology.

The thesis is divided into six chapters, which are briefly described below:

In Chapter 1, a general background along with the objective and originality of the present research is presented.

An efficient FE modelling technique is presented in Chapter 2, for the prediction of buckling response of grid stiffened composite panels having different stiffening arrangements. The laminated skin of the stiffened structure is modelled with a triangular degenerated curved shell element. An efficient curved beam element compatible with the shell element is developed for the modelling of stiffeners which may have different lamination schemes. The deformation of the beam element is completely defined in terms of the degrees of freedom of shell elements and it does not require any additional degrees of freedom.

Chapter 3 aims to extend conventional unimodal optimization to challenging multimodal optimization of composite structures, by means of newly emerged multimodal PSO using niching techniques. It has shown that the ring topology based PSO without any niching parameter is more robust and efficient for multimodal optimization of composite structures, compared with the species-based PSO (SPSO) and the fitness Euclidean-distance ratio based PSO (FER-PSO).

In Chapter 4, a random reflection boundary is proposed to replace the conventional fixed absorption boundary for the range-exceeding particles, in order to eliminate/reduce the significance and sensitivity of an empirical parameter of particles’ maximum velocity in PSO. Based on the results
obtained from the experimentation on the abovementioned test functions, empirical guidelines for appropriately using the half-range/full-range random reflection boundary are further proposed.

Chapter 5 presents an efficient methodology to conduct concept design of AGS composite panels, based on a multi-level approach where an inner 3-stage optimization process is nested within an outer 3-step optimization process. The proposed methodology is applied to a design optimization problem of an AGS composite plate against its buckling resistance, by incorporating a ring topology based multimodal PSO algorithm with an improved FE buckling analysis model.

Finally, the conclusions of the present research are summarized in Chapter 6. The limitations and the future development directions of the present study are also described in this chapter.
Statement of Originality

I, Liang Huang, hereby declare that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution in my name and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Chapter 1

Introduction

1.1 Research background on stiffened composite panels

Over past few decades, fiber reinforced polymer (FRP) composite materials have been increasingly employed for a large variety of structural applications in different engineering industries, such as aerospace, automotive, military aircraft and civil infrastructures [1-4]. For example, the world’s first vehicle bridge made entirely of fiber composite materials was traced back to 1982 in Beijing of China [5]. Undoubtedly, there are many benefits of using fiber composite materials for engineering structures. The main advantages of composite materials are their high strength/weight and stiffness/weight ratios as well as lower maintenance requirement, compared with conventional materials such as metals. Especially, a great surprising advantage of composite structures is that their material properties could be conveniently tailed to meet the various requirements of specific applications. Thus engineers are more flexible to customize the composite structures for practical applications in terms of topology, geometry and material design of structure components [6].

Despite unstiffened composite laminates are typical composite components, stiffened composite panels are more popular structural forms and are extensively used in different engineering applications. It is because composite stiffeners can effectively enhance the bending stiffness and buckling capacity of panel components without substantially increasing of structure weight. Stiffened composite panels are commonly adopted in bridge composite deck
system, hull design for composite ship, aircraft fuselage and launch vehicle fuel tanks. For instance, the combination of stiffened composite plate and steel fiber reinforced concrete was utilized to develop a new hybrid bridge deck system [7], as shown in Fig. 1.1.

![Schematic of hybrid deck system](image1.png)  ![Photograph of stiffened composite plate](image2.png)

(a) Schematic of hybrid deck system  (b) Photograph of stiffened composite plate

**Fig. 1.1.** Hybrid composites-concrete bridge deck system (adapted from [5])

Stiffened composite panels are subjected to any combination of in-plane, out-of-plane and shear load conditions during service life. Because of the geometrical property of these thin-walled structures, buckling is one of the most critical failure criteria. Buckling failure mode of a stiffened composite panel can be further classified into global structure buckling, local skin buckling and local stiffener crippling. To investigate the buckling phenomena of stiffened composite panels, many analytical and numerical methods have been developed by researchers. The analytical models can be established by discrete method [8] and the smeared stiffeners approach [9] etc.. The numerical models can be further subdivided into the finite strip method [10] and the finite element method [11-13]. Each type of computational model for buckling analysis has its advantage and disadvantage. Analytical models are attractively efficient to predict buckling loads of stiffened composite panels in terms of computational cost, but they are commonly restricted to uniform distributed load conditions as well as specific panel configurations and boundary conditions. In terms of solution accuracy and model universality, finite strip models are better than the analytical models to predict buckling capacity associated with various stiffened composite panels subjected to
different loading conditions. Fortunately, finite element models are generally the most versatile and accurate method for analyzing stiffened composite structures with arbitrary stiffener configurations, loading cases and supporting conditions.

Once the buckling prediction model for stiffened composite panels is sufficiently accurate, the practical design of stiffened composite panels subjected to buckling loads can be performed based on these buckling analysis results. With an enormous growth of computer power over the past decades, optimization techniques have become an important tool for the buckling-resistant design of stiffened composite panels. It should be noted that Schmit and Mehrinfar [14] conducted one of the first optimal design of stiffened composite panel components to seek minimum weight design subjected to buckling, stress and displacement constraints by using multilevel approach to optimization. Since early research stage in the 1980’s, the optimization method applied for composite structures have advanced from conventional gradient-based and mathematic techniques for local optimization such as feasible direction method [15-17] and sequential quadratic programming [18], to modern gradient-free and heuristic approaches for global optimization such as genetic algorithm (GA) [19, 20], particle swarm optimizer (PSO) [21-23] and artificial bee colony (ABC) algorithm [24].

Contemporary heuristic optimization approaches are able to identify many local optima with comparable performance and yield globally best designs, while traditional gradient-based optimization methods could not ensure to obtain the global optima. Due to fast development of computer technology, it has become a trend to perform gradient-free global optimization for the optimal design of composite structures. Therefore, it is not doubted that there has been a great deal of research on successful application of GA to optimal design of stiffened composite panels. For instance, Nagendra et al. [25] performed one of the representative works and they reported that somewhat better design for buckling-resistance of stiffened panels could be obtained using the GA method compared with integer programming. However, they also pointed out that the GA approach requires huge computational cost.
Unlike GA, which is a competition algorithm based on the biological theory of survival of the fittest, PSO is a corporation model based on sociobiology of insect swarms. It has been demonstrated that PSO has better computational efficiency than other stochastic methods, such as GA and simulated annealing (SA) [26]. Therefore, PSO has been recognized as a robust and efficient algorithm for global optimization problems. A number of PSO variants have been further developed and widely applied in solving various complex optimization problems [27].

Metamodels or approximation methods have also been emerged to increase the efficiency of global optimization procedure for composite structures. When computationally expensive simulation code for structural analysis could be replaced by function approximations or surrogate models, metamodels play an important role for design optimization. Metamodels significantly speed up the optimal design by decoupling the analysis problem and optimization problem. So far there has been much progress in the development of metamodel techniques ranging from Taylors’s series truncated approximation to response surface, radius based function (RBF), Kriging and artificial neural network (ANN) metamodels [28-32]. Metamodels are based on the theory of design of experiments (DOE). DOE is the techniques about how to arrange fewer sample points to achieve better coverage of the design space so that these selected sample points can gain as much information as possible for the outputs-inputs relationship approximation. DOE techniques have developed from classic methods (e.g. factorial design, central composite design and Taguchi’s orthogonal matrices) to modern methods such as “space filling” design (e.g. Latin hypercube design and uniform design) [33, 34]. For simulated computer experiments with systematic modelling errors rather than random errors as in physical experiments, Simpson et al. [35] advocated to use modern “space filling” designs which scatter the sample points over the whole design space as evenly as possible. Moreover, “space filling” designs will be suitable and reliable in cases where the form of the metamodel cannot be specified in advance due to the lack of any prior information about the relationship being approximated.
1.2 Challenge of concept design of advanced grid stiffened composite panels subjected to buckling loads

For practical engineering application of composite structures, concept structural design is first conducted to identify several design solutions having optimal performance to satisfy the requirements under consideration at the preliminary design stage. These identified design solutions out of all feasible design candidates are then further refined and finalized at the detailed design stage for the final field construction. Concept design of composite structures is a typical optimization problem with respect to topology, geometry and material optimal designs.

As mentioned by Venkataraman and Haftka [36], model complexity, analysis complexity and optimization complexity are three aspects contributing to the challenge of a structural optimization problem. Model complexity refers to computational cost associated with modelling in terms of topology, geometry and material of the structure and loading on the structure. For instance, the number of degree of freedom (DOF) in the finite element (FE) model and the topology of a structure may be used as the indications of the model complexity. In terms of the computational complexity/cost of the analysis procedure, linear elastic analysis is the simplest. Nonlinear elastic analysis, eigenvalue buckling analysis and linear dynamic analysis are more complicated. Recently, the most complex structural analyses could include time history-dependent nonlinear analysis, nonlinear dynamic analysis and nonlinear crack propagation.

The complexity/cost of optimization process is related to the number of analysis required, the number of design variables and the type of optimization. For example, the simplest type of optimization is local and gradient-based optimization approach, while global and gradient-free optimization with continuous-discrete mixed design variables is the most difficult type of optimization in that the computation cost of solutions may be expected to
increase exponentially with the number of design variables. Even if computer technology have greatly developed, however, it should be noted that it is still not possible to solve optimization problems, which have more than two of model, analysis and optimization complexity, as long as the maximal complexity in model, analysis and optimization component continues to increase [36].

For optimal design of composite structures, examples of increasing complexity are presented in terms of model, analysis and optimization complexity, respectively, as shown in Fig. 1.2. From the view point of structural optimization, optimal design of stiffened composite panels is optimization with models of intermediate complexity and the optimization problem has a combination of continuous design variables (e.g. stiffener geometric sizes) and discrete design variables (e.g. ply orientation and number of stiffeners). For buckling-resistant optimal design of simply stiffened composite panels, it is feasible to perform more complex global optimization using less expensive analysis method [37, 38]. These simple models are based on analytical closed-form solutions, shell-of-revolution solution and finite strip analysis, which are adopted in commercial software for optimal design of stiffened composite panels such as VICONOPT [39] and PANDA2 [40]. Alternatively, it is also feasible to conduct less computational expensive optimization procedure based on metamodel techniques using hundreds of nonlinear buckling analysis [41, 42].

With the advance of new manufacturing techniques, the applications of advanced grid stiffened (AGS) composite panels have been boosting. The AGS composite panels are attractive alternatives as these configurations help to enhance the structural efficiency in a more effective way in complex loading conditions compared with conventional unidirectional-stiffened composite panels. Fig. 1.3 shows typical stiffening-grid pattern of AGS composite panels, such as ortho-grid, angle-grid and triangle-grid schemes [43, 44] and Fig. 1.4 presents two examples of AGS composite panels in which iso-grid arrangement is a special case of triangle-grid pattern [45, 46].
The concept design of AGS composite panels becomes rather challenging because stiffening-grid pattern options of AGS composite panels remarkably increase the model topology complexity and search space complexity in optimization process. Due to the integration of high complexity in model, analysis and optimization aspects according to Fig. 1.2, the computation cost of the concept design of AGS composite panels subjected to buckling loads would become extremely expensive even unaffordable to simultaneously optimize topology design (i.e. to optimize grid pattern of stiffeners), geometry design (e.g. to optimize stiffener cross-section size and spacing) and material design (e.g. to optimize ply thickness and orientation), when the concept design procedure is focused on global heuristic optimization incorporating accurate but time-consuming FE buckling analysis.

Moreover, the optimization design of composite structures is usually a typical multimodal optimization problem, in which there exist multiple global optimal solutions with identical or closely comparable optima of structural performance [47]. It is necessary and desirable to find all the multiple global optimal solutions for the concept design of AGS composite panels subjected to buckling loads, because the multiple global optimal solutions will definitely provide engineers with more flexibility and freedom in decision making for the final detailed design in practices. Based on literature review, however, most of the present studies for optimal design of composite structures are only limited to unimodal optimization by identifying a single optimal design, thus to find all of the global optimal solutions in an effective and efficient way becomes another challenging and significant issue on the concept design of AGS composite panels subjected to buckling loads.
Fig. 1.2. Examples of increasing model, analysis and optimization complexity for optimal design of composite structures (adapted from [36])
Fig. 1.3. Typical grid patterns for AGS composite panels (adapted from [44])

Fig. 1.4. Examples of iso-grid stiffened composite structure
(adapted from [45, 46])

1.3 Research objectives

In order to overcome the difficulty of the concept design of AGS composite panels subjected to bucking loads, the underlying objective of the present study is to develop an effective and efficient methodology to simultaneously obtain the topology, geometry and material solutions of concept design of AGS composite panels which can be subsequently refined further to get the final detailed design used for actual construction. It is expected that the proposed methodology could efficiently solve the concept design process by decomposing the systematic level optimization problem with entire design variables into the component level optimization problems with partial design
variables. It is also expected that the proposed methodology for the concept design of AGS composite panels could fully considerate the effects of all the design variables in an effective way.

Moreover, in view of global multimodal optimization and finite element analysis (FEA) for the concept design of composite structures, there are other two aims focusing on the model complexity and the optimization complexity in buckling-resistant optimization of AGS composite panels, respectively. The first aim is to develop a computational efficient and accurate FE model to analyze AGS composite plates subjected to buckling loads. The second aim is to select and develop an efficient and robust PSO algorithm for multimodal optimization to find the multiple optimal designs of AGS composite panels.

It should be noted that the proposed multi-level methodology for the concept design of AGS composite panels can be substantially consolidated to achieve better performance further by incorporating the efficient technique components, such as FE modelling and multimodal optimization.

The main objective for the multi-level methodology and the two subordinate aims for the modelling and optimization techniques are shown in Fig. 1.5. In total, four research papers have been presented in this thesis. Accordingly, Paper 1 is relevant to the first aim; Paper 2 and Paper 3 are related to the second aim; and Paper 4 is to achieve the overall objective of this study.
1.4 Organisation of thesis

The main bodies of this thesis are composed of four papers. The titles and the corresponding chapters of these four papers are listed as follows:

**Paper 1 (Chapter 2):** An efficient finite element model for buckling analysis of grid stiffened laminated composite plates

**Paper 2 (Chapter 3):** Niching particle swarm optimization techniques for multimodal buckling maximization of composite laminates

**Paper 3 (Chapter 4):** New technique of random reflection boundary for PSO and its application in buckling optimization of composite panels

**Paper 4 (Chapter 5):** Novel multi-level methodology for concept design of advanced grid stiffened composite panels subjected to buckling loads
Paper 1 is to develop fast and accurate buckling analysis of AGS composite plates (Aim 1) using shell elements and beam elements to model the skin and the stiffeners respectively, for the sake of application in optimization of AGS composite plates where repetitive analyses of the structures are required. The laminated skin is modelled with a triangular degenerated curved shell element having 3 corner nodes and 3 mid-side nodes. An efficient curved beam element fully compatible with the shell element is developed for the modelling of stiffeners which may have different lamination schemes. The formulation of the 3 node degenerated beam element may be considered as one of the major contributions. The deformation of the beam element is completely defined in terms of the degrees of freedom of shell elements and thus it does not require any additional degrees of freedom. Unlike the usual formulation of degenerated beam elements which overestimates their torsion rigidity, a torsion correction factor is introduced for different stiffener stacking schemes. Numerical examples of AGS composite plates for buckling capacity prediction are solved by the proposed FE model to validate its accuracy. A parametric study is presented to show the effects of skin thickness, stiffener breadth and stiffener depth on the buckling capacity of AGS composite plates.

Paper 2 is to investigate the adaptability of the newly developed multimodal PSO algorithms for composite structure optimization (Aim 2), in order to extend conventional unimodal optimization to challenging multimodal optimization of composite structures with multimode characteristic in nature. The advanced multimodal PSO algorithms under investigation include the species-based PSO (SPSO) requiring niching parameter and other two algorithms without using niching parameter, namely the fitness Euclidean-distance ratio based PSO (FER-PSO) and the ring topology based PSO. By applying these three multimodal PSO algorithms to a maximum buckling capacity of composite design problem well-studied in literature, it shows that all of these multimodal algorithms have not only successfully located 7 first-best-fitness solutions, but also discovered 14 second-best-fitness optimal designs for this problem. Unexpectedly, the difference of the objective function value of this optimization between the first-best-fitness solutions and
second-best-fitness solutions was extremely close. The performances of the three multimodal PSO algorithms have been further investigated according to the numerical tests. The statistical comparisons of these testing results show that the performance of SPSO requiring user-predefined niching parameter is sensitive to the value of niching radius. Both the ring topology based PSO and the FER-PSO do not need any additional niching parameter compared with SPSO, but the ring topology based PSO exhibits appealing capability to locate multiple global and local solutions. Therefore, the ring topology based PSO is suggested for applications of multimodal optimization of composite structure designs.

**Paper 3** is to propose a random reflection boundary to replace the original fixed absorption boundary in order to enhance the efficiency and robustness PSO for multimodal/unimodal optimization of composite structures (*Aim 2*). The random reflection boundary is expected to remove/reduce the performance impact of the indispensible and sensitive parameter of particles’ velocity limit in the conventional PSO algorithms. According to the experimental results of testing functions for optimization, it is shown that PSO algorithms using the proposed random reflection boundary technique achieve better performance than corresponding PSO counterparts with the fixed absorption boundary. It is also shown that it is feasible to remove the indispensible algorithmic parameter of particles’ maximum velocity in conventional PSO by using the random reflection boundary to achieve reliable and satisfactory optimization results. Based on the experimentation of testing functions, empirical guidelines for effective usage of the half-range/full-range random reflection boundary are further established in terms of PSO topology, PSO convergent method and dimensions of optimal problems. The proposed random reflection boundary technique for PSO algorism is then successfully applied in a practical case of buckling optimization of composite panels.

**Paper 4** is to propose a novel multi-level optimization strategy in order to develop an efficient and effective methodology for concept design of AGS composite panels at the initial design stage (*Global objective*). This innovative multi-level methodology is based on an inner 3-stage optimization
process nested within an outer 3-step optimization process. The outer 3-step optimization process first decomposes the whole optimization problem of concept design into finite number of sub-optimization problems according to the identified critical discrete design variables. Each of these sub-optimization problems are then solved by the inner 3-stage optimization process through further decomposed into two inferior sub-optimization problems. For the inner 3-stage optimization process, the whole design variables excluding the identified critical discrete design variables in the outer 3-step optimization process is divided into two groups: ply-orientation variables and the other variables. The RBF metamodels combined with DOE is used to represent the intermediate optimization results which have involved the optimization effects of the design variables of ply-orientation when the inferior sub-optimization related to the other design variables (i.e. non-ply-orientation design variables) is performed in the inner 3-stage optimization process. When a buckling-resistant design problem of AGS composite plates with orth-grid, x-grid, bi-grid or iso-grid stiffener arrangement options is presented to illustrate the application of the developed methodology, the structural component of FE bucking analysis based on Paper 1 and the optimization component of multimodal PSO based on Paper 2 and Paper 3 are systematically incorporated for better consolidation of the proposed multi-level methodology. For this illustrative problem, iso-grid stiffener pattern is recommended for optimal concept design of AGS composite plates subjected to uniaxial compressive loads and the corresponding preliminary optimal design variables including stiffener width, stiffen height, stiffen spacing, ply orientations of the skin and ply orientations of the stiffener have been simultaneously obtained.

1.5 Research originality

1. Modelling improvement: for FE buckling analysis of grid stiffened composite laminates in Paper 1 (Chapter 2), an efficient curved beam element completely compatible with a shell element is developed without introducing any additional degree of freedom. The proposed finite element
modelling tool is fast and accurate, allowing for various stacking schemes of stiffener lamination and convenient correction of stiffener torsion rigidity. This computationally efficient model can be used for optimal design of grid stiffened composite at initial design stage.

2. **First application with new findings:** in *Paper 2 (Chapter 3)*, three newly developed PSO algorithms for multimodal optimization are first introduced to extend conventional unimodal optimization to challenging multimodal optimization of composite laminates in view of multimodal characteristic for optimal design of composite structures. By application of the multimodal PSO algorithms, there are new findings of 21 acceptable design solutions for a composite plate design problem well-studied in literature; therefore, this case of composite plate design can become a benchmark problem to be used for suitability testing of an optimal algorithm for composite structure optimization. Based on statistical comparison for the three multimodal PSO algorithms, the ring topology-based PSO algorithm is identified as the efficient and robust one for multimodal optimization of composite structures in practical applications.

3. **Original technique innovation:** *Paper 3 (Chapter 4)* proposes a new random reflection boundary to replace the conventional fixed absorption boundary in PSO algorithm. The random reflection boundary technique and its application guidelines benefit to enhance PSO algorithmic performance by successfully eliminating/reducing the significance and sensitivity of the empirical parameter of particles’ maximum velocity in PSO. By using the proposed random reflection boundary technique, PSO algorithms can have less empirically user-selected parameter and better searching capability in terms of implementation convenience, performance robustness and algorithmic suitability for complex real-world applications such as multimodal/unimodal optimization of composite structures.

4. **Original methodology innovation:** in *Paper 4 (Chapter 5)*, an effective and efficient methodology is synthetically developed for concept design of AGS composite structures, based on a novel multi-level optimization strategy
where the inner 3-stage optimization process is nested within an outer 3-step optimization process. The basic thoughts of this innovative multi-level optimization methodology can be conveniently and generally extended to design optimization of other engineering problems in which many topology configuration options, large number of design variables and complex system responses are involved.
References


34. VCP Chen, Tsui KL, Barton RR, Meckesheimer M. A review on design, modeling and applications of computer experiments. IIE transactions 2006; 38(4): 273-291.


Chapter 2

An efficient finite element model for buckling analysis of grid stiffened laminated composite plates

(Paper 1)

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25
Abstract

For the prediction of buckling response of grid stiffened composite panels having different stiffening arrangements, an efficient finite element modelling technique is presented. The laminated skin of the stiffened structure is modelled with a triangular degenerated curved shell element having 3 corner nodes and 3 mid-side nodes. An efficient curved beam element compatible with the shell element is developed for the modelling of stiffeners which may have different lamination schemes. The deformation of the 3-noded degenerated beam element is completely defined in terms of the degrees of freedom of shell elements and it does not require any additional degrees of freedom. This modelling strategy has helped to reduce the number of unknowns significantly compared to the usual approach where solid or shell elements are used for modelling stiffeners. As the usual formulation of degenerated beam elements overestimates their torsional rigidity, a torsion correction factor is introduced for different lamination schemes. The formulation of the beam element may be considered as one of the major contributions of this study. Numerical examples are solved by the proposed finite element technique to assess its performance and it shows that the accuracy of the proposed model is quite satisfactory. A parametric study is presented to show the effects of skin thickness, stiffener breadth and stiffener depth on the buckling capacity of grid stiffened composite plates. Though the present study is focussed on grid stiffened plates, the formulation is capable of modelling stiffened shell having any curvature. Moreover, this computationally efficient model can be used for optimal design of grid stiffened composite panels at initial stage.
2.1 Introduction

Due to high specific strength and stiffness as well as flexibility in tailoring the structural performance, fiber reinforced polymer (FRP) laminated composites have been widely used in many weight sensitive structural applications such as aerospace structures, satellite launch vehicles, automotive structures and marine vehicles as well as modern bridge decks and buildings. On the other hand, a stiffened panel always performs better in resisting loads compared to an unstiffened panel of same weight. Thus a combination of lightweight composite materials and stiffened structural forms can efficiently enhance the load resisting capability that can be buckling strength of a structure. The structural behaviour of a stiffened composite panel is quite complex due to complex structural form and anisotropic material properties of the skin and stiffeners. These types of thin-walled structures are vulnerable to lose global and local stability under compression loadings. Consequently, the buckling analysis of these structures is a critical component for in-deep understanding of their buckling performances and also a prerequisite for a reliable buckling-resistant design of these panels.

There is vast literature on stiffened plates and shells specifically for these structures made of isotropic materials and simple stiffener orientations. The research carried out up to the middle of nineteen nineties was nicely reviewed by Sinha and Mukhopadhyay [1] who mostly covered research on isotropic stiffened panels. In 1998, Bedair [2] presented an extensive review on the methods for predicting global and local buckling responses of isotropic stiffened plates. Prusty [3] compiled the research on composite stiffened shells conducted up to the end of nineteen nineties. Dawe [4] elaborately summarized the work on the use of the finite strip approach for predicting buckling response of composite structures with unidirectional stiffeners in 2002. In order to avoid any repetition, only the recent investigations in this area are mentioned in the following two paragraphs.

Bisagni and Vescovini [5] presented two analytical solutions for local buckling of longitudinally stiffened composite panels where the skin was
idealized as a thin plate and the stiffeners were modelled as torsion bars that provided torsional rigidity along unloaded edges to resist local buckling of the skin. Applying their model to calibrate buckling constraints, they [6] conducted the minimum weight design of longitudinally stiffened composite plates subjected to axial compression. Stamatelos et al. [7] proposed an analytical method to predict the local buckling load of plates with symmetrically exposed longitudinal stiffeners, while Sun and Harik [8] developed an analytical strip method for buckling analysis of composite plates with unidirectional stiffeners having anti-symmetrical exposure. A semi-analytical finite strip method was proposed by Mocker and Reimerdes [9], in which element stiffness matrices of each discretized strip of the whole structure were determined by closed-form solutions to the governing differential equations, for buckling analysis of stringer stiffened composite shells. Wang et al. [10] employed finite strip method to investigate the buckling load capacity of stringer stiffened composite panels with various number of T-shape stiffeners. Rikards et al. [11] developed a finite element model with six degrees of freedom at each node and applied to buckling analysis of an isotropic stiffened plate and a unstiffened composite cylindrical panel under axial compression. Guo et al. [12] proposed a layerwise finite element formulation to study the buckling responses of single rib stiffened composite plates according to different parameters such as plate aspect ratio and stiffener depth ratio. Chen and Soares [13] investigated the effects of plate thickness and stacking sequence on the post-buckling response of longitudinally stiffened composite panels, based on a non-linear finite element model using degenerated shell elements. Prusty [14] presented a finite element model for buckling analysis of laminated cylinder shell with a laminated central stringer and carried out a parametric study for shell thickness ratio, stiffener shape, number of layers and curvature ratio.

Some researchers used commercially available finite element packages to conduct buckling analysis and optimization of stiffened composite panels. Herencia et al. [15] performed buckling analysis using quadrilateral shell elements having 4 nodes and used an optimization aid of MSC/NASTRAN for optimum design of longitudinally stiffened composite plate with T-shaped
laminated stiffener subjected to buckling loads. A longitudinally stiffened composite panel was modelled by Lanzi and Giavotto [16] using 10120 4-node shell elements (S4R) of ABAQUS to optimise the structure under buckling loads. Mallela and Upadhyay [17] conducted a parametric study for in-plane shear buckling for uni-directionally stiffened composite plates based on 450 finite element models developed in ANSYS using 8-noded quadrilateral shell elements. Jain and Upadhyay [18] extended their work for in-plane shear buckling of angle-stiffened, T-stiffened and hat-stiffened composite plates to enrich the database for sensitivity of different parameters of shear-loaded stringer stiffened composite plates.

All these studies mentioned above have considered simple stiffener orientations and didn’t address the problem of advanced grid stiffened (AGS) panels with ortho-grid, x-grid, bi-grid or iso-grid stiffening arrangements, as shown in Figs. 2.1 and 2.2. One of the earlier works on buckling of grid stiffened composite cylindrical panels was due to Reddy et al. [19] who idealised the stiffened panel as an equivalent bare shell panel by smearing the stiffeners within the shell skin. They [19] used Donnell shell theory to model the structure and proposed an analytical solution based on Galerkin approach of the shell problem. The transverse shear deformation of stiffeners was included whereas the torsion of these ribs was ignored. A similar modelling approach was proposed by Chen and Tsai [20] where the torsion of stiffeners was incorporated in their model. Kidane et al. [21] also proposed an analytical solution based on smeared model to calculate the equivalent stiffness parameters of iso-grid stiffened composite cylinders where the stiffeners were assumed to bear uniform axial loading only to predict the global buckling load. Using this model [21], Wodesenbet et al. [22] conducted a parametric study and Rao and Lakshmi [23] optimised the buckling load capacity for iso-grid stiffened composite cylindrical shells. Zhang et al. [24] developed a stiffened plate element model for buckling analysis of ortho-grid and iso-grid stiffened composite panels, without considering the rotational compatibility of stiffeners and skin along their interface. Based on the smearing theory and the minimum potential energy principle, Shi et al. [25] presented an equivalent
stiffness model for grid stiffened conical composite shells considering a non-uniform grid distribution.

It shows that a limited number of investigations have been conducted on AGS composite panels and all these studies are based a simplified idealisation of the structural system. Moreover, an analytical solution is used in most of the cases which is attractive in terms of computational involvement but it lacks in terms of generality because an analytical model can only be applicable to simple structural geometry, loadings and boundary conditions. These limitations may be overcome by using a reliable commercially available finite element code but the computational involvement will be reasonably high if the skin and stiffeners are modelled with shell elements. This modelling strategy is applicable for blade stiffened panels only where stiffeners are made with thin laminates with their layers stacked perpendicular to the skin surface. If the stiffeners are thick and their layers are stacked parallel to the skin surface, the modelling should be done with 3D solid elements where the computational involvement will be extremely high. Thus these modelling strategies are not suitable in a situation where a repetitive analysis of the structure is required. This type of scenario is quite common in a problem of optimum design of a structure which is a part of this ongoing research.

Therefore there is genuine need for a computationally efficient as well as reliable modelling technique that can be used for buckling analysis of different grid stiffened composite panels accommodating different stiffener configurations, as well as buckling modes under arbitrary loadings and boundary conditions. In the present study an efficient finite element modelling technique having sufficient generality is proposed for the buckling analysis of grid stiffened composite panels having different grid configurations. The detail of the finite element formulation is presented in the following Section 2.2. The laminated plate skin is modelled with a shear deformable triangular curved shell element having 6 nodes where each node contains 5 degrees of freedom. An efficient curved beam element compatible with the shell element is developed for the modelling of laminated stiffeners which may have two different lamination schemes. In Section 2.3, a number
of numerical examples are investigated to validate the proposed approach. In Section 2.4, a parametric study is carried out to investigate the effects of skin thickness, stiffener breadth and stiffener depth on the buckling responses of grid stiffened composite plates. Finally, the significance of the proposed finite element model and its potential for optimal design of grid stiffened composite laminates are concluded in Section 2.5.

Fig. 2.1. Ortho-grid, x-grid and bi-grid stiffened panels

Fig. 2.2. Iso-grid stiffened panels


2.2 Finite element formulation

The governing equation for the buckling analysis of a stiffened composite panel can be expressed as

$$[K]\{\delta\} = \lambda[K_g]\{\delta\}$$  \hspace{1cm} (2.1)

where $[K]$ and $[K_g]$ are the elastic stiffness matrix and the geometric stiffness matrix of the stiffened structure, respectively, which are consisting of their contributions of the skin and stiffeners. In the above equation, $\lambda$ and $\{\delta\}$ are the buckling load parameter and the displacement vector representing the buckling mode of the structure, respectively.

2.2.1 Stiffness matrix of the shell element

The concept of the three-dimensional (3D) degenerated shell element [26, 27] is utilized to derive the triangular curved shell element having six nodes with five usual degrees of freedom per node as shown in Fig. 2.3. The present formulation differs from that of Panda and Natarajan [27] in the treatment of mapping the thickness direction. Panda and Natarajan [27] have mapped the individual layers whereas the entire laminate is mapped in the present formulation.

![6-noded triangular curved shell element](image)

**Fig. 2.3.** 6-noded triangular curved shell element
The element geometry (Fig. 2.3) can be represented by the curvilinear coordinate system \((\zeta - \eta - \zeta')\) where the coordinates of any point within the element can be expressed as

\[
\begin{bmatrix}
x \\
y \\
z 
\end{bmatrix} = \sum_{i=1}^{6} N_i(\zeta, \eta) \begin{bmatrix} x_i \\
y_i \\
z_i \end{bmatrix} + \sum_{i=1}^{6} N_i(\zeta, \eta) \frac{\zeta'}{2} \begin{bmatrix} 0 \\
0 \\
1 \end{bmatrix}
\]

(2.2)

where \(N_i(\zeta, \eta), \ t_i, \ \{v_3\}\) are the shape function [28] corresponding to node \(i\) (defined in \((\zeta - \eta)\) plane which is the shell mid-plane or reference plane), the shell thickness at node \(i\) and the vector normal to \((\zeta - \eta)\) plane (directed along \(\zeta'\) i.e. thickness direction) at node \(i\), respectively.

The displacement components \(\begin{bmatrix} u & v & w \end{bmatrix}^T\) (along \(x\), \(y\) and \(z\) axis respectively) at any point within the element are expressed in terms of reference plane (shell mid-surface) parameters (Fig. 2.3) which consist of two nodal tangential vectors \(\{v_{1i}\}\) and \(\{v_{2i}\}\) (\(\{v_{1i}\}\) passes through \((x - z)\) plane) mutually perpendicular to each other, nodal displacements \(\begin{bmatrix} u & v & w \end{bmatrix}^T\) and nodal rotations \(\begin{bmatrix} \theta_{xi} & \theta_{yi} \end{bmatrix}^T\) along the nodal tangential vectors as

\[
\begin{bmatrix}
u \\
w 
\end{bmatrix} = \sum_{i=1}^{6} N_i(\zeta, \eta) \begin{bmatrix} u_i \\
v_i \\
w_i \end{bmatrix} - \sum_{i=1}^{6} N_i(\zeta, \eta) \frac{\zeta'}{2} \begin{bmatrix} \{v_{1i}\} \\
\{v_{2i}\} \end{bmatrix} \begin{bmatrix} \theta_{xi} \\
\theta_{yi} \end{bmatrix}
\]

(2.3)

The components of strains and stresses at a surface having a specific value of \(\zeta'\) are taken along the local orthogonal axis system \(\{\tau_{1i}\}, \{\tau_{2i}\}\) and \(\{\tau_{3i}\}\), which is similar to that at the mid-surface corresponding to \(\{v_{1i}\}, \{v_{2i}\}\) and \(\{v_{3i}\}\). It is assumed that the normal stress and strain components in the thickness direction are sufficiently small and these are neglected. Given that the local axis system is defined by \((x' - y' - z')\) and the corresponding displacement components are \(u', v'\) and \(w'\), the strain components are defined as
The local strain components can be expressed in terms of those in the global coordinate system as follows.

\[
\begin{bmatrix}
\frac{\partial u'}{\partial x'} & \frac{\partial v'}{\partial x'} & \frac{\partial w'}{\partial x'} \\
\frac{\partial u'}{\partial y'} & \frac{\partial v'}{\partial y'} & \frac{\partial w'}{\partial y'} \\
\frac{\partial u'}{\partial z'} & \frac{\partial v'}{\partial z'} & \frac{\partial w'}{\partial z'}
\end{bmatrix}
= [\mathbf{H}]
\begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\
\frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\
\frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z}
\end{bmatrix}
\] (2.5)

where

\[
[\alpha] = \begin{bmatrix} \{\bar{v}_{1i}\} & \{\bar{v}_{2i}\} & \{\bar{v}_{3i}\} \end{bmatrix}
\] (2.6)

With the help of Eqs. (2.5) and (2.6) as well as the Jacobian matrix \([J]\) (Eq. (2.10)), the local strain vector can be rewritten as
where

\[ [T] = \begin{bmatrix} \{v_i\} & \{v_{2i}\} & \{v_{3i}\} \end{bmatrix} [\alpha]^{\top}, \]

and

\[
[J] = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\
\frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{1}{2} \sum N_i v_{3xi} \\
\frac{1}{2} \sum N_i v_{3yi} \\
\frac{1}{2} \sum N_i v_{3zi}
\end{bmatrix}
\]

and

\[
[J] = \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\
\frac{\partial u}{\partial \xi} & \frac{\partial u}{\partial \eta} & \frac{\partial u}{\partial \zeta} \\
\frac{\partial u}{\partial \xi} & \frac{\partial u}{\partial \eta} & \frac{\partial u}{\partial \zeta}
\end{bmatrix}
\]

\[\{\varepsilon\}' = [H]^{T} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{1} \\ \gamma_{2} \\ \gamma_{3} \\ \gamma_{4} \\ \gamma_{5} \\ \gamma_{6} \\ \gamma_{7} \\ \gamma_{8} \\ \gamma_{9} \\ \gamma_{10} \end{bmatrix} = [H]^{T} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{1} \\ \gamma_{2} \\ \gamma_{3} \\ \gamma_{4} \\ \gamma_{5} \\ \gamma_{6} \\ \gamma_{7} \\ \gamma_{8} \\ \gamma_{9} \end{bmatrix}
\]

(2.7)

\[
[J]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

(2.8)

\[
\Gamma = \begin{bmatrix} 0 & [J]^{-1} & 0 \\ 0 & 0 & [J]^{-1} \end{bmatrix}
\]

(2.9)

\[
\sum \frac{\partial N_i}{\partial \xi} \left( x_i + \frac{\gamma_i}{2} v_{3xi} \right) \]

\[
= \sum \frac{\partial N_i}{\partial \xi} \left( y_i + \frac{\gamma_i}{2} v_{3yi} \right) \]

\[
= \sum \frac{\partial N_i}{\partial \eta} \left( z_i + \frac{\gamma_i}{2} v_{3zi} \right)
\]

(2.10)
The Jacobian matrix in the above equation is used to get the different vectors. The nodal vectors \( \{ v_{ni} \} \) normal to \((\xi - \eta)\) are first calculated from the cross product of the corresponding nodal tangential vectors along \(\xi\) and \(\eta\) obtained from the first two rows of the Jacobian matrix (Eq. (2.10)) which is in turn used to calculate nodal tangential vectors \( \{ v_{ni} \} \) and \( \{ v_{nj} \} \) in the desired directions. The vectors \( \{ v_{ni} \}, \{ v_{nj} \} \) and \( \{ v_{nk} \} \) are calculated in a similar manner.

After substitution of the derivatives of displacement components \( u \), \( v \) and \( w \) (Eq. (2.3)) in Eq. (2.7), the strain components can be expressed in terms of nodal displacement vector \( \{ \delta \} \) as

\[
\{ \varepsilon \} = [H] [T] [ \Gamma ] [L] [\delta] = [B] [\delta] \\
(2.11)
\]

The shell is made of fiber reinforced laminated composites consisting of a number of orthotropic layers having different orientations. For such a layer, the stress-strain relationship in the material axis system may be given by [29]

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12} \\
\tau_{23} \\
\tau_{31}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{12} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & G_{12} & 0 & 0 \\
0 & 0 & 0 & \beta G_{23} & 0 \\
0 & 0 & 0 & 0 & \beta G_{31}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12} \\
\gamma_{23} \\
\gamma_{31}
\end{bmatrix}
\]

or

\[
\{ \sigma \} = [Q] \{ \varepsilon \} \\
(2.13)
\]

where \( Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}} \), \( Q_{22} = \frac{E_2}{1 - \nu_{12} \nu_{21}} \), \( Q_{12} = \frac{\nu_{12} E_1}{1 - \nu_{12} \nu_{21}} = \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}} \) and \( \beta \) is the shear correction factor, which is taken as 5/6.

For different ply orientations, the stress-strain relationship in the desired local axis system \((x' - y' - z')\) is determined by [29]

\[
\{ \sigma' \} = [D'] \{ \varepsilon' \} \\
(2.14)
\]

where

\[
[D'] = [T']^T [Q] [T']
\]

and
where \( \theta \) is the orientation of principle material axis 1-2 with respect to shell local \( x' - y' \) coordinate axis.

The elastic stiffness matrix of an element can be derived with the help of the virtual work technique and it may be expressed as

\[
[T_e] = \begin{bmatrix}
\cos^2 \theta & \sin \theta & \sin \theta \cos \theta & 0 & 0 \\
\sin^2 \theta & \cos^2 \theta & -\sin \theta \cos \theta & 0 & 0 \\
-2\sin \theta \cos \theta & 2\sin \theta \cos \theta & \sin^2 \theta \cos^2 \theta & 0 & 0 \\
0 & 0 & 0 & \cos \theta & \sin \theta \\
0 & 0 & 0 & -\sin \theta & \cos \theta
\end{bmatrix}
\]  

(2.16)

The integration in Eq. (2.17) is carried out numerically following Gauss quadrature integration technique where three-point integration scheme in the \( (\xi - \eta) \) plane and two-point integration scheme along \( \zeta \) direction is adopted. This is also applied to the subsequent quantities involving integration. In the present case the above integration scheme is applied for the individual layers since \([D']\) is different for different layers and their contributions are added together to get the stiffness matrix of an element.

### 2.2.2 Stiffness matrix of the stiffener element

The stiffeners are modelled as curved beam elements having 3 nodes which are placed along the edges of shell elements with an eccentricity \( (e) \) along the shell thickness. The deformation of the beam element is completely expressed in terms of degrees of freedom of the corresponding shell element nodes. The composite curved beam element is derived from the 3D solid element degenerated for small depth and breadth compared to its length. The geometry of the curved beam element shown in Fig. 2.4 can be represented by another
curvilinear coordinate system \((\xi - \eta - \zeta)\) and the coordinates of any point within the element can be expressed as

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \sum_{i=1}^{3} N_i(\xi)\{x_i\} + \sum_{i=1}^{3} N_i(\eta)\left(\frac{\xi d}{2} + e\right)\{v^S_{2i}\} + \sum_{i=1}^{3} N_i(\zeta)\left(\frac{\eta b}{2}\right)\{v^S_{3i}\} \tag{2.18}
\]

where \(\xi, \eta\) and \(\zeta\) are oriented along the beam length, breadth and depth respectively. The displacements at any point within the element can be expressed as

\[
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} = \sum_{i=1}^{3} N_i(\xi)\begin{bmatrix} T_{i1} \\ T_{i2} \\ T_{i3} \end{bmatrix}\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} - \sum_{i=1}^{3} N_i(\eta)\left(\frac{\xi d}{2} + e\right)\begin{bmatrix} \{v^S_{1i}\} \\ \{v^S_{2i}\} \end{bmatrix}\begin{bmatrix} \theta_{si} \\ \theta_{sj} \end{bmatrix} \tag{2.19}
\]

where the matrix \([T_i]\) is used to eliminate the local lateral displacement of the stiffener. This matrix can be obtained for the nodal vectors of the stiffener \((\{v^S_{1i}\}, \{v^S_{2i}\}\) and \(\{v^S_{3i}\}\) directed along \(\xi, \eta\) and \(\zeta\) respectively) as

\[
[T_i] = \begin{bmatrix} \{v^S_{1i}\} & [0] & \{v^S_{2i}\} & \{v^S_{3i}\} \end{bmatrix}^T \tag{2.20}
\]

![Fig. 2.4. 3-noded curved stiffener element](image)

These nodal vectors of the stiffener can be obtained from the corresponding nodal vectors of the shell element as

\[
\begin{align*}
\{v^S_{1i}\} &= \{v_{1i}\} \cos \theta_i + \{v_{2i}\} \sin \theta_i \\
\{v^S_{2i}\} &= -\{v_{1i}\} \sin \theta_i + \{v_{2i}\} \cos \theta_i \\
\{v^S_{3i}\} &= \{v_{3i}\}
\end{align*} \tag{2.21}
\]
where the angle $\theta_s$ is used to indicate the orientation of $\{v^s_i\} - \{v^s_{2i}\}$ with respect to $\{v_i\} - \{v_{2i}\}$.

The stiffeners may have two types of lamination schemes as shown in Fig. 2.5. The constitutive equation of the stiffener can be conveniently expressed in its local axis system $x' - y' - z'$ (oriented along $\xi'$, $\eta'$ and $\zeta'$) of the element as

$$
\begin{bmatrix}
\sigma'_{x'} \\
\tau_{x'y'} \\
\tau_{x'z'}
\end{bmatrix} =
\begin{bmatrix}
\bar{Q}_{1m} & 0 & 0 \\
0 & \beta_s \bar{Q}_{5m} & 0 \\
0 & 0 & \beta_t \bar{Q}_{6m}
\end{bmatrix}
\begin{bmatrix}
\varepsilon'_{x'} \\
\gamma'_{x'y'} \\
\gamma'_{x'z'}
\end{bmatrix}
$$

(2.22)

where $\beta_s$ is the shear correction factor which is taken as $5/6$ and $\beta_t$ is the torsion correction factor. For the two lamination schemes (Fig. 2.5), the stiffness parameters ($\bar{Q}_{1m}$, $\bar{Q}_{5m}$ and $\bar{Q}_{6m}$) and $\beta_t$ can be obtained from the 2D stiffness parameters [29] of the stiffener laminate as follows.

![Stiffener ply lamination schemes](image)

(a) Type I (parallel to the skin)  (b) Type II (perpendicular to the skin)

Fig. 2.5. Stiffener ply lamination schemes

Ply arrangement I: $\bar{Q}_{1m} = \bar{Q}_{11} + \bar{Q}_{12} \frac{\bar{Q}_{45} \bar{Q}_{26} - \bar{Q}_{21} \bar{Q}_{46}}{\bar{Q}_{22} \bar{Q}_{66} - \bar{Q}_{62} \bar{Q}_{26}} + \bar{Q}_{16} \frac{\bar{Q}_{41} \bar{Q}_{22} - \bar{Q}_{21} \bar{Q}_{42}}{\bar{Q}_{26} \bar{Q}_{62} - \bar{Q}_{66} \bar{Q}_{22}}$, $\bar{Q}_{5m} = \bar{Q}_{55} - \bar{Q}_{54} \frac{\bar{Q}_{45}}{\bar{Q}_{44}}$, $\bar{Q}_{6m} = \bar{Q}_{66}$, and

$$
\beta_t = \frac{3kb^2 \sum_{i=1}^{n} \bar{Q}_{5m}^i (s_{i+1} - s_i)}{\sum_{i=1}^{n} \bar{Q}_{6m}^i (s_{i+1}^3 - s_i^3)}
$$

(2.23)
Ply arrangement II: 
\[ \begin{align*}
\mathbf{Q}_{1m} &= \mathbf{Q}_{11} + \mathbf{Q}_{12} \left( \frac{\mathbf{Q}_{61} \mathbf{Q}_{26} - \mathbf{Q}_{21} \mathbf{Q}_{66}}{\mathbf{Q}_{22} \mathbf{Q}_{66} - \mathbf{Q}_{62} \mathbf{Q}_{26}} \right) + \mathbf{Q}_{16} \left( \frac{\mathbf{Q}_{61} \mathbf{Q}_{22} - \mathbf{Q}_{21} \mathbf{Q}_{62}}{\mathbf{Q}_{22} \mathbf{Q}_{66} - \mathbf{Q}_{62} \mathbf{Q}_{26}} \right), \\
\mathbf{Q}_{5m} &= \mathbf{Q}_{66}, \quad \mathbf{Q}_{6m} = \mathbf{Q}_{55} - \mathbf{Q}_{44} \frac{\mathbf{Q}_{45}}{\mathbf{Q}_{44}} \quad \text{and}
\end{align*} \]

This is achieved by considering some stress free conditions \((\sigma_r = \tau_{rs} = 0)\). In the above equation, \(k\) is the torsion constant of the rectangular beam section in Fig. 2.4, if it is assumed to be homogeneous. For the calculation of 2D stiffness parameters \((\mathbf{Q}_{11}, \mathbf{Q}_{12}, \ldots, \mathbf{Q}_{66})\) of the stiffener laminate, directions 1, 2 and 3 are taken along the stiffener axis, \(r\) and \(s\) (Fig. 2.5) respectively. The stiffness matrix of the stiffener element is derived using Eqs. (2.18) – (2.24).

### 2.2.3 Geometric stiffness matrix of the shell element

The initial stress vector \(\{\sigma'\}\) and nonlinear Green-Lagrangian strain vector \(\{\varepsilon'_n\}\) in the shell local axis system \(x' - y' - z'\) are respectively given as

\[ \{\sigma'\} = \begin{bmatrix} \sigma_{x'} & \sigma_{y'} & \tau_{x'y'} & \tau_{x'z'} & \tau_{y'z'} \end{bmatrix}^T \]

\[ \{\varepsilon'_n\} = \begin{bmatrix} \frac{\partial u'}{\partial x'}^2 + \frac{\partial v'}{\partial x'}^2 + \frac{\partial w'}{\partial x'}^2 & \frac{\partial u'}{\partial y'}^2 + \frac{\partial v'}{\partial y'}^2 + \frac{\partial w'}{\partial y'}^2 & \frac{\partial u'}{\partial x'} \frac{\partial v'}{\partial x'} + \frac{\partial v'}{\partial x'} \frac{\partial w'}{\partial x'} + \frac{\partial w'}{\partial x'} \frac{\partial u'}{\partial x'} & \frac{\partial u'}{\partial y'} \frac{\partial v'}{\partial y'} + \frac{\partial v'}{\partial y'} \frac{\partial w'}{\partial y'} + \frac{\partial w'}{\partial y'} \frac{\partial u'}{\partial y'} & \frac{\partial u'}{\partial z'} \frac{\partial v'}{\partial z'} + \frac{\partial v'}{\partial z'} \frac{\partial w'}{\partial z'} + \frac{\partial w'}{\partial z'} \frac{\partial u'}{\partial z'} \end{bmatrix} \]

The strain energy due to initial stress \(\{\sigma'\}\) can be expressed as
\[
U_\sigma = \int \{\sigma\}^T \{\varepsilon\} d\xi' d\eta' d\zeta' = \frac{1}{2} \int \{\varepsilon\}^T \{\varepsilon\} d\xi' d\eta' d\zeta'
\quad (2.27)
\]

where
\[
\{\varepsilon\} = \left[ \frac{\partial u'}{\partial \xi'} \frac{\partial u'}{\partial \eta'} \frac{\partial w'}{\partial \xi'} \frac{\partial w'}{\partial \eta'} \frac{\partial v'}{\partial \xi'} \frac{\partial v'}{\partial \eta'} \frac{\partial u'}{\partial \zeta'} \frac{\partial w'}{\partial \zeta'} \frac{\partial v'}{\partial \zeta'} \frac{\partial w'}{\partial \zeta'} \frac{\partial w'}{\partial \xi'} \frac{\partial w'}{\partial \eta'} \right]^T
\quad (2.28)
\]

and
\[
[r] = \begin{bmatrix}
[\varepsilon^T] & 0 & 0 \\
0 & [\varepsilon^T] & 0 \\
0 & 0 & [\varepsilon^T]
\end{bmatrix}
\quad (2.29)
\]

The sub-matrix \([\varepsilon^T]\) within the initial stress matrix \([\tau]\) in Eq. (2.29) can be expressed as
\[
[\varepsilon^T] = \begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & 0
\end{bmatrix}
\quad (2.30)
\]

The geometric strain vector as defined in Eq. (2.28) can be expressed in terms of nodal displacement vector following the techniques used for the elastic strain vector (Eq. (2.11)) as
\[
\{\varepsilon\} = [r][\Gamma][L][\delta] = [B_g][\delta]
\quad (2.31)
\]

Substituting Eq. (2.31) into Eq. (2.27) and applying the minimisation technique of the energy or principle of virtual work, the geometric stiffness matrix of the shell element can be expressed as
\[
[k_g] = \int [B_g]^T [r][B_g] d\xi' d\eta' d\zeta' = \int [B_g]^T [r][B_g] J d\xi d\eta d\zeta
\quad (2.32)
\]

### 2.2.4 Geometric stiffness matrix of the shell element

of the energy or principle of virtual work, the geometric stiffness matrix of the shell element

The procedure adopted of the derivation of the geometric stiffness matrix of the shell element in Section 2.2.3 is followed to derive the geometric stiffness
matrix of the stiffener element in a similar manner with the help of Eqs. (2.18) – (2.24) used for the derivation of elastic stiffness of the stiffener element in Section 2.2.2.

Chapter 2

2.3 Numerical examples

2.3.1 A simply supported laminated square plate under uniaxial compression

A cross-ply (0/90/90/0) square laminated plate \((a \times a)\) is subjected to uniaxial compression \((N_x)\) in the \(x\) direction. Taking thickness ratio \((h/a)\) of the plate as 0.1 and \(E_1/E_2 = 20, 30\) and 40 \((G_{12} = G_{13} = 0.6E_2, G_{23} = 0.5E_2, \nu_{12} = 0.25)\), the analysis is carried out with three different mesh sizes. The values of critical buckling load obtained in the present analysis are presented in non-dimensional form in Table 2.1 with those reported by Khdeir and Librescu [30], Noor [31] and Kam and Chang [32]. The table shows that the results have very good agreement.

Table 2.1
Non-dimensional buckling load parameter \(N_xa^2/(E_2h^3)\) of the simply supported cross-ply (0/90/90/0) square laminate subjected to uniaxial compression

<table>
<thead>
<tr>
<th>References</th>
<th>(E_1/E_2 = 20)</th>
<th>(E_1/E_2 = 30)</th>
<th>(E_1/E_2 = 40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present model ((8\times8))</td>
<td>15.187</td>
<td>19.565</td>
<td>23.244</td>
</tr>
<tr>
<td>Present model ((10\times10))</td>
<td>15.181</td>
<td>19.558</td>
<td>23.237</td>
</tr>
<tr>
<td>Present model ((12\times12))</td>
<td>15.178</td>
<td>19.556</td>
<td>23.235</td>
</tr>
<tr>
<td>Noor [31]</td>
<td>15.019</td>
<td>19.304</td>
<td>22.880</td>
</tr>
</tbody>
</table>

\(a: mesh\) division
2.3.2 A simply supported stiffened square plate under uniaxial compression

A square plate \((a \times a)\) having a stiffener along the plate centreline parallel to \(x\) axis is subjected to uniaxial compressive stress \(\sigma_x\). The stability of the isotropic stiffened panel having same material for the plate and the stiffener is studied by taking the stiffener parameters corresponding to cross-sectional area \(\gamma = A_s/bh\) as 0.05, 0.1 and 0.2 and moment of inertia \(\beta = EI_s/Db\) as 5, 10 and 15 \((b\) is the plate width = \(a\), \(D\) is the flexural rigidity of the plate) as defined by Timoshenko and Gere [33]. The eccentricity and torsional rigidity of the stiffener are taken as zero [33]. Taking \(h/a = 0.01\) for the plate and \(\nu = 0.3\), the analysis is carried out with three different mesh sizes in each case and the values of buckling stress obtained are presented in non-dimensional form with those of Timoshenko and Gere [33] in Table 2.2 which shows that the agreement between the results is quite good.
Table 2.2

Non-dimensional buckling stress parameter $\sigma h a^2 / (p^2 D)$ of a simply supported square plate having a central stiffener

<table>
<thead>
<tr>
<th>Stiffener parameter</th>
<th>Present model (8×8)</th>
<th>Present model (10×10)</th>
<th>Present model (12×12)</th>
<th>Timoshenko and Gere [33]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0.05, \beta = 5$</td>
<td>11.882</td>
<td>11.790</td>
<td>11.746</td>
<td>12.0</td>
</tr>
<tr>
<td>$\gamma = 0.05, \beta = 10$</td>
<td>17.710</td>
<td>16.904</td>
<td>16.490</td>
<td>16.0</td>
</tr>
<tr>
<td>$\gamma = 0.05, \beta = 15$</td>
<td>17.632</td>
<td>16.834</td>
<td>16.424</td>
<td>16.0</td>
</tr>
<tr>
<td>$\gamma = 0.1, \beta = 5$</td>
<td>11.116</td>
<td>11.039</td>
<td>11.002</td>
<td>11.1</td>
</tr>
<tr>
<td>$\gamma = 0.1, \beta = 10$</td>
<td>17.439</td>
<td>16.904</td>
<td>16.409</td>
<td>16.0</td>
</tr>
<tr>
<td>$\gamma = 0.1, \beta = 15$</td>
<td>17.632</td>
<td>16.834</td>
<td>16.424</td>
<td>16.0</td>
</tr>
<tr>
<td>$\gamma = 0.2, \beta = 5$</td>
<td>9.7444</td>
<td>9.6878</td>
<td>9.6600</td>
<td>9.72</td>
</tr>
<tr>
<td>$\gamma = 0.2, \beta = 10$</td>
<td>15.652</td>
<td>15.525</td>
<td>15.464</td>
<td>15.8</td>
</tr>
<tr>
<td>$\gamma = 0.2, \beta = 15$</td>
<td>17.632</td>
<td>16.834</td>
<td>16.424</td>
<td>16.0</td>
</tr>
</tbody>
</table>
2.3.3 A simply supported rectangular stiffened plate under in-plane shear

An angle ply [(45/-45)₄], rectangular laminated plate (aspect ratio $a/b = 2$) having a cross ply laminated stiffener [(0/90)₄], as shown in Fig. 2.6 is subjected to in-plane shear stress $\tau_{xy}$. The plate skin as well as the stiffener are made of 0.125 mm thick sixteen layers where the stiffener layers are perpendicular to the skin surface. The stiffener depth ($d$) is defined in terms of a factor $D_r = 100d/b$ ($b$ is the plate width) which is taken as 2, 4, 6 and 8 in the present study. Taking $h/a = 0.01$, the analysis is carried out in a similar manner and the values of buckling stress obtained are presented in non-dimensional form with those reported by Loughlan [34] and Prusty and Satsangi [35] in Table 2.3, which shows that the results have reasonably good agreement. The material properties used for the plate and the stiffener are: $E_1 = 140$ GPa, $E_2 = 10$ GPa, $G_{12} = 5$ GPa and $\nu_{12} = 0.3$.

![Fig. 2.6. Simply supported laminated rectangular stiffened plate under in-plane shear](image-url)
Table 2.3
Non-dimensional buckling stress parameter $\tau_{xy} h b^2 \left(\frac{p^2 D_{44}}{4} \right)$ of a simply supported laminated rectangular plate having a laminated central stiffener

<table>
<thead>
<tr>
<th>References</th>
<th>$D_r = 2$</th>
<th>$D_r = 4$</th>
<th>$D_r = 6$</th>
<th>$D_r = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present model (8×4)</td>
<td>9.9436</td>
<td>11.5538</td>
<td>13.2539</td>
<td>14.5012</td>
</tr>
<tr>
<td>Present model (12×6)</td>
<td>9.3535</td>
<td>11.0279</td>
<td>12.6294</td>
<td>13.5270</td>
</tr>
<tr>
<td>Present model (16×8)</td>
<td>9.1495</td>
<td>10.7728</td>
<td>12.2817</td>
<td>13.0749</td>
</tr>
<tr>
<td>Loughlan [34]</td>
<td>9.33</td>
<td>12.22</td>
<td>13.55</td>
<td>13.77</td>
</tr>
</tbody>
</table>

2.3.4 Simply supported grid stiffened composite plates under uniaxial compression

Grid stiffened composite plates having different grid or stiffening arrangements such as ortho-grid, x-grid, bi-grid and iso-grid as shown in Fig. 2.7 are analysed taking $a = 1.732$ m and $b = 1$ m in all cases. The plate as well as the stiffeners are made with 64 layers of graphite/epoxy symmetric laminates having a stack sequence of $[(45/-45)_{16}]$. The thickness of each graphite/epoxy layer is 0.127 mm and having the following material properties: $E_1 = 138$ GPa, $E_2 = 10.3$ GPa, $G_{12} = G_{13} = 6.6$ GPa, $G_{23} = 2.6$ GPa and $\nu_{12} = 0.21$. The rectangular stiffener section has an aspect ratio of 3 (depth/breadth = 3) and it is made with Type II lamination stacking scheme (Fig. 2.5(b)). The stiffened panels are subjected to uniform compressive loads in the longitudinal direction (along $x$ axis). The critical buckling loads ($N_s$) obtained by the present finite element model are presented in Table 2.4. In order to validate the proposed model, these grid stiffened composite plates are analysed using a commercially available finite element code ABAQUS where the linear 4-noded quadrilateral shell element S4R having 6 degrees of
freedom per node is used to model the plate skin and stiffeners [16]. Based on the convergence study, the results from ABAQUS choosing the linear isoparametric elements at size 25mm are compared with the outcomes obtained from the proposed model using quadratic isoparametric elements at size 50mm, as shown in Table 2.4. As a specific case, the bare plate is analysed and the buckling load predicted by the proposed model is presented with the analytical solution [29] and finite element solution (ABAQUS) in Table 2.4. The table shows a reasonable agreement between the results for such complex structural forms which is quite encouraging.

Fig. 2.7. Simply supported grid stiffened composite plates
Table 2.4
Uniaxial buckling load $N_x$ ($10^5$ N/m) of grid stiffened composite plates having different configurations

<table>
<thead>
<tr>
<th>Stiffening configuration</th>
<th>Analytical solution [29] (size 25mm)</th>
<th>ABAQUS (size 125mm)</th>
<th>Present model (size 62.5mm)</th>
<th>Present model (size 50mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare plate</td>
<td>1.3308</td>
<td>1.3248</td>
<td>1.3494</td>
<td>1.3287</td>
</tr>
<tr>
<td>Ortho-grid</td>
<td>--</td>
<td>1.7581</td>
<td>1.7702</td>
<td>1.7464</td>
</tr>
<tr>
<td>X-grid</td>
<td>--</td>
<td>1.9028</td>
<td>1.8916</td>
<td>1.8675</td>
</tr>
<tr>
<td>Bi-grid</td>
<td>--</td>
<td>2.2271</td>
<td>2.3095</td>
<td>2.2826</td>
</tr>
<tr>
<td>Iso-grid</td>
<td>--</td>
<td>2.3577</td>
<td>2.3271</td>
<td>2.3154</td>
</tr>
</tbody>
</table>

2.4 Parametric study

The proposed finite element model is used to conduct a parametric study on buckling load prediction of grid stiffened composite plates (Fig. 2.7) considered in Section 2.3.4. The effect of any parameter is investigated by varying the value of that parameter while the values of other parameters are kept unchanged and these values are same as those of the examples in Section
2.3.4. The parameters examined in this section include skin thickness, stiffeners breadth and stiffener depth.

2.4.1 Effect of number of ply of the plate skin

The skin thickness is an indispensable design parameter for grid stiffened composite panels. As the laminae are of equal thickness, the thickness of the skin will change with the variation of its number of ply. In this study the number of ply is varied from 16 to 120 for the skin of the grid stiffened composite plates shown in Fig. 2.7 and the variation of buckling load obtained in the present analysis is plotted in Fig. 2.8 which shows a monotonic and significant increase of the buckling load capacity of all these grid stiffened panels as expected. It has been observed that the number of ply or thickness of the skin is a dominant parameter for the improvement of buckling capacity of grid stiffened composite plates. For instance the buckling load of ortho-grid, x-grid, bi-grid and iso-grid stiffened composite plates is increased by 5.03, 4.65, 3.64 and 3.57 times, respectively, when the number of ply for the skin is double from 48 to 96 in the present case.

![Figure 2.8](image_url)  
*Fig. 2.8. Effect of the number of ply for the plate skin*
2.4.2 Effect of number of ply for the blade stiffeners

The aspect ratio of the rectangular cross-section of stiffeners is one of the basic geometry parameters for grid stiffened composite panels. The effect of stiffener breadth which depends on the number of equally thick ply (Fig. 2.5(b)) is studied in this section. The variation of the buckling load capacity of the grid stiffened plates (Fig. 2.7) due to the variation of the number of ply for their stiffeners is plotted in Fig. 2.9. It shows more or less a linear variation of the buckling load capacity of all the cases but the rate of increase is different for different stiffening arrangements and it is highest for the iso-grid stiffened plate. It has been also observed that the effect of the stiffener breadth on the buckling load capacity of grid stiffened composite plates is not that significant compared to the effect of the skin thickness observed in Section 2.4.1. For an example the buckling capacities of ortho-grid, x-grid, bi-grid and iso-grid stiffened composite plates are increased by 15%, 18%, 30% and 31% respectively by increasing the number of ply of the stiffeners from 16 to 48. Similarly the buckling capacities of ortho-grid, x-grid, bi-grid and iso-grid stiffened composite plates are improved by 26%, 31%, 50% and 52%, respectively, if the number of ply is raised from 32 to 96.

Fig. 2.9. Effect of the number of ply for the stiffeners
2.4.3 Effect of the stiffener depth

The effect of the depth of stiffeners on the buckling responses of grid stiffened composite plates is investigated by varying their depth in the form of a ratio ranging from 1 to 12. This is defined as the ratio of the stiffener depth to the skin thickness in the present study. The variation of buckling load capacity of the grid stiffened panels (Fig. 2.7) due to the increase of their stiffener depth or depth ratio is plotted in Fig. 2.10 which shows that the buckling capacity can be substantially improved by increasing this ratio. It is observed that the buckling load of ortho-grid, x-grid, bi-grid and iso-grid stiffened composite plates is raised by 2.20, 1.66, 3.10 and 3.17 times higher, respectively, if the depth ratio of the stiffeners is increased from 3 to 6. It is also observed that the effect of the increase of stiffener depth on the improvement of buckling capacity is almost exhausted after a certain value of this depth ratio which is 9 in the present case.

![Fig. 2.10. Effect of stiffener depth ratio](image)
2.5 Conclusions

An efficient finite element modelling technique for buckling analysis of grid stiffened composite laminated plates is proposed which can be used for the design of these structures. In order to have a better representation of the structural deformation, the skin of the panels and the stiffeners are modelled discretely where a 6-noded triangular curve shell element based on three-dimensional degenerated shell theory is used to model the skin, while a compatible 3-noded curved beam element based on a similar hypothesis is used for the modelling of the stiffeners. It is to be noted that stiffener elements share the same degrees of freedom of shell elements and do not require any additional unknowns/degrees of freedom to represent their deformations. This is achieved by placing the stiffener elements along the edges of shell elements and the full compatibility conditions between their deformations are used to formulate the stiffener elements with necessary modifications for their representation as a beam. For the laminated stiffeners, the stacking sequence of their layers may be parallel or perpendicular to the skin surface.

In order to validate the proposed modelling technique and investigate its performance, a number of numerical examples are solved. The performance of the model is found to be very good in majority of these test problems which have covered grid stiffened plates with ortho-grid, x-grid, bi-grid and iso-grid stiffening arrangements. Based on this observation, the model can be recommended with confidence for the analysis of grid stiffened composite plates having any stiffening configurations with reasonable accuracy and reliability.

From the parametric study for the buckling load capacity of ortho-grid, x-grid, bi-grid and iso-grid stiffened composite plate made with angle-ply laminates, it has been observed that skin thickness, stiffener breadth and stiffener depth have significant effects on the improvement of buckling resistance of these panels. The skin thickness and the stiffener depth are found to have more dominant influence than the stiffener breadth on the buckling capacity enhancement. It is also observed that the improvement of the buckling
capacity of these grid stiffener stiffened plates is saturated after exceeding a certain stiffener depth.

Though the present study is focussed on grid stiffened plates, the formulation is capable of being extended to model grid stiffened shell having any curvature. Moreover, this computationally efficient and accurate model can be utilised to optimise these grid stiffened composite panels subjected to buckling loads which is part of this ongoing research.
Chapter 2

References


Chapter 3

Niching particle swarm optimization techniques for multimodal buckling maximization of composite laminates

(Paper 2)

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Applied Soft Computing (submitted for publication)
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Abstract

This chapter/paper aims to extend conventional unimodal optimization to challenging multimodal optimization of composite structures, by means of newly emerged multimodal particle swarm optimization (PSO) using niching techniques. The advanced multimodal PSO algorithms investigated in the present study include the species-based PSO (SPSO), the fitness Euclidean-distance ratio based PSO (FER-PSO) and the ring topology based PSO, which are applied to a multimodal buckling maximization problem of composite panels. All of the three multimodal PSO algorithms succeed to simultaneously identify seven first-best-fitness designs (global optima) and fourteen second-best-fitness designs (global sub-optima) in a single optimization process of this buckling optimization problem. It is found that the fourteen second-best-fitness solutions of this problem are discovered for the first time, and that the buckling-resistance difference between the first-best-fitness designs and second-best-fitness designs is definitely negligible from a viewpoint of practical applications. The performance of SPSO which requires an empirically pre-specified niching parameter is sensitive to distribution uniformity of multiple optima. The ring topology based PSO without any niching parameter is more robust and efficient for multimodal optimization of composite structures, compared with SPSO and FER-PSO.
3.1 Introduction

In the recent decades, fiber reinforced composite materials have been growingly adopted in different engineering applications, such as civil, aerospace, maritime, and automotive, because of their high specific stiffness and light weight characteristics as compared with traditional materials such as metals. Moreover, composite laminates provide great flexibility in tailing structural performances to meet specific requirements for real applications. One of the challenging issues of taking advantage of this tailing flexibility is that optimal designs of composite laminated structures are usually typical multimodal optimization problems, in which there are multiple global optimal solutions with identical fitness function values. In addition fitness functions of optimization objectives normally have complex search landscapes due to highly nonlinear mechanic responses of composite structures. A lot of studies in the literature have successfully solved unimodal optimization of composite structures to find one optimal solution in a single optimization process by using global unimodal optimization methods, such as evolutionary algorithms (EAs) [1-3].

More importantly, however, it is to extend conventional unimodal optimization to challenging multimodal optimization of composite structures by simultaneously identifying all of global optima (first-best-fitness solutions) and even global sub-optima (second-best-fitness solutions) for practical designs. This is because multimodal optimization of composite structures is able to not only provide engineers with better insights into multimodal nature of composite structures, but also offer more alternative designs to allow for final appropriate applications of composite structures in various circumstances, especially in case second-best-fitness solutions have almost the same fitness-value as that of first-best-fitness solutions.

To solve multimodal optimization problem for composite structures, the EA based on population search mechanism has natural advantages over traditional optimization techniques. For example, the EA based optimization methods do not require gradient information and hence it avoids the possibility of
solutions being trapped by local optimums. In addition the EA based optimization methods can be applied to highly non-linear optimization problems and the optimization results are independent on the initial guess of the design parameters. It is a robust search technique in solving optimization problems. Techniques to detect and preserve multiple solutions for multimodal optimization are commonly referred to “niching” methods. Standard EAs, such as genetic algorithm (GA), evolution strategies (ES), differential evolution (DE) and particle swarm optimization (PSO), could be modified to tackle the multimodal optimization by incorporating niching techniques in order to simultaneously identify multiple global optima in a single optimization process [4].

PSO has been recognized as one of the robust and efficient algorithms in solving optimization problems since it was first introduced by Kennedy and Eberhart [5] in 1995. A lot of researchers have developed a number of variants of the basic algorithms for solving various complex optimization problems [6]. Based on the successful developments and applications of PSO algorithms in unimodal optimization problems, it has been extended to deal with the challenging multimodal problems. Multimodal PSO algorithms and their applications have become an active research area in the last decade. Multimodal PSO algorithms using niching techniques have been proposed, including NichePSO [7], dynamic niching PSO (DNPSO) [8], k-means-based PSO (kPSO) [9] and multi-grouped PSO (MGPSO) [10]. The review paper [11] provided an extensive survey of PSO niching algorithms in detail. Among the developed niching PSO algorithms, species-based PSO (SPSO) [12] proposed by Li in 2004 was a representative multimodal PSO, which was reported having better performance than NichePSO [7], sequential Niched GA (SNGA) [13] and species conservation GA (SCGA) [14]. In SPSO, the swarm population is divided into individual subpopulations consisting of the species seed and the particles within the $r_s$ distance from the species seed.

The aforementioned multimodal PSO algorithms involve empirically pre-specifying a niching parameter, which might limit the practical applications of the algorithms as they require choosing suitable value of niching parameter in
different optimization problems. Hence an adaptive niching parameters PSO (ANPSO) [15] was developed to avoid pre-specifying the niching parameter by determining the size of a niche that is sensitive to the performance of the niching algorithms. However ANPSO does not work well in high dimensional multimodal problems, which limits its application to optimization design of composite structures. A multimodal PSO based on fitness Euclidean-distance ratio (FER-PSO) [16] was then proposed to eliminate the need of any niching parameter without prior knowledge for real world optimization problems. FER-PSO exhibited competitive performance in different challenging test functions. In 2010, Li proposed a niching PSO algorithm with ring topology [17]. The ring topology based PSO algorithm does not require the niching parameters. In the literature, the results of the numerical experiments showed that the ring topology PSO outperforms the niching PSO with a fixed niching radius calculated based on the average number of global optima found in a single optimization process.

To the best of the authors’ knowledge, however, multimodal design optimization of composite structures using multimodal optimization algorithms such as niching PSO techniques has not yet been available in the public literature. Based on the recent development of niching PSO for multimodal optimization summarized above, thus it is natural and necessary to employ newly developed multimodal PSO techniques for multimodal optimization design of composite laminated structures, as well as investigate their adaptability and performance of these advanced multimodal PSO algorithms.

In present study, one multimodal PSO algorithm with niching parameter (SPSO) and two multimodal PSO algorithms without niching parameter (FER-PSO and the ring topology PSO) are selected and applied to a buckling optimization design of composite panels. This buckling design problem has high dimensional design variables and multiple global optima. Using GA optimization, Riche and Haftka [18] in 1993 and Sooremekun [19] in 2001 solved the same optimal design problem and it was found that there were five and six multimodal global optima (first-best-fitness solutions) identified in
their studies respectively. After that Erdal and Sommez [20] found the seventh global optimum of this problem using the simulated annealing (SA) algorithm in 2005. In the present study, by using advanced niching PSO for multimodal optimization, new fourteen second-best-fitness solutions as well as known seven first-best-fitness solutions have been first simultaneously discovered in a single optimization process for this buckling optimization of composite panels. It is also found that the fitness value of the fourteen second-best-fitness solutions is extremely close to that of the seven first-best-fitness solutions in this example. The performance and adaptability of these multimodal PSO algorithms are investigated, and then the suitable niching PSO algorithm for solving multimodal optimization of composite structures is recommended according to the robustness of algorithmic performance.

This chapter/paper first describes the standard unimodal PSO and its extension to multimodal algorithms in Section 3.2. In Section 3.3, the buckling optimization design problem of the composite laminates is then outlined. The solutions obtained by the niching PSO algorithms for this buckling optimization are presented in Section 3.4 and the performances of the investigated niching PSO algorithms are assessed by statistical comparison in Section 3.5. Finally, the significance of using niching PSO for multimodal optimization of composite structures is concluded.

### 3.2 PSO and multimodal PSO algorithms

This section first describes the principle of the standard PSO for unimodal optimization. Three niching PSO for multimodal optimization, namely SPSO, FER-PSO and the ring topology based PSO, are then introduced in detail. Among them, SPSO requires pre-specifying the niching parameter, while FER-PSO and the ring topology based PSO are niching algorithms without niching parameters. These unimodal and multimodal PSO algorithms are
applied to solve the optimal design of composite laminated structures described in Section 3.3.

3.2.1 PSO

The mechanism of PSO is inspired by the social and cooperative behavior of animals, such as bird flocking and fish schooling. The PSO algorithm is based on a population (swarm) of potential solutions called particles. These particles explore through the search domain with a specified velocity to find the optimal solution. Each particle maintains a memory, which helps it in keeping the track of its previous best position. The positions of the particles are distinguished as personal best of each individual and global best of the whole swarm. In the past decade, PSO has been successfully applied in many research areas and real-world applications. It has been demonstrated that PSO has better computation efficiency compared to other meta-heuristic methods, such as GA and SA [21], etc.

The basic steps of the PSO algorithm are summarized as below [22]:
1. Set the swarm’s initial positions and velocities randomly for a given size of population.
2. Determine the velocity vector for each particle in the swarm by learning from its own best experience and other particle’s best past experience according to Eq. (3.1).
3. Update the current position of each particle using the velocity vector and the previous position of each particle according to Eq. (3.2).
4. Repeat Steps 2 and 3 until stopping criteria are achieved.

The velocity vector $V^i_k = (V^i_{1,k}, V^i_{2,k}, \cdots, V^i_{d,k})$ and the position vector $X^i_k = (X^i_{1,k}, X^i_{2,k}, \cdots, X^i_{d,k})$ of the $i$-th particle in $d$-dimensional search space at $k$-th iteration step are updated according to the following formula [23]:

$$V^i_k = \omega V^i_{k-1} + c_1 r_1 (p^i_{k-1} - X^i_{k-1}) + c_2 r_2 (p^g_{k-1} - X^i_{k-1}) \quad (3.1)$$

$$X^i_k = X^i_{k-1} + V^i_k \quad (3.2)$$
where $r_1$ and $r_2$ represent uniformly distributed random numbers in the interval $[0, 1]$. $c_1$ and $c_2$ stand for cognitive and social learning factor, respectively. $\omega$ is the inertia weight to balance the global and local search performance. $p^i_{k-1}$ denotes the personal best (pbest) position attained by the $i$-th particle in the swarm and $p^g_{k-1}$ refers to the global best (gbest) position attained by the whole swarm or the local best (lbest) position of the surrounding subswarm of the $i$-th particle so far until iteration $k-1$.

According to the method for choosing $p^g_{k-1}$, there are two versions of PSO, namely the global (gbest) and the local (lbest) PSO. In the gbest version, all particles adjust their velocities according to the same gbest solution found so far by the population as well as their personal best solutions. The global version can be described as a standard PSO, due to the simplicity and uniqueness of swarm population communication structure and the standard PSO is developed for unimodal optimization to find the single global optimal solution. While in lbest PSO, each particle chooses the local best solution within its neighbors which are composed of part of the whole swarm. Though local version of PSO was initially proposed for unimodal optimization, local PSO is able to simultaneously search various feasible regions and converge into the locations of multiple optima. Local PSO is thus more suitable to be extended for solving multimodal optimization problem compared with the standard and global PSO version for unimodal optimization.

### 3.2.2 Niching PSO for multimodal optimization

#### 3.2.2.1 SPSO

Li [12] first proposed SPSO to deal with multimodal problems based on the idea of species in 2004. At each iteration step of the standard PSO algorithm, a number of species are dynamically indentified by similarity of particles according to Euclidean distance. Each specie comprises a dominated specie
seed, and the Euclidean distance between the subordinate particles and species seed is within niching radius parameter $r_s$. The species seed is assigned as the $lbest$ for all the individuals identified in the same species.

To determine the species seeds, all the particles are sorted in decreasing order based on their fitness values. The first dominating species seed is the best-fit particle of the whole population and the second dominating species seed is the best-fit particle in the remaining particles by removing the first dominating species seed from the whole particles and its neighborhood particles within the radius $r_s$. The identification procedure for dominating species seed and its neighbors is repeated until each particle is assigned as a species seed or a member belonging to certain species.

The specified niching radius parameter is critical to the performance of SPSO. If the species radius for niching is too large, the species could contain two or even more global optima. This reduces the probability of identifying all global optima. On the contrary, too small niching radius may lead to decease in the population diversity of a niche, resulting in premature convergence. Assuming that the distribution of multiple optima was uniform, Deb and Goldberg [24] proposed a method to determine the species radius as

$$r_s = \sqrt{\frac{\sum_{k=1}^{d} (x_k^u - x_k^l)^2}{2dP}} \quad (3.3)$$

where $x_k^u$ and $x_k^l$ are the upper and lower boundary of the $k$-th design variable in $d$-dimensional space. $P$ is the number of global optima.

### 3.2.2.2 FER-PSO

Inspired by the fitness-distance-ratio based PSO (FDR-PSO) for identifying a single global optimum, a multimodal PSO called FER-PSO [16] was proposed in 2007, in which a memory-swarm formed by the personal best of the population provides a table network to retain the best positions detected so far by the particles, while the current particles acting as an explore-swarm to
explore the search space extensively. Instead of moving toward the same single global best, each particle is adjusted to its “fittest and closest” neighbor particle identified by its maximum FER value, and hence, the lbest of each particle is determined naturally without any subjective judgment of users. FER-PSO is an attractive PSO algorithm for solving multimodal design optimization problems as it does not need to pre-specify niching parameter, which makes it practical for being applied in real applications. The FER value is defined as

$$ FER_{(i,j)} = \alpha \frac{f(p_i) - f(p_j)}{\|p_j - p_i\|} \quad (3.4.1) $$

$$ \alpha = \frac{\|s\|}{f(p_g) - f(p_w)} \quad (3.4.2) $$

$$ \|s\| = \sqrt{\sum_{k=1}^{d} (x^k_i - x^k_j)^2} \quad (3.4.3) $$

where $p_g$ and $p_w$ are the best-fit and the worst-fit particles in the current population, respectively. $p_i$ and $p_j$ are the personal best points of $i$-th and $j$-th particles, respectively. $f(p_i)$ is the fitness evaluation of the $i$-th particle. $\alpha$ is a scaling factor to avoid either fitness or Euclidean distance becoming dominated. $\|s\|$ is the diagonal distance of $d$-dimensional search space.

### 3.2.2.2 Ring topology based PSO

The third advanced multimodal PSO algorithm investigated in this study was proposed by Li [17] in 2010. The algorithm was based on the concept of ring neighborhood topology. Instead of determining the local neighborhood of a particle according to the Euclidean distance between the particles in the solution space, e.g. SPSO and FER-PSO algorithms, the ring topology based PSO utilizes particles indices to identify their neighbors in the population located on a ring-shaped topology. Using a ring topology, the particle with index 1 ($X_1$) has the immediate right neighbor particle with index 2 ($X_2$) and the immediate left neighbor particle with index $N$ ($X_N$), where $N$ is the population size. The lbest of each particle is the best-fit personal solution in
its index neighborhood particles. In the standard ring topology based PSO, named as “r3pso” [17], each particle interacts with its immediate neighbor on its left and right. “r2pso” [17] is a variant lbest PSO with ring topology and each particle is only influenced by its closest neighbor to its right. The ring topology based PSO does not require any prior knowledge of the niching parameter because the niches naturally consist of three local members (r3pso) or two local members (r2pso).

3.3 Composite laminate design problem

A simply supported composite laminated panel consists of \( N \) identical plies with different ply orientations, as shown in Fig. 3.1. The laminated panel is subjected to a combination of in-plane compressive loads \( N_x \) and \( N_y \) in the \( x \) and \( y \) axis directions. It is assumed that the laminated panel was symmetric, balanced and made up of building blocks composed of 2-ply stacks with 0\( ^\circ \) (two 0\( ^\circ \) plies), ±45 (a pair of +45\( ^\circ \) and -45\( ^\circ \) plies) and 90\( ^\circ \) (two 90\( ^\circ \) plies). As the stacking sequence of the composite laminate is symmetric about the mid-plane and made up of 2-ply stacks, the stacking sequence of the laminated panel is completely defined by \( N/4 \) design variables, in which each design variable represents the orientation of an individual 2-ply stack at one half of the laminated panel.
According to the assumption of classical lamination theory and by neglecting normal-shear extension coupling and extension-bending coupling, the governing differential equation for the buckling load $\lambda N_x$ and $\lambda N_y$, where $\lambda$ is a scalar parameter, is defined as

$$
D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} = \lambda \left( N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} \right)
$$

(3.5)

where $D_{ij}$ are the bending stiffness coefficients, which depends on the stacking sequence of the composite laminate; $w$ is the out-plane deflection of the composite laminate. The laminate buckles into $m$ and $n$ half-waves along $x$ and $y$ axes, respectively, when the applied loadings reach the buckling value determined by

$$
\lambda_b(m, n) = \pi^2 \left( \frac{m^4 D_{11} + 2(D_{12} + 2D_{66})(rmn)^2 + (rn)^4 D_{22}}{(am)^2 N_x + (ran)^2 N_y} \right)
$$

(3.6)

where $r = a/b$ is the plate aspect ratio that is the ratio of the length $a$ to the width $b$. The critical buckling load factor $\lambda_b = \min \lambda_b(m, n)$ is the minimum value of any combination of $m$ and $n$, for a given circumstance of $N_x$ and $N_y$. 

---

**Fig. 3.1.** Simply supported composite laminate with symmetric stacking sequence under in-plane compressive loads
In this study, the optimization design problem is formulated as optimizing the stacking sequence of the 2-ply stacks to maximize the critical buckling factor $\lambda_{cb}$ of the composite laminates. Considering a 64-ply composite laminate made of graphite-epoxy layers with ply thickness $t_i = 0.127$ mm subjected to loading condition of $N_x/N_y = 1.0$. The laminate has length $a = 0.508$ m and width $b = 0.254$ m with elastic properties $E_1 = 127.59$ GPa, $E_2 = 13.03$ GPa, $G_{12} = 6.41$ GPa and $v_{12} = 0.3$. This composite laminate design problem has been widely adopted in different research work focusing on using unimodal optimization algorithms [18-20]. According to those previous studies [18-20], it is found that this composite laminate design is an example of multimodal optimization and seven global optima (first-best-fitness solutions) have been reported.

This optimization design problem has 16 stacking sequence components of a design variable vector, in which each variable component specifies the orientation of a 2-ply stack with three possible discrete orientations: $0_2$, $\pm 45$ and $90_2$. This discrete optimization can be converted into continuous variable optimization in the $[0 1]^{16}$ search space. In each dimension, intervals $[0 1/3)$, $[1/3 2/3)$, and $[2/3 1]$ represent discrete orientation $0_2$, $\pm 45$ and $90_2$, respectively. There are $4.305 \times 10^7$ possible designs in this optimization problem. In the present study, SPSO, FER-PSO and the ring topology based PSO including r3pso and r2pso are applied to identify the multiple optimal designs using the following PSO parameters: $\omega = 1.0$, $c_1 = 2.0$, $c_2 = 2.0$. To consider the non-uniform distribution of multiply global optima, Eq. (3.3) proposed to calculate the niching radius of SPSO is modified as

$$r_s = \beta \left[ \frac{\sum_{k=1}^{d} (x_k^* - x_k')^2}{2P} \right]$$

(3.7)

where $\beta$ is the correction factor $(0 < \beta \leq 1.0)$. SPSO ($\beta$) stands for a SPSO variant using Eq. (3.7) with the correction factor $\beta$ to determine the niching radius value. In this study, SPSO ($\beta = 1.0$), SPSO ($\beta = 0.5$) and SPSO ($\beta = 0.25$) are three SPSO variants used to assess the parameter influence of the niching radius $r_s$. Additionally take $d=16$, $P=7$, $x_k^* = 1$ and $x_k' = 0$ to
compute the niching radius value in Eq. (3.7) for this 16-dimension optimization design problem of composite panels with seven global optima.

### 3.4 Results

Using fully sufficient particle population size \( n_{\text{pop}} \) and generation number \( n_{\text{gen}} \), such as \( n_{\text{pop}} = 5,000 \) and \( n_{\text{gen}} = 1,000 \), the optimization results show that all of SPSO, FER-PSO and the ring topology based PSO not only simultaneously identified seven first-best-fitness designs (global optima) reported in the literature, but also newly reveal fourteen second-best-fitness designs (global sub-optima) in a single optimization process for this problem described in Section 3.3. Alternatively, all of SPSO, FER-PSO and the ring topology based PSO successfully identify seven first-best-fitness designs and fourteen second-best-fitness designs according to the comprehensive results of 50 independent optimization procedures using large particle population size \( n_{\text{pop}} \) and generation number \( n_{\text{gen}} \), such as \( n_{\text{pop}} = 500 \) and \( n_{\text{gen}} = 500 \).

The first-best-fitness designs with buckling load factor 3973.014 and the first discovered second-best-fitness designs with buckling load factor 3972.996, as identified by SPSO, FER-PSO or the ring topology based PSO, are listed in Tables 3.1 and 3.2 respectively. It is observed that the relative difference of maximum critical buckling load factor between the first-best-fitness solutions and the new finding of second-best-fitness designs is only \( 4.53 \times 10^{-6} \). From the view of engineering practice, the number of acceptable optima of this problem for practical applications is the sum of all the first-best-fitness designs and the second-best-fitness designs. Thus there are 21 acceptable designs (acceptable optima) for this multimodal optimization problem.
Table 3.1
The first-best-fitness solutions (global optima) with symmetric 64 plies at $0, \pm 45 \text{ or } 90$ for $N_x/N_y = 1.0, a/b = 2.0$

<table>
<thead>
<tr>
<th>Design No.</th>
<th>Stacking sequence</th>
<th>Buckling load factor $\lambda_{cb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(\pm 45/90_{10}/\pm 45/90_{b}/\pm 45/90_{b})_S$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$(\pm 45/90_b/\pm 45/90_{18}/\pm 45)_S$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$(90_{10}/\pm 45/90_{b}/\pm 45/90_{b}/\pm 45_4)_S$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$(90_2/\pm 45/90_{b}/\pm 45/90_{b}/\pm 45/90_{10})_S$</td>
<td>3973.014</td>
</tr>
<tr>
<td>5</td>
<td>$(90_4/\pm 45/90_{16}/\pm 45/90_b)_S$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$(90_{10}/\pm 45/90_{b}/\pm 45/90_{b}/\pm 45)_S$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$(90_b/\pm 45/90_{2}/\pm 45/90_{2}/\pm 45/90_{b}/\pm 45_6)_S$</td>
<td></td>
</tr>
</tbody>
</table>

In order to check the independence of second-best-fitness designs from the first-best-fitness designs in terms of the location difference in the 16-dimension design space, the stacking sequence components of each second-best-fitness design are compared with those of each first-best-fitness design. For every first-best-fitness design, the number of the stacking sequence components different from those of a second-best-fitness design is recorded, and the first-best-fitness design with the minimum number of different components is defined as the nearest the first-best-fitness design to this second-best-fitness design, as listed in Table 3.2. For instance, the first-best-fitness solution of Design No. 4 listed in Table 3.1 is the nearest first-best-fitness design to the second-best-fitness solution of Design No. 10 listed in Table 3.2 and any other first-best-fitness solutions have more than five stacking sequence components distinct from those of the second-best-fitness solution of Design No. 10.
## Table 3.2

The second-best-fitness solutions (global sub-optima) with symmetric 64 plies at $0_2$, $\pm 45$ or $90_2$ for $N_s/N_y = 1.0, \ a/b = 2.0$

<table>
<thead>
<tr>
<th>Design No.</th>
<th>Stacking sequence</th>
<th>Nearest first-best-fitness designs in Table 1 to a second-best-fitness design</th>
<th>Number of different components between a second-best-fitness design and its corresponding nearest first-best-fitness designs</th>
<th>Buckling load factor $\lambda_{cb}$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>$(\pm 45/90_0/\pm 45/90_0/\pm 45/90_0/\pm 45)$</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$(\pm 45/90_1/\pm 45/90_1/\pm 45/90_0/\pm 45)$</td>
<td>1, 2, 5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$(90_2/\pm 45/90_1/\pm 45/90_0/\pm 45/90_0)$</td>
<td>4, 5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$(90_2/\pm 45/90_1/\pm 45/90_2/\pm 45/90_0)$</td>
<td>1, 5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$(90_2/\pm 45/90_0/\pm 45/90_0/\pm 45/90_0)$</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$(90_1/\pm 45/90_2/\pm 45/90_2/\pm 45/90_0)$</td>
<td>5</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$(90_0/\pm 45/90_2/\pm 45/90_2/\pm 45/90_0)$</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$(90_0/\pm 45/90_2/\pm 45/90_0/\pm 45/90_0)$</td>
<td>4, 5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$(90_0/\pm 45/90_0/\pm 45/90_0/\pm 45/90_0)$</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$(90_2/\pm 45/90_0/\pm 45/90_0/\pm 45/90_0)$</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>$(90_0/\pm 45/90_0/\pm 45/90_0/\pm 45/90_0)$</td>
<td>1, 4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>$(90_0/\pm 45/90_0/\pm 45/90_0/\pm 45/90_0)$</td>
<td>2, 4, 5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>$(90_0/\pm 45/90_0/\pm 45/90_0/\pm 45/90_0)$</td>
<td>1, 5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>$(90_0/\pm 45/90_2/\pm 45/90_1)$</td>
<td>1</td>
<td>5</td>
<td>3972.996</td>
</tr>
</tbody>
</table>
As shown in Table 3.2, though 3 second-best-fitness designs (Design No. 1, 5 and 7 in Table 2) have one stacking sequence component distinct from their corresponding nearest first-best-fitness design, the rest 11 of 14 second-best-fitness designs are apparently independent to each first-best-fitness designs because they are not mathematically located in the neighborhood of any first-best-fitness design in the design space. Therefore, it is shown that SPSO, FER-PSO and the ring topology based PSO have distinctive capability of simultaneously identifying multiple independent global optima and global sub-optima in an optimization process.

3.5 Algorithmic performance assessments

A series of numerical experiments to solve the multimodal buckling maximization of composite panels presented in Section 3.3 are conducted by varying the particle population size \textit{n\_pop} (\textit{n\_pop} = 75, 150 or 250) as well as the generation number \textit{n\_gen} (\textit{n\_gen} = 150, 250, 500 or 1000) in order to investigate the multimodal optimization performance of SPSO, FER-PSO and the ring topology based PSO including r3pso and r2pso algorithms. The number of the global optima (first-best-fitness designs) \textit{N}_g and the number of the acceptable optima (first-best-fitness designs plus second-best-fitness designs) \textit{N}_a, which are simultaneously identified by a niching PSO algorithm in a single optimization process, are two measures to evaluate the algorithmic performance in the present study. Due to the stochastic nature of PSO performance, numerical experiments for all cases of \textit{n\_pop} and \textit{n\_gen} are independently repeated over 50 times to assure the reliability of optimization results in the statistical sense.

In terms of \textit{n\_pop} = 75, 150 and 250, respectively, Tables 3.3 to 3.5 list the mean value \textit{m} and the corresponding variation coefficient \textit{δ} of \textit{N}_g and \textit{N}_a for SPSO (\textit{β} = 1.0, 0.5 and 0.25), FER-PSO and the ring topology based PSO.
(r3pso and r2pso) algorithms as well as the standard unimodal PSO. The maximum mean \( m \) of \( N_g \) and \( N_a \) in Tables 3.3 to 3.5 are presented in bold for each combination of population size \( n_{pop} \) and evolutionary generation \( n_{gen} \). The performances of standard unimodal PSO, SPSO, FER-PSO and the ring topology based PSO are discussed in the following four paragraphs, respectively, according to these experimental results.

### Table 3.3
The mean value \( m \) and variation coefficient \( \delta \) of the number of global optima \( N_g \) and acceptable optima \( N_a \) found by unimodal and multimodal PSO algorithms over 50 independent runs at population size 75

<table>
<thead>
<tr>
<th>Generations</th>
<th>Results</th>
<th>Standard PSO</th>
<th>SPSO</th>
<th>FER-PSO</th>
<th>Ring topology based PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>k=1.0</td>
<td>k=0.5</td>
<td>k=0.25</td>
<td>r3pso</td>
</tr>
<tr>
<td>N_g</td>
<td>N_a</td>
<td>N_g</td>
<td>N_a</td>
<td>N_g</td>
<td>N_a</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>150</td>
<td>m</td>
<td>0.34</td>
<td>0.86</td>
<td>0.46</td>
<td>1.10</td>
</tr>
<tr>
<td>( \delta )</td>
<td>(1.41)</td>
<td>(1.07)</td>
<td>(1.33)</td>
<td>(0.85)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>250</td>
<td>m</td>
<td>0.32</td>
<td>0.94</td>
<td>0.58</td>
<td>1.50</td>
</tr>
<tr>
<td>( \delta )</td>
<td>(1.72)</td>
<td>(0.97)</td>
<td>(0.93)</td>
<td>(0.68)</td>
<td>(0.65)</td>
</tr>
<tr>
<td>500</td>
<td>m</td>
<td>0.60</td>
<td>1.32</td>
<td>0.44</td>
<td>0.96</td>
</tr>
<tr>
<td>( \delta )</td>
<td>(1.35)</td>
<td>(0.89)</td>
<td>(1.14)</td>
<td>(0.87)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>1000</td>
<td>m</td>
<td>0.42</td>
<td>1.20</td>
<td>0.48</td>
<td>1.26</td>
</tr>
<tr>
<td>( \delta )</td>
<td>(1.37)</td>
<td>(0.87)</td>
<td>(1.21)</td>
<td>(0.75)</td>
<td>(0.49)</td>
</tr>
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Table 3.4
The mean value $m$ and variation coefficient $\delta$ of the number of global optima $N_g$ and acceptable optima $N_a$ found by unimodal and multimodal PSO algorithms over 50 independent runs at population size 150.

<table>
<thead>
<tr>
<th>Generations</th>
<th>Results</th>
<th>Standard PSO</th>
<th>SPSO</th>
<th>FER-PSO</th>
<th>Ring topology based PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$N_g$</td>
<td>$N_a$</td>
<td>$N_g$</td>
<td>$N_a$</td>
</tr>
<tr>
<td>150</td>
<td>$m$</td>
<td>0.48</td>
<td>1.36</td>
<td>0.48</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>($\delta$)</td>
<td>(1.35)</td>
<td>(0.90)</td>
<td>(1.21)</td>
<td>(0.85)</td>
</tr>
<tr>
<td>250</td>
<td>$m$</td>
<td>0.54</td>
<td>1.44</td>
<td>0.38</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>($\delta$)</td>
<td>(1.36)</td>
<td>(0.94)</td>
<td>(1.40)</td>
<td>(0.82)</td>
</tr>
<tr>
<td>500</td>
<td>$m$</td>
<td>0.62</td>
<td>1.78</td>
<td>0.58</td>
<td>1.44</td>
</tr>
<tr>
<td></td>
<td>($\delta$)</td>
<td>(0.97)</td>
<td>(0.66)</td>
<td>(1.05)</td>
<td>(0.74)</td>
</tr>
<tr>
<td>1000</td>
<td>$m$</td>
<td>0.44</td>
<td>1.48</td>
<td>0.56</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td>($\delta$)</td>
<td>(1.31)</td>
<td>(0.70)</td>
<td>(1.03)</td>
<td>(0.61)</td>
</tr>
</tbody>
</table>
Table 3.5
The mean value $m$ and variation coefficient $\delta$ of the number of global optima $N_g$ and acceptable optima $N_a$ found by unimodal and multimodal PSO algorithms over 50 independent runs at population size 250

<table>
<thead>
<tr>
<th>Generations</th>
<th>Results</th>
<th>Standard PSO</th>
<th>SPSO $k=1.0$</th>
<th>SPSO $k=0.5$</th>
<th>SPSO $k=0.25$</th>
<th>FER-PSO</th>
<th>Ring topology based PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_g$</td>
<td>$N_a$</td>
<td>$N_g$</td>
<td>$N_a$</td>
<td>$N_g$</td>
<td>$N_a$</td>
<td>$N_g$</td>
</tr>
<tr>
<td>150</td>
<td>$m$</td>
<td>0.84 (1.03)</td>
<td>1.92 (0.74)</td>
<td>0.54 (1.00)</td>
<td>1.58 (0.69)</td>
<td>2.50 (0.31)</td>
<td>5.82 (0.28)</td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td>0.72 (1.12)</td>
<td>1.92 (0.84)</td>
<td>0.66 (1.13)</td>
<td>1.66 (0.68)</td>
<td>2.96 (0.26)</td>
<td>7.08 (0.28)</td>
</tr>
<tr>
<td>250</td>
<td>$m$</td>
<td>0.70 (1.01)</td>
<td>1.92 (0.70)</td>
<td>0.74 (0.81)</td>
<td>1.84 (0.61)</td>
<td>3.08 (0.30)</td>
<td>7.72 (0.28)</td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td>0.76 (1.08)</td>
<td>2.02 (0.78)</td>
<td>0.48 (1.35)</td>
<td>1.56 (0.72)</td>
<td>3.28 (0.20)</td>
<td>8.80 (0.19)</td>
</tr>
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</table>

As shown in Tables 3.3 to 3.5, the average values of $N_g$ and $N_a$ identified by the standard unimodal PSO are much less than those identified by SPSO ($\beta = 0.5$ or 0.25), FER-PSO, r3pso and r2pso in a single optimization process. In comparison with the niching PSO, the limitation of the standard unimodal PSO in identifying multiple global optima and sub-optima becomes more obvious when large swarm population size for optimization is used. Taking $n_{\text{pop}} = 250$ and $n_{\text{gen}} = 500$ for example, the standard unimodal PSO averagely identifies only 0.70 global optima and 1.92 acceptable optima,
while r2pso finds 3.54 global optima and 9.76 acceptable optima in a single optimization process. As expected it is difficult for the standard unimodal PSO to search multiple global optima because the swarm of particles of the standard unimodal PSO always tends to converge into one of the identified global optimum.

The numerical experiment outcomes have showed that the performance of SPSO is sensitively dependent on the pre-specified niching radius value or the distribution characteristics of multiple optima. Taking Deb and Goldberg’s method (Eq. (3.3)) to determine niching radius for uniform distribution of multiple global optima, SPSO (β = 1.0) for multimodal optimization even cannot overwhelm the standard unimodal PSO which is hard to locate more than one multimodal global optima of this problem. Thus it is indicated that Eq. (3.3) for calculation of the niching radius value should be modified by taking into account the non-uniform distribution of the multiple global optima in an optimization problem. Reducing the value of Eq. (3.3) for niching radius calculation by 50% and 75% (i.e. taking β=0.5 and β =0.25 in Eq. (3.7)), both SPSO (β =0.5) and SPSO (β =0.25) have significantly enhanced the capability in searching multiple optima due to considering the uneven distribution of multiple global optima, compared with SPSO (β=1.0). In a single optimization process using $n_{pop}=150$ and $n_{gen}=500$, for example, SPSO (β =0.5) and SPSO (β =0.25) achieve 2.68 and 2.98 global optima respectively as well as 5.66 and 6.68 acceptable optima respectively, while SPSO (β =1.0) just find 0.58 global optima and 1.44 acceptable optima. Moreover, SPSO (β = 0.25) identifies the maximum average numbers of global optima and acceptable optima for the case of $n_{gen}=1000$ and $n_{pop}=75, 150$ or 250 compared with the other PSO algorithms under investigation. It is shown that as long as the niching radius value is appropriately selected, SPSO can be successfully applied in multimodal optimization of composite structures even if the multiple global optima are not uniformly distributed.

According to the experiment results, FER-PSO without niching parameter outperforms the standard unimodal PSO and SPSO (β = 1.0), but FER-PSO
cannot overwhelm SPSO ($\beta = 0.5$), SPSO ($\beta = 0.25$) and the ring topology based PSO including r3pso and r2pso. In the case of $n_{pop} = 150$ and $n_{gen} = 250$ as shown in Table 3.4, FER-PSO averagely detects 1.00 global optima and 2.76 acceptable optima which are 1.9 times of those identified by the standard unimodal PSO, but FER-PSO is inferior to SPSO ($\beta = 0.5$) which identifies 2.64 global optima and 5.42 acceptable optima, as well as r3pso which identifies 2.34 global optima and 6.16 acceptable optima in a single optimization process.

The experimental results also show that the ring topology PSO including r3pso and r2pso without any niching parameter exhibit superior performance and robustness for various combination cases of $n_{pop}$ and $n_{gen}$ in this multimodal buckling optimization of composite panels. As shown in Table 3.3, r3pso simultaneously identifies the maximum mean number of acceptable optima in a single optimization process using $n_{pop} = 75$ and $n_{gen} = 150, 250$ or 500. In the cases of $n_{pop} = 150$ or 250 and $n_{gen} = 150, 250$ or 500, r2pso overwhelms the other niching PSO algorithms in terms of the average number of acceptable optima identified in a single optimization process as shown in Tables 3.4 and 3.5. Overall, r3pso and r2pso show comparably superior performance to SPSO ($\beta = 0.5$) and SPSO ($\beta = 0.25$) according to the observation about the experimental results. Thus it is indicated that r3pso and r2pso are desirably robust and adaptive for the multiply buckling optimization of composite panels, because the ring topology based PSO does not require any prior knowledge of the multiple optima for multimodal optimization of composite structures, in comparison with SPSO which needs to empirically user-specify a niching radius value.

In addition, by comparing the case of $n_{pop} = 75$ and $n_{gen} = 500$ listed in Table 3.3 with the case of $n_{pop} = 150$ and $n_{gen} = 250$ listed in Table 3.4, it is observed that all of the investigated niching PSO variants using larger population size perform better than their corresponding algorithms using smaller population size, when the total numbers of function evaluation are
kept same in these two cases. Similar observation is also found by comparing
the case of \( n_{\text{pop}} = 75 \) and \( n_{\text{gen}} = 1000 \) listed in Table 3.3 with the case of
\( n_{\text{pop}} = 150 \) and \( n_{\text{gen}} = 500 \) listed in Table 3.4. Therefore, it is indicated
using large population size benefits the efficiency of the niching PSO
algorithms to achieve superior performance in multimodal optimization of
composite structures.

3.6 Conclusions

Advanced multimodal optimization of PSO using niching techniques has been
introduced and explored in order to extend conventional unimodal
optimization to challenging multimodal optimization of composite structures,
since optimal designs of composite structures are usually typical multimodal
optimization problems.

Three niching PSO algorithms including SPSO, FER-PSO and the ring
topology PSO have been applied in a multimodal optimization problem, by
maximizing the buckling capacity of a composite panel subjected to a
combination of in-plane compressive loads. In a single optimization process
of this multimodal problem, SPSO, FER-PSO and the ring topology PSO have
simultaneously identified 14 second-best-fitness solutions (global sub-optima)
having buckling load factor 3972.996 as well as 7 first-best-fitness solutions
(global optima) with optimal buckling load factor 3973.014, respectively. It
should be noted that these 14 second-best-fitness solutions are first discovered.
Thus it is shown that these niching PSO algorithms under investigation have
strong power in dealing with multimodal optimization of composite structures.
It also found that most second-best-fitness designs are mathematically far
away from the neighborhood of any first-best-fitness design in the search
space. Moreover, it is observed that the buckling resistance difference
between the 7 first-best-fitness solutions and the 14 second-best-fitness
solutions is extremely close and can be negligible from a viewpoint of
engineering applications. Consequently, there are 21 acceptable optimal
designs for this buckling optimization problem. Based on the new finding of 21 multiple optimal designs, this buckling maximization can become one benchmark design problem of composite panels to test the adaptability of multimodal/unimodal optimization algorithms used for optimal design of composite structures.

The performance of SPSO relies on the size of the niching radius or the distribution uniformity of multiple optima. SPSO with suitable niching radius value exhibits superior ability in locating multiple global optima even if the optima in multimodal optimization of composite structures are non-uniformly distributed. The niching radius value proposed by Deb and Goldberg [24] for SPSO needs to multiply a correction factor less than 1.0, taking account of the uneven distribution of global optima for multimodal optimization of composite structures. The ring topology PSO without any niching parameter has encouraging capability and robustness to identify multiple global optima. Due to the superior performances of simultaneously identifying multiple optima in a single optimization process, the ring topology PSO algorithms including r3pso and r2pso are recommended for solving multimodal optimization design of composite structures, compared with SPSO and FER-PSO. It should be noted that the ring topology PSO does not require any prior knowledge about the number and distribution of global optima, and hence, the ring topology PSO without any empirically user-selected niching parameter is in great demand for engineers who have no/limited experiences in multimodal optimization application of composite structures. In addition, taking large swarm population size benefits to efficiently achieve successful applications of niching PSO in multimodal optimization of composite structures.
References


Chapter 4

New technique of random reflection boundary for PSO and its application in buckling optimization of composite panels

(Paper 3)

Liang Huang, Ching-Tai Ng, Abdul H. Sheikh & Michael C. Griffith

School of Civil, Environmental and Mining Engineering, the University of Adelaide, Adelaide SA 5005, Australia

Manuscript in publication style
## Statement of Authorship

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### Author Contributions

By signing the Statement of Authorship, each author certifies that their stated contribution to the publication is accurate and that permission is granted for the publication to be included in the candidate's thesis.

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<th>Name of Principal Author (Candidate)</th>
<th>Liang Huang</th>
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<tr>
<td>Contribution to the Paper</td>
<td>Proposed and realised the technique, conducted numerical experiments, analysis and application, wrote manuscript</td>
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<td>Helped in manuscript evaluation</td>
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Abstract

Particle swarm optimization (PSO) has been widely used in various practical problems due to its efficient and robust performance. In PSO, the value of particles’ maximum velocity is found to be indispensable and critical. Moreover, the control of maximum velocity of particles is problem-dependent, which sensitively influences the robustness of PSO. In this study, a random reflection boundary is proposed to replace the conventional fixed absorption boundary for the range-exceeding particles, in order to eliminate/reduce the significance and sensitivity of an empirical parameter of particles’ maximum velocity in PSO. Based on the statistical comparisons of optimization on testing functions, it has shown that the PSO algorithms with the proposed random reflection boundary achieve better performance than the corresponding PSO algorithms with the conventional fixed absorption boundary. It is also observed that it is feasible in some cases to remove the control of particles’ maximum velocity in PSO by using the random reflection boundary without affecting the achievement of reliable and satisfactory optimization performance. Based on the results obtained from the experimentation on the abovementioned test functions, empirical guidelines for appropriately using the half-range/full-range random reflection boundary are further proposed in terms of the particle communication topology of PSO, the control method of PSO convergence and the dimensionality of optimization problems. The empirical guidelines are applied and validated by a practical example of a laminated composite plate where the stacking sequence of the layers of the composite plate is optimized to achieve a maximum buckling load capacity of the structure. In conclusion, the proposed random reflection boundary technique for PSO allows for using less user-selected algorithmic parameters for more reliable and more convenient applications of PSO in various real-world practices.
4.1 Introduction

Particle swarm optimization (PSO) is population-based evolutionary algorithm mimicking swarm behavior of birds flock, animal herd and fish school searching for food in a cooperative manner [1, 2]. Due to the robustness, efficiency and easy implementation, PSO has become a promising and powerful optimization technique and researchers have made a lot of efforts to develop new variants successfully applied to solve real world problems [3-8].

In PSO algorithm, a number of simple particles representing feasible solutions in the \( d \)-dimensional search space and the global optimal solutions are identified by the swarm of particles. The location of each particle \( x \) is adjusted by a moving velocity \( v \) to change its searching trajectory, according to best-fitness history of itself and neighborhood particles in the swarm. The basic equations for update of velocity \( v_{k}^{d} \) and position \( x_{k}^{d} \) of \( i \)-th particle in \( d \)-th dimension at \( k \)-th iteration are shown as follows:

\[
v_{k}^{d} = v_{k-1}^{d} + c_{1}r_{1}(p_{k-1}^{d} - x_{k-1}^{d}) + c_{2}r_{2}(p_{k-1}^{d} - x_{k-1}^{d}) \quad (4.1)
\]

\[
x_{k}^{d} = x_{k-1}^{d} + v_{k}^{d} \quad (4.2)
\]

where \( c_{1} \) and \( c_{2} \) stand for cognitive and social learning factor, respectively; \( r_{1} \) and \( r_{2} \) represent uniformly distributed random numbers in the interval \([0, 1]\); \( p_{k-1}^{d} \) denotes the personal best position attained by the \( i \)-th particle so far and \( p_{k-1}^{d} \) refers to the neighborhood best position of the \( i \)-th particle at iteration \( k - 1 \). In basic PSO [9], each component of particle speed \( v_{k}^{d} \) is clamped to a range \([-v_{\text{max}}^{d}, v_{\text{max}}^{d}]\), in which \( v_{\text{max}}^{d} \) is the maximum velocity of particles in the \( d \)-th direction of the search space.

In terms of the topology of neighbors of a particle in the swarm, PSO can be classified into global and local version. In global version, the trajectory of each particle is adjusted by the global best point found by the any member in the whole swarm, in which all particles are neighbors to every other. In local
version of the PSO the velocity of a particle is influenced by local best experience of its adjacent members, which is certain subset of the population. Typical population configuration for local PSO is a ring topology and each individual is connected to only two neighbors with one on each side according to the indexes of particles.

Global PSO tends to rapidly converge toward the peak of fitness landscape with risk of the solution being trapped into local optima. Local PSO can explore the solution space more extensively with lower convergent speed because sub-swarms with smaller number of particles can simultaneously visit different regions of the searching space [10]. The global PSO for unimodal optimization conveys information in the quickest way as all the particles are fully connected to one another. On the contrary, the ring topology based local PSO for multimodal optimization has the slowest and the most indirect commutation characteristic.

It is an appealing advantage of original PSO for practical problems that there are quite few parameters to be set, and hence, PSO is easy to be controlled by the users who have no/little experiences of optimization. First of all, the parameters of population size and evolution generation are empirically dependent on the dimensionality and perceived complexity of a problem, but large population size and evolution generation usually ensure better performance of PSO. Secondly, the parameter  \( c_1 \) and  \( c_2 \) determine the balance between the influence of individual and that of the neighbor group. Fortunately Kennedy [11] conducted the theoretical analysis of a simplified PSO algorithm and noted that if  \( c_1 r_1 + c_2 r_2 < 4.0 \), the trajectories of deterministic particles in one dimension maintain periodical regularities, otherwise, the behavior of the particle becomes unstable and eventually causes swarm explosion (particle moving linearly or exponentially toward infinity). Setting  \( c_1 = c_2 = 2.0 \) is commonly adopted for most of applications. The maximum velocity of particles  \( v_{\text{max}} \) is the third indispensible parameter affecting the performance of optimization since it appears to balance rough exploration and fine exploitation. Increasing the value of  \( v_{\text{max}} \) allows for
particles to quickly fly away from the past good region while decreasing $v_{\text{max}}$ enhances particles’ local search ability. However, the value of $v_{\text{max}}$ is empirically pre-specified and even somewhat arbitrary chosen by the users based on some ad hoc rules of thumb since there is no theoretical analysis of PSO algorithm to determine a proper $v_{\text{max}}$. Usually a component of $v_{\text{max}}$ is related to the maximum dynamic range of the variable $x_{\text{max}}$ in each dimension, according to the expression [12]

$$v_{\text{max}} = \lambda x_{\text{max}}$$

where $\lambda$ is a proportional factor to be specified, empirically between 0.1 and 1.0.

To reduce the significance of $v_{\text{max}}$ for better convergence control of exploration and exploitation, an inertia weight $\omega$ was introduced by Shi and Eberhart [13]. The velocity was modified according to

$$v_{i,d}^{k} = \omega v_{i,d}^{k-1} + c_1 r_1 (p_{i,d}^{k-1} - x_{i,d}^{k-1}) + c_2 r_2 (p_{g,d}^{k-1} - x_{i,d}^{k-1})$$

Parameter selection and empirical study have been conducted to investigate the performance of the inertia weight approach [14, 15]. The inertia weight $\omega$ is typically set to 0.9 and it is decreasing linearly to 0.4 during a run [14]. It was found that the performance of the improved algorithm is satisfactory when $v_{\text{max}} = 1.0 x_{\text{max}}$ is the setting rule for $v_{\text{max}}$, but the PSO still requires $v_{\text{max}}$ to be defined [9].

Alternatively, Clerc [16] proposed a constriction factor to control the behavior of particles in the warm for convergence control, and hence, it aimed to avoid the arbitrary parameter $v_{\text{max}}$ being used in the algorithm. The new velocity with the constriction factor was defined as [17]:

$$v_{i,d}^{k} = \chi (v_{i,d}^{k-1} + c_1 r_1 (p_{i,d}^{k-1} - x_{i,d}^{k-1}) + c_2 r_2 (p_{g,d}^{k-1} - x_{i,d}^{k-1}))$$

$$\chi = \frac{2}{\phi - 2 + \sqrt{\phi^2 - 4\phi}} , \quad \text{where} \quad \phi = c_1 + c_2 > 4$$
For instance, the formulation above is commonly applied with $c_1 = c_2 = 2.05$, $\varphi = 4.1$ and $\chi = 0.7298$. The constriction factor method ensures the convergence of the optimization process, and hence, the authors claimed that the velocity boundary $v_{\text{max}}$ was not necessary any more. However, Eberhart and Shi [18] concluded that the algorithm still requires the velocity boundary $v_{\text{max}}$ and needs to limit the $v_{\text{max}}$ to $x_{\text{max}}$ (i.e. $\lambda = 1.0$ in Eq.(4.3)) as a rule of thumb for a better application of the constriction factor method. More recently the proportional factor $\lambda$ to adjust $v_{\text{max}}$ was suggested using value of 0.05 for unimodal functions and 0.5 for multimodal functions in PSO with constriction factors [12].

Since the problem-dependent and indispensible parameter $v_{\text{max}}$ plays a critical role on the performance of PSO, the aim of the present study is to explore the possibility of eliminating the empirical parameter $v_{\text{max}}$ from PSO, in order that the PSO algorithms only requiring two easily controlled parameters (i.e. population size and evolutionary generation) can still achieve robust and satisfactory optimization performance. In Section 4.2, a random reflection boundary technique applied to particles without velocity restriction is proposed to explore the possibility of removing the parameter $v_{\text{max}}$ from PSO. A set of benchmark functions for unimodal and multimodal optimization are solved by PSO algorithms to assess the random reflection boundary technique in Section 4.3. Based on experimentation assessment, the empirical guidelines of the random reflection boundary technique are deduced in Section 4.4 and then applied by a practical buckling optimization of composite panels in Section 4.5. Finally, Section 4.6 draws the conclusions that it is feasible to achieve robust performance of PSO without the sensitive and empirical parameter $v_{\text{max}}$ by applying the random reflection boundary technique.
4.2 Random reflection boundary technique

If a particle flies outside the range of a search space in basic PSO, the range-exceeding location component of the particle is limited to the range boundary in this direction, that is, let

\[ x^{i,d}_k = x^d_{\max}, \quad \text{when } x^{i,d}_k > x^d_{\max} \]  \hspace{1cm} (4.7)

and

\[ x^{i,d}_k = x^d_{\min}, \quad \text{when } x^{i,d}_k < x^d_{\min} \]  \hspace{1cm} (4.8)

This conventional treatment of exceeding design parameter range, called fixed absorption boundary technique in this paper for comparison, is to set the location component of a breaking-through particle to the corresponding design variable limit of feasibility in that direction.

In this study, a technique of random reflection boundary is proposed to replace the original fixed absorption boundary technique. This is motivated by eliminating the parameter \( v_{\max} \) in PSO or reducing the influence of \( v_{\max} \) on the performance of PSO. The random reflection boundary is classified into two types as the half-range one and the full-range one.

When the half-range random reflection boundary is applied to outside-range particles with or without velocity limit, any location component of a particle exceeding the variable boundary will be evenly reinitialized between that limit passed across and the center of dynamic range in this dimension. Interval \((x^d_{\min} + x^d_{\max}/2, x^d_{\max})\) and \((x^d_{\min}, x^d_{\min} + x^d_{\max}/2)\) are the positive and negative range of the \(d\)-th location component of search space (or \(d\)-th design variable), respectively. The implement of the half-range random reflection boundary is expressed as follows:

\[ x^{i,d}_k = \text{random}(x^d_{\min} + x^d_{\max}/2, x^d_{\max}), \quad \text{if } x^{i,d}_k > x^d_{\max} \]  \hspace{1cm} (4.9)

and

\[ x^{i,d}_k = \text{random}(x^d_{\min}, x^d_{\min} + x^d_{\max}/2), \quad \text{if } x^{i,d}_k < x^d_{\min} \]  \hspace{1cm} (4.10)
where \( \text{random}(a,b) \) denotes a random value drawn from a uniform distribution on the interval \([a, b]\).

Accordingly the full-range random reflection boundary is expressed as follows:

\[
x_k^i d = \text{random}(x_{\min}^d, x_{\max}^d), \quad \text{if} \quad x_k^i d < x_{\min}^d \quad \text{or} \quad x_k^i d > x_{\max}^d
\] (4.11)

Obviously the new random reflection boundary tends to maintain diversity and randomness of particles in PSO in order to prevent premature for better search capacity when it is compared with the original fixed absorption boundary.

If a particle tends to fly away from the solution space in positive or negative direction along certain dimension of the search space, it implies that the location component of the particle in that dimension needs to become greater or less than the previous value to achieve a better fitness value, and thus the potential better value of this location component could more likely lie in corresponding positive or negative range in that dimension of the search space. It is expected that the half-range random reflection boundary may comprise diversity maintenance and local exploitation capacity of PSO, while the full-range random reflection boundary can take full advantage of diversity to keep global exploration ability of particles.

### 4.3 Experimentation, rank and assessment

#### 4.3.1 Testing functions

There are many variants of PSO algorithms in terms of the neighborhood topology of particles, the method for convergence control and the treatment on range-exceeding particles. For the sake of expression in clarity and conciseness, a format \( \text{PSO} : V - M(\lambda) - B \) is used to denote a corresponding variant of PSO. \( V = g \) or \( l \) means the global or local version of PSO, respectively. \( M(\lambda) = w(\lambda) \) or \( cf(\lambda) \) stands for the inertia weight or
constriction factor method with proportional factor $\lambda$ to specify $v_{\text{max}}$, given $\lambda = \infty$ refers to no velocity limit applied to particles. $B = a$, $h$ or $f$ represents the fixed absorption boundary, the half-range random reflection boundary or the full-range random reflection boundary, respectively. $PSO : 1 - cf(1.0) - a$, for instance, denotes the local version PSO variant using the constriction factor approach with fixed absorption boundary by limiting $v_{\text{max}}$ to $x_{\text{max}}$ (i.e. $\lambda = 1.0$ in Eq.(4.3)).

In total there are twenty-four PSO variants and they are classified into three groups in the experiments. The PSO variants in each group are further divided into two sub-classes according to the convergence control method, as shown in Table 4.1. Group 1 (G1) consists of four original PSO algorithms with $v_{\text{max}}$ constriction and the fixed absorption boundary. Eight PSO variants with $v_{\text{max}}$ specification and the random reflection boundary are included in Group 2 (G2). Group 3 (G3) involves the PSO variants without any $v_{\text{max}}$ control. Group 3(a) and (b) (G3(a) and G3(b)) are related to the random reflection boundary and the fixed absorption boundary, respectively.

Global version and local ring neighborhood version of PSO are adopted for experiments. This is because global version of PSO has unique particle population topology of fully connected particles to pass the information among the swarm in the fastest way. On the other hand local versions of PSO with the ring lattice population structure results in the slowest communication pattern in the particle swarm compared with other population topologies, such as von Neumann neighborhood and pyramid neighborhood [10]. Therefore, the PSO algorithms under investigation cover the full range of communication pattern from the most direct to indirect way.
Table 4.1
PSO variants tested for experimentation

<table>
<thead>
<tr>
<th>Group</th>
<th>PSO with inertia weight</th>
<th>PSO with constriction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>( g - w(\lambda) - a, l - w(\lambda) - a )</td>
<td>( g - cf(\lambda) - a )</td>
</tr>
<tr>
<td></td>
<td>( l - cf(\lambda) - a )</td>
<td>( l - cf(\lambda) - h ),</td>
</tr>
<tr>
<td></td>
<td>( g - cf(\lambda) - h )</td>
<td>( g - cf(\lambda) - f ),</td>
</tr>
<tr>
<td></td>
<td>( l - cf(\lambda) - f )</td>
<td>( g - cf(\infty) - h )</td>
</tr>
<tr>
<td>G2</td>
<td>( g - w(\lambda) - h, l - w(\lambda) - h )</td>
<td>( l - cf(\lambda) - h )</td>
</tr>
<tr>
<td></td>
<td>( g - w(\lambda) - f, l - w(\lambda) - f )</td>
<td>( g - cf(\lambda) - f )</td>
</tr>
<tr>
<td></td>
<td>( l - cf(\lambda) - f )</td>
<td>( g - cf(\infty) - f )</td>
</tr>
<tr>
<td>G3(a)</td>
<td>( g - w(\infty) - h, l - w(\infty) - h )</td>
<td>( l - cf(\infty) - h )</td>
</tr>
<tr>
<td></td>
<td>( g - w(\infty) - f, l - w(\infty) - f )</td>
<td>( g - cf(\infty) - f )</td>
</tr>
<tr>
<td></td>
<td>( l - cf(\infty) - f )</td>
<td>( l - cf(\infty) - h )</td>
</tr>
<tr>
<td>G3(b)</td>
<td>( g - w(\infty) - a, l - w(\infty) - a )</td>
<td>( g - cf(\infty) - a )</td>
</tr>
<tr>
<td></td>
<td>( l - cf(\infty) - a )</td>
<td></td>
</tr>
</tbody>
</table>

As shown in Table 4.2, ten benchmark functions including five unimodal and five multimodal problems for minimum optimization in 10-dimension and 30-dimension space are tested to evaluate the influence of parameter \( \nu_{\text{max}} \) and random reflection boundary technique on the performances of various PSO algorithms. For nondeterministic PSO, if the fitness difference between the global optima and a solution is not greater than a user-specified acceptance value, the solution is regarded to successfully determine the global peak at this corresponding acceptance level of accuracy. An appropriate acceptance threshold of accuracy may vary in a quite large range, as seen in Table 4.2, which is related to complexity of a problem.
<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>D</th>
<th>Search Space</th>
<th>$f_{\text{min}}$</th>
<th>Acceptance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unimodal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphere [19, 22]</td>
<td>$f_1(x) = \sum_{i=1}^{D} x_i^2$</td>
<td>10, 30</td>
<td>[-100,100]$^D$</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>Schwefel’s P2.22 [19, 22]</td>
<td>$f_2(x) = \sum_{i=1}^{D}</td>
<td>x_i</td>
<td>+ \prod_{i=1}^{D}</td>
<td>x_i</td>
<td>$</td>
</tr>
<tr>
<td>Quadric [19, 22]</td>
<td>$f_3(x) = \sum_{i=1}^{D} (\sum_{j=1}^{i} x_j)^2$</td>
<td>10, 30</td>
<td>[-100,100]$^D$</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Rosenbrock [19]</td>
<td>$f_4(x) = \sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$</td>
<td>10, 30</td>
<td>[-10,10]$^D$</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Quadric Noise [19, 22]</td>
<td>$f_5(x) = \sum_{i=1}^{D} ix^4 + \text{random}[0,1]$</td>
<td>10, 30</td>
<td>[-1.28,1.28]$^D$</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Noncontinuous</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rastrigin [19, 22]</td>
<td>$f_6(x) = 418.9829 \times D - \sum_{i=1}^{D} x_i \sin(\sqrt{</td>
<td>x_i</td>
<td>})$</td>
<td>10, 30</td>
<td>[-500,500]$^D$</td>
</tr>
<tr>
<td>Rastrigin [19, 22, 23]</td>
<td>$f_7(x) = \sum_{i=1}^{D} (y_i^2 - 10 \cos(2\pi y_i) + 10)$</td>
<td>10, 30</td>
<td>[-5.12,5.12]$^D$</td>
<td>0</td>
<td>50</td>
</tr>
</tbody>
</table>
| Griewank [19, 22]           | $f_8(x) = \sum_{i=1}^{D} y_i^2 - 10 \cos(2\pi y_i) + 10$ | 10, 30 | [-5.12,5.12]$^D$ | 0                | 50         | where $y_i = \begin{cases} \frac{x_i}{\text{round}(2\pi x_i)} & |x_i| < 0.5 \\ \frac{\sqrt{|x_i|}}{2} & |x_i| \geq 0.5 \end{cases}$
| **Multimodal**              |            |   |              |                  |            |
| Schewefel [19, 22]          | $f_9(x) = \sum_{i=1}^{D} x_i^2 - 10 \cos(2\pi x_i) + 10$ | 10, 30 | [-500,500]$^D$ | 0                | 100        |
| Generalized Penalized [19, 22] | $f_{10}(x) = \frac{\pi}{D} \left[ 10 \sin^2(\sqrt{y_{j+1}}) + \sum_{i=1}^{D-1} (y_i - 1)^2 [1 + 10 \sin^2(\sqrt{y_{j+1}})] \right] + (y_D - 1)^2 + \sum_{i=1}^{D} u(x_i, 10, 100, 4)$ | 10, 30 | [-50,50]$^D$ | 0                | 0.01       | where $y_i = 1 + \frac{1}{4}(x_i + 1)$, $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a < x_i < a \\ k(-x_i - a)^m, & x_i < -a \end{cases}$ |
The parameters of PSO variants to solve these problems are stated as follows. For PSO with inertia weights, \( c_1 = c_2 = 2.0 \) and \( \omega \) linearly reduce from 0.9 to 0.4 for Eq. (4.4) are used during an optimization run [13], while \( c_1 = c_2 = 2.05 \) and \( \chi = 0.7298 \) are used in Eqs. (4.5) and (4.6) for PSO with constriction factors [17]. In each PSO algorithm, the population sizes for 10-dimension and 30-dimension search space are 20 and 30, respectively, whereas the evolutionary generations for 10-dimension and 30-dimension functions are 5000 and 10000, respectively. For PSO with inertia weights let \( \lambda = 0.2 \), \( \lambda = 0.5 \), \( \lambda = 1.0 \) and \( \lambda = \infty \) to specify parameter \( \nu_{\text{max}} \), respectively, whereas \( \lambda = 1.0 \), \( \lambda = 10^3 \) and \( \lambda = \infty \) for PSO with constriction factors.

Solution accuracy, convergence speed and successful reliability [19-21] are three performance measures for evaluating the average performance of each algorithm based on 30 independent runs of each testing function. Solution accuracy (i.e. the best function fitness value obtained in a run) represents the local search ability of an algorithm. Successful reliability is percentage of trials reaching acceptable solution, which serves as a measure of global research ability by indicating the possibility of global optimal region to be discovered in multiple runs. Convergence speed is the minimum number of evolution generations for achieving an acceptable solution in a single optimization run.

The experimentation is conducted to investigate the advantage of random boundary technique as well as the sensitivity of \( \nu_{\text{max}} \) on PSO performance.

### 4.3.2 Rank and assessment

#### 4.3.2.1 Rank based on solution accuracy and success reliability

Table 4.3 to Table 4.10 list the mean and standard deviation of solution accuracy and converge speed as well as success reliability of the experiment
on test functions over 30 independent runs. Because $g - w(\lambda) - f$, $g - cf(\lambda) - f$, $l - w(\lambda) - f$ and $l - w(\lambda) - f$ obtain similar statistical outcomes compared with $g - w(\lambda) - h$, $g - cf(\lambda) - h$, $l - w(\lambda) - h$ and $l - w(\lambda) - h$, respectively. Therefore, the results of $g - w(\lambda) - f$, $g - cf(\lambda) - f$, $l - w(\lambda) - f$ and $l - w(\lambda) - f$ are not shown for the sake of conciseness and clarity in expression.

The results are classified into the following four main investigation categories for discussion: 10-dimension problems for global PSO (Case 1), 10-dimension problems for local PSO (Case 2), 30-dimension problems for global PSO (Case 3) and 30-dimension problems for local PSO (Case 4). Further, each of the main investigation categories is divided into two sub-categories in terms of the inertia weight (notated with “A”) or constriction factor (notated with “B”) method. For example, Case 2(B) means the group of local PSO with constriction factors for 10-dimension problem discussion. In other words, there are eight investigation or comparison cases for discussion in total according to the combination of particles’ topology type (global or local version), control method of particle convergence (inertia weight or constriction factor method) and the dimension of search space (10d or 30d problems).

For each case of PSO with inertia weight, the difference of solution accuracy between the fixed absorption boundary and random reflection boundary in Section 4.2 is compared based on t-test over 30 independent runs at a 0.05 level of significance. Among $\lambda = 0.2$, $\lambda = 0.5$ and $\lambda = 1.0$ on parameter $v_{max}$ specification, the minimum mean of test function obtained by the PSO with inertia weights and the fixed absorption boundary is taken as a “benchmark solution” and denoted as “0”, for the comparison with any other PSO variants with inertia weight. The t-test results are shown as “+1”, “-1” and “0”, which means the accuracy of the result for comparison is significantly larger than, significantly less than, and almost indifferent from the corresponding “benchmark solution”.
The amount of “+1” and “-1” for each PSO algorithm with inertia weights over the 10 test functions are summed, respectively. The subtraction of the amount of “-1” minus the amount of “+1” is defined as “merit”, which is introduced to assess the overall performance of PSO algorithm with inertia weights on the test functions. Apparently the algorithm getting high merits relatively tends to get better optimization accuracy. Therefore, the rank is organized according to the merits in descending order, as shown in Tables 4.3, 4.5, 4.7 and 4.9.

The same “merit” ranking approach is also applied to investigate the overall performances of solution accuracy for each case of PSO algorithms with constriction factors, as shown in Tables 4.3, 4.5, 4.7 and 4.9.

For each case of PSO variants with inertia weights, the average reliability of successfully reaching global optima is listed and the rank based on the average success reliability is arranged in descending order as shown in Tables 4.4, 4.6, 4.8 and 4.10. Naturally higher average success reliability results in better rank. Similarly, the average and rank of successful reliability for PSO algorithms with constriction factors are presented as well for each case.

The convergence speed of a PSO variant for each test case is also listed. If a PSO variant cannot achieve an acceptable global optimum within the maximum evolution generations (i.e., the success reliability is zero), the convergence speed is noted as “--”, as shown in Tables 4.4, 4.6, 4.8 and 4.10.
### Table 4.3

Solution accuracy of the 10d testing functions as well as the corresponding merits and ranks obtained from the global version of PSO using population size 20 and evolutionary generation 5000 over 30 independent runs.
## Table 4.4

Convergence speed and successful reliability of the 10d testing functions as well as the corresponding $R_{\text{average}}$ and ranks obtained from the global version of PSO using population size 20 and evolutionary generation 5000 over 30 independent runs.

<table>
<thead>
<tr>
<th>Function</th>
<th>Case (1A)</th>
<th>Case (1B)</th>
<th>Case (1D)</th>
<th>Case (1E)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g-w(r)-a</td>
<td>g-w(r)-h</td>
<td>g-w(r)-B</td>
<td>g-w(r)-B</td>
</tr>
<tr>
<td>$R_{\text{average}}$</td>
<td>$R_{\text{average}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphere</td>
<td>± 65</td>
<td>± 78</td>
<td>± 78</td>
<td>± 78</td>
</tr>
<tr>
<td></td>
<td>± 65</td>
<td>± 78</td>
<td>± 73</td>
<td>± 73</td>
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<td>± 69</td>
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<td></td>
<td>± 65</td>
<td>± 78</td>
<td>± 73</td>
<td>± 73</td>
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<tr>
<td>R</td>
<td>± 100</td>
<td>± 100</td>
<td>± 100</td>
<td>± 100</td>
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<td></td>
<td>± 100</td>
<td>± 100</td>
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<td>± 100</td>
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<td>± 100</td>
<td>± 100</td>
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<td>± 100</td>
</tr>
<tr>
<td>Schwefel’</td>
<td>± 105</td>
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<td>± 70</td>
<td>± 70</td>
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<td>± 105</td>
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<td>± 48</td>
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<tr>
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<td>± 1576</td>
<td>± 1576</td>
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<tr>
<td></td>
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### Table 4.5

Solution accuracy of the 10d testing functions as well as the corresponding merits and ranks obtained from the local version of PSO using population size 20 and evolutionary generation 5000 over 30 independent runs

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**Note:** The table presents results from the local version of PSO using population size 20 and evolutionary generation 5000 over 30 independent runs, comparing the performance of different functions and algorithms against each other.
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**Table 4.6**

Convergence speed and successful reliability of the 10d testing functions as well as the corresponding $R_{average}$ and ranks obtained from the local version of PSO using population size 20 and evolutionary generation 5000 over 30 independent runs.
### Table 4.7

Solution accuracy of the 30d testing functions as well as the corresponding merits and ranks obtained from the global version of PSO using population size 30 and evolutionary generation 10000 over 30 independent runs

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- **Schwefel's H**: This column represents the Schwefel's H function values for each case. The values are given in scientific notation, where the exponent ranges from ±1.51e-61 to ±4.03e+1.
- **Quadric**: This column represents the Quadric function values for each case. The values are given in scientific notation, where the exponent ranges from ±1.51e-60 to ±4.03e+1.
- **Rosenbrock**: This column represents the Rosenbrock function values for each case. The values are given in scientific notation, where the exponent ranges from ±1.51e-60 to ±4.03e+1.
- **Noise**: This column represents the Noise function values for each case. The values are given in scientific notation, where the exponent ranges from ±1.51e-60 to ±4.03e+1.
- **H**: This column represents the H function values for each case. The values are given in scientific notation, where the exponent ranges from ±1.51e-60 to ±4.03e+1.
- **Schwefel**: This column represents the Schwefel function values for each case. The values are given in scientific notation, where the exponent ranges from ±1.51e-60 to ±4.03e+1.
## Table 4.8

Convergence speed and successful reliability of the 30d testing functions as well as the corresponding $R_{\text{average}}$ and ranks obtained from the global version of PSO using population size 30 and evolutionary generation 10000 over 30 independent runs.

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Table 4.9
Solution accuracy of the 30d testing functions as well as the corresponding merits and ranks obtained from the local version of PSO using population size 30 and evolutionary generation 10000 over 30 independent runs

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<td>2.49e-3</td>
<td>4.71e-2</td>
<td>5.92e-2</td>
<td>3.50e-1</td>
<td>1e-1</td>
<td>1e-1</td>
<td>1e-1</td>
<td>1e-1</td>
<td>1e-1</td>
</tr>
<tr>
<td>H</td>
<td>±1.08e-3</td>
<td>±1.80e-3</td>
<td>±3.60e-3</td>
<td>±2.70e-3</td>
<td>±4.12e-3</td>
<td>±2.49e-3</td>
<td>±4.71e-2</td>
<td>±5.92e-2</td>
<td>±3.50e-1</td>
<td>±1.08e-3</td>
<td>±1.80e-3</td>
<td>±3.60e-3</td>
<td>±2.70e-3</td>
<td>±4.12e-3</td>
</tr>
<tr>
<td>Penalized</td>
<td>±1.08e-3</td>
<td>±1.80e-3</td>
<td>±3.60e-3</td>
<td>±2.70e-3</td>
<td>±4.12e-3</td>
<td>±2.49e-3</td>
<td>±4.71e-2</td>
<td>±5.92e-2</td>
<td>±3.50e-1</td>
<td>±1.08e-3</td>
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<td>±4.12e-3</td>
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<td>±0</td>
<td>±0</td>
<td>±0</td>
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<td>Merit</td>
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<td>3</td>
</tr>
</tbody>
</table>
Table 4.10

Convergence speed and successful reliability of the 30d testing functions as well as the corresponding $R_{\text{average}}$ and ranks obtained from the local version of PSO using population size 30 and evolutionary generation 10000 over 30 independent runs

<table>
<thead>
<tr>
<th></th>
<th>Case 4(A)</th>
<th>Case 4(B)</th>
<th>Case 4(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_{-}(\lambda=0.2)$</td>
<td>$I_{-}(\lambda=0.5)$</td>
<td>$I_{-}(\lambda=1.0)$</td>
</tr>
<tr>
<td>Sphere $\pm 114$</td>
<td>$\pm 115$</td>
<td>$\pm 100$</td>
<td>$\pm 91$</td>
</tr>
<tr>
<td>Schwefel $\pm 5953$</td>
<td>$\pm 6057$</td>
<td>$\pm 5716$</td>
<td>$\pm 5933$</td>
</tr>
<tr>
<td>R $\pm 5953$</td>
<td>$\pm 5716$</td>
<td>$\pm 5933$</td>
<td>$\pm 6057$</td>
</tr>
<tr>
<td>Quadratic $\pm 5$</td>
<td>$\pm 5$</td>
<td>$\pm 5$</td>
<td>$\pm 5$</td>
</tr>
<tr>
<td>Rosenbrock $\pm 1000$</td>
<td>$\pm 1000$</td>
<td>$\pm 1000$</td>
<td>$\pm 1000$</td>
</tr>
<tr>
<td>R $\pm 1000$</td>
<td>$\pm 1000$</td>
<td>$\pm 1000$</td>
<td>$\pm 1000$</td>
</tr>
<tr>
<td>Quade $\pm 3000$</td>
<td>$\pm 3000$</td>
<td>$\pm 3000$</td>
<td>$\pm 3000$</td>
</tr>
<tr>
<td>Noise $\pm 1104$</td>
<td>$\pm 1000$</td>
<td>$\pm 1000$</td>
<td>$\pm 1000$</td>
</tr>
<tr>
<td>R $\pm 1000$</td>
<td>$\pm 1000$</td>
<td>$\pm 1000$</td>
<td>$\pm 1000$</td>
</tr>
<tr>
<td>R $\pm 0.000$</td>
<td>$\pm 0.000$</td>
<td>$\pm 0.000$</td>
<td>$\pm 0.000$</td>
</tr>
<tr>
<td>Rastrigin $\pm 393$</td>
<td>$\pm 393$</td>
<td>$\pm 393$</td>
<td>$\pm 393$</td>
</tr>
<tr>
<td>R $\pm 1.000$</td>
<td>$\pm 1.000$</td>
<td>$\pm 1.000$</td>
<td>$\pm 1.000$</td>
</tr>
<tr>
<td>Non- $\pm 3731$</td>
<td>$\pm 3731$</td>
<td>$\pm 3731$</td>
<td>$\pm 3731$</td>
</tr>
<tr>
<td>Rastrigin $\pm 1291$</td>
<td>$\pm 1291$</td>
<td>$\pm 1291$</td>
<td>$\pm 1291$</td>
</tr>
<tr>
<td>Griewank $\pm 6158$</td>
<td>$\pm 6158$</td>
<td>$\pm 6158$</td>
<td>$\pm 6158$</td>
</tr>
<tr>
<td>R $\pm 0.967$</td>
<td>$\pm 0.967$</td>
<td>$\pm 0.967$</td>
<td>$\pm 0.967$</td>
</tr>
<tr>
<td>Generalized $\pm 5499$</td>
<td>$\pm 5499$</td>
<td>$\pm 5499$</td>
<td>$\pm 5499$</td>
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<tr>
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<td>$\pm 1.00$</td>
<td>$\pm 1.00$</td>
<td>$\pm 1.00$</td>
</tr>
</tbody>
</table>

$R_{\text{average}}$ 0.897 0.733 0.583 0.890 0.790 0.747 0.680 0.743 0.483 0.727 0.703 0.800 0.797 0.767 0.873 0.700

Rank 1 4 8 2 3 5 7 6 9 3 6 2 3 4 1 7

116
4.3.2.1.1 10-dimension problems for global PSO (Case 1(A) and Case 1(B))

As to solution accuracy for Case 1(A) as shown in Table 4.3, $g - w(0.5) - a$, $g - w(0.5) - h$ and $g - w(\infty) - h$ are listed in the 1st rank group followed by $g - w(0.2) - a$, $g - w(1.0) - a$, $g - w(1.0) - h$ and $g - w(\infty) - f$ in the 4th rank group. For Case 1(B) of Table 4.3, $g - cf(1.0) - a$ and $g - cf(\infty) - f$ are listed in the 1st rank group followed by $g - cf(1.0) - h$, $g - cf(10^3) - h$ and $g - cf(\infty) - h$ in the 3rd rank group.

On the average successful reliability for Case 1(A) shown in Table 4.4, $g - w(0.2) - h$, $g - w(\infty) - h$ and $g - w(\infty) - f$ are listed in the 1st rank group followed by $g - w(0.2) - a$, $g - w(0.5) - a$, $g - w(0.5) - h$ and $g - w(1.0) - h$ in the 4th rank group. For Case 1(B), $g - cf(\infty) - f$ lies in the 1st rank place followed by $g - cf(10^3) - h$ and $g - cf(\infty) - h$ in the 2nd rank group. However, conventional $g - cf(1.0) - a$ and $g - cf(10^3) - a$ take the 4th and 6th place, respectively.

In terms of Case 1, it is seen that global PSO with inertia weights and the random reflection boundary i.e. $g - w(\infty) - h$ and $g - w(\infty) - f$ achieve close performance of solution accuracy and better success performance of reliability, compared with the original global PSO with inertia weights and the absorption boundary i.e. $g - w(\lambda) - a$ respectively. The similar phenomena can be found in the comparison between global PSO with constriction factors and the random reflection boundary i.e. $g - cf(\infty) - h$ and $g - cf(\infty) - f$ and the original global PSO with constriction factors and the absorption boundary i.e. $g - cf(\lambda) - a$. Especially $g - w(\infty) - h$ is the sole variant that is able to locate the minimal optima of function Schwefel within 30 trials, although the success reliability merely 0.033. Moreover, only $g - w(0.2) - h$, $g - w(\infty) - f$ and $g - cf(10^3) - h$ can possibly find the minimum of function Griewank within 30 trials. It is shown that the introduction of random
reflection boundary can maintain the diversity of particles and thus increase the possibility of finding global optima in search space, even if there is not any requirement on $v_{\text{max}}$.

According to Case 1, global PSO with no limitation of $v_{\text{max}}$ and the fixed absorption boundary i.e. $g - w(\infty) - a$ and $g - cf(\infty) - a$ get the worst solution accuracy and success reliability. It obvious that the original fixed absorption boundary of PSO is not applicable without incorporation of $v_{\text{max}}$ constriction.

### 4.3.2.1.2 10-dimension problems for local PSO (Case 2(A) and Case 2(B))

According to Tables 4.5 and 4.6 on Case 2(A), $l - w(0.5) - h$ has the 1$^{\text{st}}$ rank of solution accuracy as well as 3$^{\text{rd}}$ place of success reliability, respectively. While $l - w(\infty) - h$ has the 1$^{\text{st}}$ rank of success reliability as well as 4$^{\text{th}}$ place of solution accuracy, respectively. For $l - w(\infty) - f$, the ranks for solution accuracy and success reliability are both 4$^{\text{th}}$.

According to Tables 4.5 and 4.6 on Case 2(B), $l - cf(1.0) - h$ achieves the best performance on both solution accuracy and success reliability. Variant $l - cf(10^{3}) - h$ takes 1$^{\text{st}}$ rank on solution accuracy and 2$^{\text{nd}}$ rank on $l - cf(1.0) - h$, while $l - cf(\infty) - h$ takes 1$^{\text{st}}$ rank on solution accuracy and 6$^{\text{th}}$ rank on success reliability.

It should be noted that the success reliability on Schwefel function achieved by $l - w(\infty) - h$ reaches 0.267, which is much higher than other local PSO algorithm.

It is also seen that the original local PSO algorithm without limit of $v_{\text{max}}$ i.e. $l - w(\infty) - a$ and $l - cf(\infty) - a$ show the worst performance on both solution accuracy and success reliability, as shown in Case 2.
4.3.2.1.3 30-dimension problems for global PSO (Case 3(A) and Case 3(B))

For Case 3(A) shown in Table 4.7, \( g - w(0.5) - h \) takes the 1\(^{st} \) place, and then \( g - w(1.0) - h \) and \( g - w(\infty) - h \) take the 2\(^{nd} \) rank followed by the 4\(^{th} \) place of \( g - w(\infty) - f \) on the solution accuracy. According to Case 3(A) in Table 4.8, \( g - w(1.0) - h \), \( g - w(1.0) - h \) and \( g - w(1.0) - h \) take the 1\(^{st} \), 2\(^{nd} \) and 3\(^{rd} \) place on success reliability respectively.

For Case 3(B), \( l - cf(1.0) - h \), \( l - cf(10^3) - h \), \( l - w(\infty) - h \) and \( l - w(\infty) - f \) are tied for first rank on the solution accuracy as seen in Table 4.7. According to Case 3(B) in Table 4.8, \( g - w(1.0) - h \), \( g - w(1.0) - h \) and \( g - w(1.0) - h \) take the 1\(^{st} \), 2\(^{nd} \) and 3\(^{rd} \) place on success reliability.

Without the control of \( v_{\text{max}} \), the global version of PSO with conventional fixed boundary method, i.e. \( g - w(\infty) - a \) and \( g - cf(\infty) - a \), has the most inferior performance as to both solution accuracy and success reliability, as shown in Case 3.

4.3.2.1.4 30-dimension problems for local PSO (Case 4(A) and Case 4(B))

For Case 4(A), \( l - w(0.2) - a \) and \( l - w(0.2) - h \) are tied for the 1\(^{st} \) place on solution accuracy, followed by \( l - w(0.5) - a \) and \( l - w(\infty) - f \) at 3\(^{rd} \) and 4\(^{th} \) rank respectively, as seen in Table 4.9. According to Table 4.10, \( l - w(0.2) - a \), \( l - w(0.2) - h \) and \( l - w(0.5) - h \) take the 1\(^{st} \), 2\(^{nd} \) and 3\(^{rd} \) place on success reliability.
For Case 4(B), \( l - cf (10^3) - h \) takes the 1\textsuperscript{st} place followed by \( l - cf (\infty) - h \) and \( l - cf (\infty) - f \) in the 2\textsuperscript{nd} rank of group on solution accuracy as shown in Table 4.9. On success reliability, \( l - cf (\infty) - f \), \( l - cf (1.0) - h \) and \( l - cf (10^3) - h \) take the 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} place respectively.

### 4.3.2.2 Integrated rank and grade assessment

In order to evaluate the comprehensive performance of solution accuracy and success reliability for each variant of PSO, an “integrated rank” for a variant of PSO is adopted, which is the simple addition of solution accuracy rank and corresponding success reliability rank. Due to the influence of parameter \( v_{\text{max}} \) setting, the integrated rank of a variant algorithm fluctuates and the range of the integrated rank can be recorded in the form of “\( R_{\text{best}} - R_{\text{worst}} \)”, in which \( R_{\text{best}} \) and \( R_{\text{worst}} \) are the best and worst integrated rank of a variant for an investigation case. The center and the radius of this rank range is defined as 
\[
m = \frac{(R_{\text{best}} + R_{\text{worst}})}{2} \quad \text{and} \quad r = \frac{(R_{\text{worst}} - R_{\text{best}})}{2},
\]
respectively. \( m \) and \( r \) can directly represent the average performance of solution accuracy and success reliability as well as performance uncertainty of a algorithm, respectively. On one hand, a good algorithm is obviously expected to achieve \( m \) and \( r \) as low as possible. On the other hand, the influence of \( m \) is relatively higher than \( r \) because \( m \) is the premise and essence for assessing an optimal algorithm compared with \( r \). Considering these two aspects, a measure, named after “combined debit” is defined as \( \text{debit} = m^2 r \) to grade the algorithms for comprehensive comparison, in which a good algorithm has low debit value. According to the value of “combined debit” in an ascending order, the performance of the algorithms in an investigation case is graded into “A”, “B” or “C”. Table 4.11 summarizes the integrated rank, the range of integrated rank, debit value and grade assessment of the algorithms except the variants \( g - w(\infty) - a \), \( g - cf (\infty) - a \), \( l - w(\infty) - a \) and \( l - cf (\infty) - a \), because the PSO variants having the fixed absorption boundary but without the control of parameter \( v_{\text{max}} \) have the worst performances in the corresponding comparison.
cases. It has fully demonstrated that the original fixed absorption boundary should be incorporated with the control of maximum particle velocity \( v_{\text{max}} \). So it should be noted that the boundary techniques for PSO variants without \( v_{\text{max}} \) control only cover the half-range and full-range random reflection boundaries in Table 4.11.

In terms of the proposed random reflection boundary for the PSO without limitation of \( v_{\text{max}} \), \( g - w(\infty) - B \), \( l - w(\infty) - B \) and \( g - cf(\infty) - B \) are assessed as A-grade in case of 10-dimension testing functions and \( g - cf(\infty) - B \) achieves A-grade over 30 dimensional testing functions. Therefore, it is shown that the proposed random reflection boundary for the PSO make it possible to eliminate \( v_{\text{max}} \) when applying PSO to achieve reliable and satisfactory performance of optimizations.

### Table 4.11

<table>
<thead>
<tr>
<th></th>
<th>Case 1 and Case 3</th>
<th>Case 2 and Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Global PSO</strong></td>
<td>( g-w(\lambda)-a )</td>
<td>( g-w(\lambda)-h )</td>
</tr>
<tr>
<td>( \lambda = 0.2 )</td>
<td>( \lambda = 0.5 )</td>
<td>( \lambda = 1.0 )</td>
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<tr>
<td>Integrated rank</td>
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<td>12</td>
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<tr>
<td>Range</td>
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<td>5-9</td>
</tr>
<tr>
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<tr>
<td>Grade</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>**10d</td>
<td>Integrated rank</td>
<td>9</td>
</tr>
<tr>
<td>Range</td>
<td>9-16</td>
<td>2-7</td>
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<tr>
<td>Combined debit</td>
<td>546.9</td>
<td>50.6</td>
</tr>
<tr>
<td>Grade</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>**30d</td>
<td>Integrated rank</td>
<td>13</td>
</tr>
<tr>
<td>Range</td>
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<td>4-9</td>
</tr>
<tr>
<td>Combined debit</td>
<td>500.0</td>
<td>105.6</td>
</tr>
<tr>
<td>Grade</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td><strong>Local PSO</strong></td>
<td>( l-w(\lambda)-a )</td>
<td>( l-w(\lambda)-h )</td>
</tr>
<tr>
<td>( \lambda = 0.2 )</td>
<td>( \lambda = 0.5 )</td>
<td>( \lambda = 1.0 )</td>
</tr>
<tr>
<td>Integrated rank</td>
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<td>7</td>
</tr>
<tr>
<td>Range</td>
<td>2-16</td>
<td>3-13</td>
</tr>
<tr>
<td>Combined debit</td>
<td>567.0</td>
<td>320.0</td>
</tr>
<tr>
<td>Grade</td>
<td>C</td>
<td>A</td>
</tr>
</tbody>
</table>
As shown in Table 4.11, the grade assessments of PSO with half-range random reflection boundary and $v_{\text{max}}$ control are superior to those of according PSO with fixed absorption boundary and $v_{\text{max}}$ control for each comparison group. It is shown that replacing fixed absorption boundary with random reflection boundary can improve the performance of PSO with $v_{\text{max}}$ control.

The performance assessments of original PSO with fixed absorption boundary and various proportional factor $\lambda$ to specify $v_{\text{max}}$ are graded as “C” class, which shows that parameter $v_{\text{max}}$ is quite sensitive to accurate and robust performance of PSO and the right value of $v_{\text{max}}$ is hard to estimated for a problem without any “pilot trails”.

### 4.4 Guidelines of the random reflection boundary technique for PSO

Based on the integrated rank and grade assessment of PSO algorithms with the half-range/full-range random reflection boundary technique listed in Table 4.11, the empirical guidelines are further proposed in order to instruct the proper selection of the half-range/full-range random reflection boundary technique used for PSO algorithms. The guidelines for the random reflection boundary are deduced in terms of the dimensionality of optimization problems, the topology structure of PSO and the control method of PSO convergence, shown in Table 4.12.

For some cases as shown in Table 4.12, the parameter $v_{\text{max}}$ is not needed any more for PSO using the random reflection boundary technique. For example, when using the half-range random reflection boundary for PSO with inertia weights in solving low dimension problems, there is no necessity of controlling the maximum velocity of particles $v_{\text{max}}$. Similar case is to apply
the full-range random reflection boundary for PSO with constriction factors in solving high dimension problems. Moreover, parameter $v_{\text{max}}$ can be removed for global PSO using the constriction factor method and the full-range random reflection boundary in solving low dimension problems.

For the other cases as shown in Table 4.12, the parameter $v_{\text{max}}$ may be incorporated in PSO using the random reflection boundary technique for better optimization performance. When applying PSO with inertia weights to solve high dimension optimization problems, for instance, it is recommended to simultaneously use the random reflection boundary and the parameter $v_{\text{max}}$. Moreover, the parameter $v_{\text{max}}$ can be introduced to achieve better performance for local PSO using the constriction factor method and the half-range random reflection boundary in solving low dimension problems.

Table 4.12

Guidelines of the random reflection boundary technique for PSO

<table>
<thead>
<tr>
<th>Low dimension problems</th>
<th>Global version</th>
<th>Local version</th>
<th>High dimension problems</th>
<th>Global version</th>
<th>Local version</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia weights</td>
<td>$h^*$</td>
<td>$h^*$</td>
<td>Inertia weights</td>
<td>$h^+$</td>
<td>$f^+$</td>
</tr>
<tr>
<td>Constriction factors</td>
<td>$f^*$</td>
<td>$h^-$</td>
<td>Constriction factors</td>
<td>$f^*$</td>
<td>$f^*$</td>
</tr>
</tbody>
</table>

Note:
Letter “h” and “f” represent the half-range and full-range random reflection boundary, respectively;
subscript “*” refers to the scenario where the performance of PSO using the random reflection boundary technique is reliably satisfactory, if the parameter $v_{\text{max}}$ is not introduced;
subscript “-” refers to the scenario where the performance of PSO using the random reflection boundary technique can effectively improved, if the parameter $v_{\text{max}}$ is introduced.
Chapter 4

4.5 Practical application

In this section, a practical engineering problem on the buckling optimization design of composite laminates is considered to apply the empirical guidelines for the random reflection boundary technique induced from experimentation of PSO algorithms on testing functions in Section 4.3.

4.5.1 Mechanical model of composite laminates

A simply supported rectangular laminated plate (a×b) consisting of N plies with symmetrical stacking sequence is shown as Fig. 4.1. The laminated plate is subjected to uniform in-plane compressive loads \( N_x \) and \( N_y \) in the \( x \) and \( y \) directions. It is assumed that the laminated panel is composed of a sequence of 2-ply stacks with three orientations of \( 0^\circ \), \( \pm 45^\circ \) and \( 90^\circ \), respectively, to maintain symmetry and balance. Given that the plate is loaded by buckling force \( \eta N_x \) and \( \eta N_y \), where \( \eta \) is a scalar parameter, the laminated panel is instable and buckled into \( m \) and \( n \) half-waves along \( x \) and \( y \) axes when the load parameter \( \lambda \) reaches the buckling value determined by

\[
\eta_b(m,n) = \pi^2 \left( \frac{m^4D_{11} + 2(D_{12} + 2D_{66})(rnn)^2 + rn^4D_{22}}{(am)^2N_x + (rn)^2N_y} \right)
\]

(4.8)

where \( r = a/b \) is the plate aspect ratio; \( D_{ij} \) are the bending stiffness coefficients of the structure, which depend on the stacking sequence of the composite laminate. The critical buckling load factor \( \eta_{cb} = \min \eta_b(m,n) \) is the minimum value of any combination of \( m \) and \( n \), for a given combination of \( N_x \) and \( N_y \). The optimal design problem of the plate on buckling strength is to maximize the critical buckling load factor \( \eta_{cb} \) (i.e. \( \max \eta_{cb} = \max \{ \min \eta_b(m,n) \} \)), by varying the stacking sequence of the laminated plate and the aspect ratio, material property and loading condition remain unchanged.
Fig. 4.1. A simply supported and symmetric laminated panel under in-plane loads

Considering a 64-ply composite laminated plate subjected to loading condition of $N_x/N_y = 1.0$ and made of graphite-epoxy layers in the shape of $a = 0.508$ m and $b = 0.254$ m. The material properties are $E_1 = 127.59$ GPa, $E_2 = 13.03$ GPa, $G_{12} = 6.41$ GPa, $v_{12} = 0.3$ and thickness $t = 0.127$ mm. To achieve the maximum buckling strength, the stacking sequence has to be optimized. This problem has been studied by using genetic algorithm (GA) [24, 25], simulated annealing (SA) algorithm [26] and multimodal PSO. It has found that the maximal buckling load factor $\eta_{cb}$ of this composite plate has seven global optima with the value of 3973.014 and fourteen global sub-optima with the value of 3972.996. Refer to Chapter 3 for more details.

4.5.2 Results and discussion

To apply PSO algorithms, this discrete optimization can be converted into continuous variable optimization in $[0 1]^{16}$ search space. In each dimension, intervals $[0 1/3)$, $[1/3 2/3)$, and $[2/3 1]$ represent orientation $0^\circ$, $\pm 45^\circ$ and $90^\circ$, respectively. Particle population and evolution generation for computation are 25 and 500, respectively. The PSO algorithms along with their corresponding parameter settings and comparison measures for performance assessment are the same as experimentation conduction in Section 4.3.1. Cases A, B, C and D
for validation of the empirical guideline refer to \( g \cdot pso \cdot \omega(\lambda) - B \), \( g \cdot pso \cdot cf(\lambda) - B \), \( l \cdot pso \cdot \omega(\lambda) - B \) and \( l \cdot pso \cdot cf(\lambda) - B \) (\( \lambda = 1/3, 2/3, 1, \infty \)), respectively.

According to 50 independent runs, the statistical results on accuracy and reliability to find global optimum of each PSO algorithm are listed in Tables 4.13 and 4.14, in terms of the fixed absorption and random reflection boundaries. The \( t \)-test on accuracy at a 0.05 level of significance is carried out in a similar way for testing function in Section 4.3.2. The \( t \)-test results are shown as “+1”, “-1” and “0”, which means the accuracy of the compared result is significantly better than, significantly worse than, and almost the same as the best performance of PSO with the fixed absorption boundary, respectively, in maximizing the critical buckling load factor \( \eta_{cb} \).

**Table 4.13**

Statistical results of global PSO variants to maximize the buckling capacity of a 64-ply composite panel

<table>
<thead>
<tr>
<th>B</th>
<th>Measures</th>
<th>Case A: ( g \cdot pso \cdot \omega(\lambda) - B )</th>
<th>Case B: ( g \cdot pso \cdot cf(\lambda) - B )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda = 1/3 )</td>
<td>( \lambda = 2/3 )</td>
<td>( \lambda = 1 )</td>
</tr>
<tr>
<td>B=a</td>
<td>Accuracy</td>
<td>3939.39 ( \pm 49.98 )</td>
<td>3896.03 ( \pm 90.70 )</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>0*</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>Reliability</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>B=h</td>
<td>Accuracy</td>
<td>3972.31 ( \pm 1.04 )</td>
<td>3972.77 ( \pm 0.28 )</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td></td>
<td>Reliability</td>
<td>0.16</td>
<td>0.30</td>
</tr>
<tr>
<td>B=f</td>
<td>Accuracy</td>
<td>3972.28 ( \pm 1.20 )</td>
<td>3972.66 ( \pm 0.34 )</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td></td>
<td>Reliability</td>
<td>0.28</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Table 4.14
Statistical results of local PSO variants to maximize the buckling capacity of a 64-ply composite panel

<table>
<thead>
<tr>
<th>B Measures</th>
<th>Case C: l-pso-ω(λ) -B</th>
<th>Case D: l-pso-cf (λ) -B</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ = 1/3</td>
<td>λ = 2/3</td>
<td>λ = 1</td>
</tr>
<tr>
<td>Reliability</td>
<td>0.14</td>
<td>0.06</td>
</tr>
<tr>
<td>H</td>
<td>0^</td>
<td>-1</td>
</tr>
<tr>
<td>B=a</td>
<td>3972.12</td>
<td>3965.28</td>
</tr>
<tr>
<td>B=h</td>
<td>3972.73</td>
<td>3972.84</td>
</tr>
<tr>
<td>B=f</td>
<td>3972.84</td>
<td>3972.73</td>
</tr>
<tr>
<td>Reliability</td>
<td>0.34</td>
<td>0.46</td>
</tr>
<tr>
<td>Accuracy</td>
<td>± 0.26</td>
<td>± 0.23</td>
</tr>
<tr>
<td>Reliability</td>
<td>0.58</td>
<td>0.44</td>
</tr>
<tr>
<td>H</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>B=a</td>
<td>3972.12</td>
<td>3965.28</td>
</tr>
<tr>
<td>B=h</td>
<td>3972.73</td>
<td>3972.84</td>
</tr>
<tr>
<td>B=f</td>
<td>3972.84</td>
<td>3972.73</td>
</tr>
<tr>
<td>Reliability</td>
<td>0.34</td>
<td>0.46</td>
</tr>
<tr>
<td>Accuracy</td>
<td>± 0.26</td>
<td>± 0.47</td>
</tr>
<tr>
<td>Reliability</td>
<td>0.58</td>
<td>0.44</td>
</tr>
<tr>
<td>H</td>
<td>+1</td>
<td>+1</td>
</tr>
</tbody>
</table>

As shown in Tables 4.13 and 4.14 for t-test on accuracy, each PSO algorithm with the random half-range/full-range reflection boundary remarkably outperforms its corresponding PSO algorithm with the fixed absorption boundary. First, every PSO algorithm with random reflection boundary has better solution accuracy than its corresponding PSO algorithm with fixed absorption boundary for a certain value of proportional factor $\lambda$. Moreover, the success reliability of each PSO algorithm with random half or full range reflection boundary undoubtedly overwhelm its corresponding PSO algorithm with fixed absorption boundary. For example, the solution accuracy of g-pso-$\omega(1/3)$ -a is 3939.39 ±49.98, which is statistically worse than those of g-pso-$\omega(1/3)$ -h (3972.31 ±1.04) and g-pso-$\omega(1/3)$ -f (3972.28 ±1.20), respectively. The success reliability of g-pso-$\omega(1/3)$ -a is 0, while the success reliability of g-pso-$\omega(1/3)$ -h and g-pso-$\omega(1/3)$ -f reach 0.16 and 0.28, respectively.
Therefore, PSO algorithm with the half-range/full-range random reflection boundary is definitely much better than its corresponding original PSO algorithm with the fixed absorption boundary in terms of solution accuracy and success reliability listed in Tables 4.13 and 4.14. This practical problem further confirms that replacing the fixed absorption boundary with the random reflection boundary can substantially enhance the performance of PSO.

As for validating the empirical guidelines of random reflection boundary, the stacking sequence optimal design of 64-ply composite laminate has a solution search space of 16-dimension, which is between 10 dimensions and 30 dimensions. To some extent, therefore, it is believed that this 16-dimension optimal design should simultaneously possess the features of low and high dimension optimizations.

As to Case A (i.e. g-psó- ω(λ) -B) in Table 4.13, the average success reliability of g-psó- ω(λ) -h (λ = 1/3, 2/3, 1, ∞) is 0.31, which is 41% higher than that of g-psó- ω(λ) -f (λ = 1/3, 2/3, 1, ∞). Moreover, g-psó- ω(∞) -h achieves much better solution accuracy and success rate (3972.81 ±0.34 and 0.48, respectively) than g-psó- ω(∞) -f (3971.91 ±1.25 and 0.12, respectively). These data confirm that the half-range random reflection boundary is applicable for the global PSO with inertia weights in both low and high dimension problems. This agrees with the empirical guidelines of the random reflection boundary on these cases as shown in Table 4.12.

As shown in Table 4.13, g-psó- cf(∞) -f achieves the best solution accuracy with highest average and lowest deviation among Case B (i.e. g-psó- cf(λ) -B). Furthermore, the success reliability of g-psó- cf(∞) -f has no obvious difference in the average success reliability of g-psó- cf(λ) -h and g-psó- cf(λ) -f (λ = 1/3, 2/3, 1, ∞), because these three rates reach 0.12, 0.10 and 0.14, respectively. These outcomes coincide with the suggestions of empirical guidelines that the full-range reflection boundary and no restrictions to particle’s velocity are recommended when applying global PSO with
According to Table 4.12, “h” and “f” are recommended for low and high dimensional problems, respectively, if local version of PSO variant with inertia weights is applied. For Case C (i.e. l-pso- $\omega(\lambda)$ -B) in Table 14, the success reliability of l-pso- $\omega(\infty)$ -h and the average success reliability l-pso- $\omega(\lambda)$ -f ( $\lambda = 1/3,2/3,1$ ) are 0.50 and 0.46, respectively. The solution accuracy of l-pso- $\omega(\infty)$ -h is also very close to that of l-pso- $\omega(\lambda)$ -f ( $\lambda = 1/3,2/3,1$ ). These show that this 16-dimension optimal practice can be classified into either a low or high dimension problem because PSO variants using the half-range or full-range reflection boundary get similar performances. Taking low dimension problem category for Case C in Table 4.14, the performance of l-pso- $\omega(\infty)$ -h is the same with the best performance of l-pso- $\omega(\lambda)$ -h ( $\lambda = 1/3,2/3,1$ ), which confirms that it is not necessary to use the parameter $v_{\text{max}}$ to acquire reliable performance for local PSO with inertia weights and the half-range random reflection boundary in low dimension optimizations as shown in Table 4.12. While considering high dimension problem category for Case C in Table 4.14, the solution accuracy and success reliability of l-pso- $\omega(\infty)$ -f are not superior than those of each l-pso- $\omega(\lambda)$ -f ( $\lambda = 1/3,2/3,1$ ). This justifies that the incorporation of $v_{\text{max}}$ can improve the optimization performance when solving high dimension problems by local PSO using the inertia weigh method and the full-range reflection boundary, as shown in Table 4.12.

According to Table 4.12, “h” and “f” are recommended local version of PSO variant with constriction factors in terms of low and high dimensional optimal problems, respectively. For Case D (i.e. l-pso- $cf(\lambda)$ -B) in Table 4.14, the average success reliability l-pso- $cf(\lambda)$ -h ( $\lambda = 1/3,2/3,1$ ) and the success reliability of l-pso- $cf(\infty)$ -f are 0.28 and 0.26, respectively. These two close success rate also show that this 16-dimension optimal practice has the characteristics of a low dimension problem and a high dimension problem at
the same time. Considering low dimension problem category for Case D in Table 4.14, the solution accuracy and success reliability of l-pso- $cf(\infty)$ -h are not superior than those of each l-pso- $cf(\lambda)$ -h ($\lambda = 1/3, 2/3, 1$). This confirms that the incorporation of $\nu_{\text{max}}$ can achieve better optimal results when solving low dimension problems by local PSO using the constriction factor method and the half-range reflection boundary as shown in Table 4.12. While considering the high dimensional problem category for Case D in Table 4.14, the performance of l-pso- $cf(\infty)$ -f is almost the same as the best performance of l-pso- $cf(\lambda)$ -f $\lambda = 1/3, 2/3, 1$. This confirms that there is no requirement of parameter $\nu_{\text{max}}$ to obtain satisfactory performance for local PSO with constriction factors and the full-range random reflection boundary in high dimension optimizations, as shown in Table 4.12.

Based on the above results and discussion for the buckling capacity optimization of a 64-ply composite laminate, of the random reflection boundary in Section 4 are examined and justified by the practical engineering design. Thus it is possible to remove the empirical parameter $\nu_{\text{max}}$ in PSO according to the guidelines for properly using the random reflection boundary technique.

### 4.6 Conclusions

In the present study the possibility of eliminating the empirically pre-specified and sensitive parameter $\nu_{\text{max}}$ of PSO has been investigated by replacing the conventional fixed absorption boundary with the proposed random reflection boundary, in order to enhance the performance of the robustness and adaptability of PSO for various real-world applications.

According to the results of the testing function experiments and the practical engineering problem on buckling optimization of composite panels, it has shown that PSO using the proposed random reflection boundary can achieve
better optimization performance compared with PSO using the conventional fixed absorption boundary. The experimental results have shown that the parameter $v_{\text{max}}$ is indispensible and sensitive to PSO using the conventional fixed absorption boundary as expected. The experimental results have also shown that PSO using the random reflection boundary may not require the parameter $v_{\text{max}}$ anymore and can still achieve satisfactory performance.

Based on the experimentation of testing functions, the empirical guidelines of the random reflection boundary have be conducted in terms of the particle communication topology of PSO, the control method of PSO convergence and the dimensionality of optimization problems. According to the guidelines and its application of a buckling optimization of composite panels, it indicated that removing the user-selected parameter $v_{\text{max}}$ of PSO is feasible, when using the half-range random boundary for PSO with inertia weights in solving low dimension optimization problems and using the full-range random boundary for PSO with constraint factor in solving high dimension optimization problems.

It should be noted that there are only two user-selected parameters left (i.e., population size and evolutionary generation) for the PSO algorithms without $v_{\text{max}}$ control by using the random reflection boundary. The performance of PSO without $v_{\text{max}}$ control could be robustly improved by simply increasing population size and evolutionary generation, because using larger population size and evolutionary generation generally leads to better optimization performance of PSO.

In conclusion, the proposed random reflection boundary technique for PSO has highly attractive advantage of eliminating/reducing the significance and sensitivity of an empirical parameter of particles’ maximum velocity in PSO. This substantially contributes to more reliable and more convenient applications of PSO in various real-world practices, such as buckling optimization of composite structures.
References


Chapter 5

Novel multi-level methodology for concept design of advanced grid stiffened composite panels subjected to buckling loads

(Paper 4)

Liang Huang, Abdul H. Sheikh, Ching-Tai Ng & Michael C. Griffith

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Manuscript in publication style
# Statement of Authorship

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<td>Liang Huang, Abdul H. Sheikh, Ching-Tai Ng &amp; Michael C. Griffith</td>
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## Author Contributions

By signing the Statement of Authorship, each author certifies that their stated contribution to the publication is accurate and that permission is granted for the publication to be included in the candidate's thesis.

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<th>Liang Huang</th>
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<th>Michael C. Griffith</th>
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Abstract

Concept design of advanced grid stiffened (AGS) composite panels is a challenging topology optimization because of a large number of possible design options and time consuming structural analysis process. In this chapter/paper an efficient methodology is proposed to conduct concept design of AGS composite panels which is used for the actual construction design with some further details. The methodology developed in this study is based on a multi-level approach where an inner 3-stage optimization process is nested within an outer 3-step optimization process. The outer 3-step optimization process decomposes the concept design of AGS composite panels into sub-optimization processes according to the critical discrete design variables. Each sub-optimization process identified within the outer 3-step optimization process is further decomposed into the inner 3-stage optimization process having inferior sub-optimization processes according to ply-orientation and other design variables. For the inner 3-stage optimization process, radial basis functions (RBF) metamodeling techniques are used to consider the effects of interaction the ply-orientation and other design variables. The proposed methodology is applied to a design optimization problem of an AGS composite plate against its buckling resistance, by incorporating a ring topology based multimodal particle swarm optimization (PSO) algorithm with an improved finite element (FE) buckling analysis model. The results show that the AGS composite plate with iso-grid stiffening scheme has higher stiffening efficiency compared with those having ortho-grid, x-grid and bi-grid stiffening arrangements, and the optimal stiffener width for maximum unit-stiffening-ratio has the lower bond within its feasible range for all types of grid studied in this research. It should also be noted that the proposed multi-level methodology is suitable for parallel computing, and the concept of the proposed methodology can be easily and generally extended to optimal design of other engineering problems with many design variables and complex system responses.
5.1 Introduction

5.1.1 Optimization of advanced grid stiffened (AGS) composite panels

The use of laminated composite stiffened panels has been popular in many engineering applications, such as launch vehicle fuel tanks, aircraft fuselages, bridge decks and auditorium roof panels. This is because a stiffened composite panel can always significantly enhance the load resistance compared to an unstiffened composite panel of same weight. Figs. 5.1 and 5.2 show a number of stiffening configurations commonly found in practice. Compared to a panel having a simple stiffening arrangement such as ring, stringer or diagonally stiffened panel (Fig. 5.1), the AGS composite panels with ortho-grid, x-grid, bi-grid or iso-grid stiffening configurations (Fig. 5.2) are attractive alternatives as these configurations help to enhance the structural efficiency in a more effective way in complex loading conditions [1, 2].

The buckling load capacity of these complex structures under compressive loads is one of the major concerns of designers and the researchers can help by proving a better insight of the behavior of these advanced grid stiffened composite panels based on concept design study. Thus there is need for an effective methodology for challenging concept design of these structures which can identify the optimal stiffening configuration including topology design (i.e. selection of the type of optimal stiffener-grid), geometry design (e.g. stiffener size and spacing optimization) and material design (e.g. stiffener ply-orientation optimization) for AGS panels subjected to in-plane compressive loads.

The design optimization of unstiffened and stiffened composite panels to achieve maximum buckling load capacity satisfying the necessary design constraints has drawn a considerable research interest. A number of approaches have been developed to achieve the optimal design such as parametric study, conventional gradient-based methods and modern gradient-
free methods. Wodesenbet et al. [3] conducted a parametrical study to investigate the optimal buckling load capacity of an iso-grid stiffened composite cylinder. The design parameters considered in their study were skin thickness, skin winding angle, stiffener orientation and longitudinal modulus. Bruyneel and Fleury [4] combined mathematical programming and convex approximation to calculate the gradient between two successive iterations to simultaneously optimize the ply thickness and orientation of unstiffened composite laminates. However, the gradient-based optimization algorithms could not guarantee to identify the global optimal design of composite structures, if there are many local optimal designs in the design problem. Moreover, the gradient-based optimization methods are not suitable for optimization of AGS composite structures with a number of discrete design variables [5].

Riche and Haftka [6] applied the genetic algorithm (GA) technique to optimize the stacking sequence of a composite laminate for its buckling load capacity maximization. They assumed that the skin is symmetrical, balanced and made of 2-layers stacks with 3 possible discrete orientations: 0°, ±45 and 90°. Jaunky et al. [7] extended the GA for minimum weight design of grid stiffened composite panels with buckling constraints. The design variables considered in this study were stiffener height, stiffer breadth, stiffener spacing and type of stiffener-grid. In spite of successful applications of the GA [8, 9] in a wide range of problems, there has been a number of newly developed powerful gradient-free optimization algorithms such as evolution strategy (ES) [10], simulated annealing (SA) [11], particle swarm optimization (PSO) [12] and ant colony optimization (ACO) [13] for better efficiency. Thus it indicates that a successful optimal design of AGS composite structures would require more efficient gradient-free algorithms to find the global optima in a large and complex search space. This is due to a large number of structural analyses required by the gradient-free algorithms generally to identify the global optimal design and each structural analysis of the complex AGS composite structures could be time-consuming.
5.1.2 Metamodeling techniques

Due to the complexity associated with the buckling characteristic of advanced grid stiffened composite panels, metamodeling techniques, which is based on statistical methods of metamodel fitting and design of experiments (DOE), have been employed to provide a surrogate tool to replace the computationally expensive simulations of structural analyses during the optimization process. The response surface method (RSM) in the form of a low order polynomial and the experimental design with D-optimality were employed by Rikards et al. [14] to build a metamodel to surrogate the finite element analysis (FEA) in the optimization process for optimal buckling-resistant design of stiffened composite panels. Todoroki and Ishikawa [15] applied a second order polynomial response surface to reduce the evaluation cost of GA for maximizing the buckling load capacity of composite structures by optimizing the stacking sequence. A new D-optimal design of experiments was proposed to construct a response surface having four design parameters of composite laminates as inputs and the buckling load capacity as the output using 36 sampling points. Rikards et al. [16] used the space-filling design of experiments to arrange 27 sample points to build a second order polynomial metamodel for four design variables (i.e. number of stiffeners, stiffener height, panel inner radius and panel length). Bisagni and Lanzi [17] employed a multi-layer forward artificial neural network (ANN) technique with the back propagation learning rule and FEA data to predict the buckling resistance of stiffened composite panels with a reduced computational cost of the optimization process. The radial basis function (RBF) model with experimental design of orthogonal arrays (OA) was proposed by Hao et al. [18] to construct a relatively high confidence surrogate model with less computational cost for the optimal design of complex stiffened structures. An adaptive surrogate-based optimization procedure based on Kriging approximation model and full-factorial design (FFD) of experiments was developed for advanced grid stiffened structures to improve the efficiency of the optimization process [19]. However, the usage of the aforementioned metamodeling techniques is limited to an efficient buckling load capacity
evaluation of a structural system and there is a simple interaction between a metamodel and an optimization procedure.

5.1.3 Multi-level optimization approaches

To overcome the challenge of global optimal design of stiffened composite structures in a high dimensional search space, multi-level optimization approaches have been proposed in the literature. Liu et al. [20] proposed a bi-level optimization strategy for global optimum design of stiffened composite panels. At panel level optimization, the continuous cross-sectional geometry of the skin and stiffeners were optimized by treating the laminates as equivalent orthotropic plates. At laminate level optimization, the discrete stacking sequences were selected to satisfy the design rules and buckling constraints. They applied the bi-level approach for the design of a Z-stiffened plate and a long wing cover panel. However, this method is not reliable for optimal design of laminates with a small number of plies because the equivalent orthotropic material properties could be obtained consistently from the extension stiffness matrix [A] and the bending stiffness matrix [D] only when the laminates have more than 40 plies. In addition, this approach cannot guarantee that the eventual optimal design is a global optima approximation because the equivalent orthotropic material properties introduced as intermediate variables for the panel level optimization were calculated from a limited number of typical stacking sequences.

Recently, Hao et al. [18] presented an alternative bi-step optimization formwork for the optimal design of structures with plenty of design variables and used for a metallic iso-grid stiffened cylindrical shell optimization. In the first step, the original optimization problem was divided into several sub-optimization problems according to different design variable categories and each sub-optimization problem was solved individually based on the corresponding group of variables. In the second step, a synthetic design with preliminary optimal values obtained from the first step was taken as the initial design for the second step and the final optimal design was identified in a reduced design space based on the synthetic design. However, this approach
optimized each group of design variables separately and ignored the coupling and interaction between them. Thus the synthetic model for the second step may result in a local optimum rather than the global optimum approximation.

A bi-level optimization technique for the lay-up design of composite plates in lamination parameter space was initially proposed by Yamazaki [21]. First a gradient-based optimization was used to obtain the optimal cross-section size and lamination parameters of a composite plate. GA was then applied to find the laminate stacking sequence that had a close match with the optimal lamination parameters from the first step. This method was adopted by Herencia et al. [22] for the optimization of a long composite plate with T-shaped laminated stiffeners. Using the lamination parameters as intermediate design variables, the lay-up optimization significantly reduced the number of design variables compared to using individual ply orientation and thickness. However, the feasible region of lamination parameters had to be derived for predefined ply orientations [23] because the lamination parameters were interrelated. This restricts this bi-level optimization to satisfactorily identify the optimal design of composite panels with non-conventional ply orientations. Moreover, there is an obvious discrepancy between the final discrete stacking sequence at the second level and the optimal continuous lamination parameters at the first level [24].

In summary, it shows that the existing multi-level optimization procedures can only optimize a part of the entire design variables at each optimization step. Therefore, the full interaction of all design variables for the final optimal result cannot be considered in these multi-level optimization approaches.

In this study a new methodology for concept design optimization is proposed for identification of the optimal topology, geometry and material design of AGS composite panels for buckling-resistance. The proposed multi-level methodology is composed of an outer 3-step optimization procedure based on the critical discrete design variables and an inner 3-stage optimization procedure based on metamodeling techniques as described in Section 5.2. The selection strategies of sampling points and surrogate models for metamodeling
technique used in the inner 3-stage optimization process are then elaborated in Section 5.3. An efficient FEA tool used for buckling simulation of AGS composite plates is concisely described in Section 5.4. A multimodal PSO serves as an optimization algorithm in the inner 3-stage optimization process is briefly presented in Section 5.5. In Section 5.6, an application of the proposed methodology is illustrated in details by evaluating the stiffening efficiency of the stiffening configurations of an AGS composite plate subjected to in-plane compressive load. Finally, the advantage and significance of the developed methodology is summarized in Section 5.7.

Fig. 5.1. Simply stiffened composite panels
Fig. 5.2. AGS composite panels
5.2 Methodology for concept design of AGS composite panels

AGS composite laminates have different design parameters such as type of stiffener-grid, stiffener size and ply orientation at the initial design stage. The main task of concept design of AGS composite laminates is to efficiently investigate the performance of all feasible designs and select few outstanding designs which are subsequently refined further to get the final design used for actual construction. The methodology developed in the present study for the concept design of AGS composite panels is based on a strategy of multi-level optimization process with a double-layer framework that consists of an inner 3-stage optimization scheme nested within an outer 3-step optimization procedure as shown in Fig. 5.3.

(a) The outer 3-step optimization process
Chapter 5

(b) The inner 3-stage optimization process

Fig. 5.3. The proposed methodology: an inner 3-stage optimization process nested within an outer 3-step optimization process for concept design of AGS composite panels

5.2.1 Outer 3-step optimization process

For an optimal structure design, an objective fitness of interest for optimization and assessment is pre-defined by designers according to practical demand. In the present study, a non-dimensional/normalized unit-stiffening-ratio ($\mu$) between an AGS composite laminate and a reference un-stiffened composite laminate is introduced in the form of

$$\mu = \frac{c_g/c_r}{m_g/m_r}$$  \hspace{1cm} (5.1)

where $c_g$ and $c_r$ are the buckling-resistant capacity (or other characteristics of interest for optimization) of the grid-stiffened laminates and the reference un-stiffened laminates, respectively; $m_g$ and $m_r$ are the masses of the grid-stiffened laminate and the reference un-stiffened laminate, respectively. The
optimization of the buckling-resistant design of AGS composite laminates is then defined by maximizing the unit-stiffening-ratio ($\mu$) as

$$\text{Max } \mu = f(X) = f(\theta_s, \theta_s^t, t, b, d, s, T) \quad X \in F(X)$$  (5.2)

where $X = (\theta_s, \theta_s^t, t, b, d, s, T)$ is the design variable vector and its components are ply orientations ($\theta_s$) of the layers of its skin, ply orientations ($\theta_s^t$) of the stiffener layers, skin thickness ($t$), stiffener width ($b$), stiffener height ($d$), stiffener spacing ($s$) and stiffener-grid type ($T$); and $F(X)$ is the feasible domains of $X$ subjected to the constraints of the problem under consideration.

As the structural performance of an AGS composite panel will be significantly affected by some critical discrete design variables, such as type of stiffener-grid $T$ and the skin thickness $t$, it is convenient to decompose the original optimization task of concept design (Eq. (5.2)) into some independent sub-optimizations based on the identified critical discrete design variables and compare the results obtained from all sub-optimizations to find the global optimal solutions of the concept design of AGS composite panels. Thus a 3-step optimization process is proposed as outlined below.

**Step 1:** to identify critical discrete design variables of the original optimization problem;

**Step 2:** to conduct finite number of sub-optimizations separately where the identified critical discrete design variables have feasible combinatory values which reduces the search space of design variables for optimization;

**Step 3:** to rank the results of all sub-optimization problems and identify the global optimal solutions of the original optimization problem.

Assuming that the type of stiffener-grid $T$ is only identified as the critical discrete design variable in the original optimization problem (Eq. (5.2)), for example, the corresponding sub-optimization problems in Step 2 for a certain value of stiffener-grid type $T_i$ ($i=1,2,\ldots,n$) can be expressed as
Max \( \mu_i = f_i(X) = g(x) = g(\theta_x, \theta_{\theta}, t, b, d, s) \quad x \in F_i(x) \) \hspace{1cm} (5.3)

where \( x = (\theta_x, \theta_{\theta}, t, b, d, s) \) is the design variable vector which excludes the components of the identified critical discrete design variable \( T \) in the sub-optimization and \( F_i(x) \) is a feasible domain of \( x \) in the \( i \)-th sub-optimization problem (or the \( i \)-th stiffener-grid type \( T=T_i \)). In Step 3, these optimal unit-stiffening-ratio \( \mu_i^{best} \) (\( i=1,2,\cdots,n \)) obtained from the sub-optimizations are ranked to identify the candidate stiffening configurations for design refinement such as the optimal stiffener-grid type \( T^* \) and the corresponding other optimal design variables \( (\theta_x^*, \theta_{\theta}^*, t^*, b^*, d^*, s^*) \).

To solve the sub-optimization problems in Step 2 of the outer optimization procedure for the concept design of AGS composite panels, a 3-stage optimization process is proposed to further reduce the search space of each sub-optimization. For a convenient representation, the 3-step optimization procedure based on the critical discrete design variables is defined as “outer 3-step optimization” (Fig. 5.3(a)), while the 3-stage optimization process embedded in the outer 3-step optimization is defined as “inner 3-stage optimization” (Section 5.2.2).

### 5.2.2 Inner 3-stage optimization process

As shown in Fig. 5.3(b), the inner 3-stage optimization process is outlined to find the global solution to the Step 2 sub-optimization of Eq. (5.3) in the outer 3-step optimization procedure. In the inner 3-stage optimization, the design variable vector \( x = (\theta_x, \theta_{\theta}, t, b, d, s) \) is divided into two sub-vectors: ply-orientation sub-vector \( x_1 = (\theta_x, \theta_{\theta}) \) and non-ply-orientation (or geometry-size) sub-vector \( x_2 = (t, b, d, s) \). In terms of these two design sub-vectors, the Step 2 sub-optimization of Eq. (5.3) could be further decomposed and systematically coupled based on a reduced subspace of ply-orientation
variables \( S(x_i) = S(\theta_i, \theta_u) \) and a reduced subspace of non-ply-orientation variables \( S(x_2) = S(t, b, d, s) \).

There are six procedures (Procedures 1-6) in the inner 3-stage optimization and every stage of the inner 3-stage optimization consist of two procedures, as listed below

**Stage 1 (Procedures 1 and 2)**

Procedure 1: the sampling points \( P_i(x_2) = (t_i, b_i, d_i, s_i) \) \( (i = 1, 2, \cdots, m) \), which uniformly scatter in the reduced subspace \( S(x_2) = S(t, b, d, s) \), are chosen by design of experiments;

Procedure 2: for a fixed value of sub-vector \( x_2 = (t, b, d, s) \) at each sampling point, the optima of objective function value \( y_i^* \) \( (i = 1, 2, \cdots, m) \) is to be obtained by only considering the design variables of the ply-orientation sub-vector \( x_1 \) in the reduced subspace \( S(x_1) = S(\theta_i, \theta_u) \) during the optimization process, that is, to maximize \( y = g(x) = g(x_1, x_2) = g_1(x_1) = g_1(\theta_i, \theta_u) \) repeatedly for every sampling point \( P_i(x_2) = (t_i, b_i, d_i, s_i) \) \( (i = 1, 2, \cdots, m) \);

**Stage 2 (Procedures 3 and 4)**

Procedure 3: based on a serial of \( y_i^* \) and \( P_i(x_2) = (t_i, b_i, d_i, s_i) \) \( (i = 1, 2, \cdots, m) \), a group of metamodels \( \hat{y} = g_2(x_2) = g_2(t, b, d, s) \) are constructed, which represents the approximate relationship between optimal objective function value and sub-vector \( S(x_2) = S(t, b, d, s) \). It should be noted that the estimated objective function value \( \hat{y} \) of the metamodel has already incorporated the optimization through sub-vector \( x_1 = (\theta_i, \theta_u) \);

Procedure 4: the most appropriate metamodel \( \hat{y} = g_2(x_2) = g_2(t, b, d, s) \) among the group of alternative metamodels is statistically identified and then
utilized to find the optimal estimated objective function value $\hat{y}^*$ of this metamodel and the corresponding intermediate optimal point $P^*(x_2) = (t^*, b^*, d^*, s^*)$ or optimal design sub-vector $x_2^* = (t^*, b^*, d^*, s^*)$ in the reduced non-ply-orientation subspace $S(x_2) = S(t, b, d, s)$.

**Stage 3 (Procedures 5 and 6)**

Procedure 5: as similarly conducted in procedure 2, $y = g(x) = g(x_1, x_2) = g_1(x_i) = g_1(\theta_s, \theta_m)$ is optimized at the intermediate optimal point $P^*(x_2) = (t^*, b^*, d^*, s^*)$ to obtain the final optima of objective function value $y^*$ in the reduced design subspace $S(x_1) = S(\theta_s, \theta_m)$ and the corresponding optimal design sub-vector $x_1^* = (\theta_s^*, \theta_m^*)$.

Procedure 6: the final optima of objective value $y^*$ determined in the Procedure 5 is treated as the approximated global optimal solution to the optimization problem in Eq. (5.3). The corresponding final optimal design vector $x^* = (x_1^*, x_2^*) = (t^*, b^*, d^*, s^*, \theta_s^*, \theta_m^*)$ is the combination of the optimal sub-vector $x_2^* = (t^*, b^*, d^*, s^*)$ determined in the procedure 4 and the optimal sub-vector $x_1^* = (\theta_s^*, \theta_m^*)$ determined in the Procedure 5.

In the Stage 1 of the inner 3-stage optimization, the sub-optimization depicted in Eq. (5.3) is conducted in the reduced subspace of ply-orientation variables $S(x_1) = S(\theta_s, \theta_m)$, while non-ply-orientation sub-vector $x_2 = (t, b, d, s)$ take constant values at each sampling point $P_i(x_2) = (t_i, b_i, d_i, s_i)$ ($i = 1, 2, \cdots, m$) in the reduced subspace of non-ply-orientation variables $S(x_2) = S(t, b, d, s)$ during the sub-optimization. In the following Stage 2, a group of metamodels is constructed to represent the relationship between the non-ply-orientation sub-vector $x_2 = (t, b, d, s)$ and the estimated value of the objective function $\hat{y}$ which has already incorporated the optimization contributions of ply-orientation variables. The most statistically accurate metamodel among the alternatives is identified and then used to locate the optima of $\hat{y}$ of the
selected metamodel and the corresponding intermediate optimal point $P'(x_2) = (t^*, b^*, d^*, s^*)$ in the subspace of non-ply-orientation variables $S(x_2) = S(t, b, d, s)$. In the Stage 3, an optimization process is conducted in the subspace of ply-orientation variables $S(x_1) = S(\theta^*, \theta^*_n)$ at the final optimal point $P'(x_2)$ to obtain the global approximation of the solution, i.e. the optima of the objective function value $y^*$ and the corresponding optimal design vector $x^* = (x_1^*, x_2^*) = (t^*, b^*, d^*, s^*, \theta^*_1, \theta^*_2)$ to the sub-optimization problem defined in Eq. (5.3).

It should be noted that the proposed inner 3-stage optimization process is different to other multi-level optimization methods [18, 20-22], because the proposed inner 3-stage approach does not introduce any intermediate variable and allow the consideration of full interactions between all design variables simultaneously and consistently for the sub-optimization in Eq. (5.3). Therefore, the final optimal design obtained from the 3-stage optimization approach could be treated as a real approximation that closes to the global optima of the sub-optimization in Eq. (5.3).

Moreover, the application of the metamodel in the inner 3-stage optimization approach is to represent the intermediate optimal results in a subspace of design variables for finding the final solutions of global optimization in the whole search space of design variables of the sub-optimization in Eq. (5.3), rather than to represent the structural modelling in normal surrogate-based optimization procedure for reducing the computational cost of structural analysis [14-18].

The key idea of the inner 3-stage optimization is to further decompose the sub-optimization in Eq. (5.3) into two more inferior sub-optimizations. One is the inferior sub-optimization in the reduced ply-orientation subspace $S(x_1) = S(\theta^*_1, \theta^*_2)$, which is solved on the basis of a structural analysis model. The other is the inferior sub-optimization in the reduced non-ply-orientation subspace $S(x_2) = S(t, b, d, s)$, which is solved on the basis of a constructed
statistical metamodel. Metamodeling techniques and structural analysis model for the inner 3-stage optimization are discussed in Sections 5.3 and 5.4, respectively. An improved multimodal PSO algorithm adopted to solve the two inferior sub-optimizations is introduced in Section 5.5.

5.3 Metamodeling techniques

One of the major tasks of conducting analyses or experiments for an unknown complex system is to obtain an output vector of system responses $y_{output}$ when an input vector of design variables $x_{input}$ is given. Generally, the functional relationship between $x_{input}$ and $y_{output}$ is implicit, and the experiments are expensive and/or time-consuming. Therefore, statistical techniques are broadly used to construct a surrogate model or metamodel for describing the input-output relationship of the experiment based on a number of suitable experiment arrangements (i.e. a serial of sample points in design space of input variables) [25]. Once the approximation model is constructed, it can be used to surrogate the original engineering analysis or physical experiment, and facilitate design space exploration, system optimization, and reliability analysis. For the inferior sub-optimization of Procedures 1, 3 and 4 in the inner 3-stage optimization, therefore, metamodel techniques are used to represent the relationship of input-output variables for optimization purpose.

Two essential issues for metamodeling techniques are (a) the design of experiments for generating the sample points in the design parameter space, and (b) the construction of a metamodel to fit the sampled input-output data [26, 27]. The quality of metamodeling technique depends on the property of sampling points and the accuracy of the surrogate model [26]. A good design of experiments allows better coverage of the design space using fewer sample points, which gain insight of the output-input relationship as much as possible. An accurate metamodel is able to provide a reliable fitting of complex non-linear relationships and the correctly predict responses at un-sampled points in
the design space of input variables. Detailed reviews of metamodeling developments refer to [25-28].

5.3.1 Design of experiments

The task of design of experiments is to select the sample points to set up a batch of experimental scenarios controlled by some input variables. Each input or “factor” of the experiment may have many discrete values or “levels” to be examined.

For real physical experiments with random measurement error, there are some common classic experimental designs, such as factorial design, central composite design, Plackett-Burman design and Box-Behnken design. These classic experimental designs tend to generate more sample points around the boundaries of the design space with a few sample points falling in the center domain of the design space of input variables [29]. For example, full-factorial design $2^k$ and $3^k$ for $k$ factors at 2 and 3 levels are the most basic experimental designs. A $2^2$ full-factorial design that contains all the vertices of the 2-factor design space is listed in Table 5.1.

<table>
<thead>
<tr>
<th>Sample points No.</th>
<th>Input 1 (Factor 1)</th>
<th>Input 2 (Factor 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 (L)</td>
<td>1 (L)</td>
</tr>
<tr>
<td>2</td>
<td>1 (L)</td>
<td>2 (H)</td>
</tr>
<tr>
<td>3</td>
<td>2 (H)</td>
<td>1 (L)</td>
</tr>
<tr>
<td>4</td>
<td>2 (H)</td>
<td>2 (H)</td>
</tr>
</tbody>
</table>

Note:
“L” and “H” represent the lowest level (lower bound) and the highest level (upper bound) of an input, respectively.

For computer-based simulation experiments with modelling system error, rather than random test error as in real physical experiments, it is advocated to
use modern “space filling” designs that evenly spread the sample points over the whole design space to capture the design behavior [25]. Moreover, “space filling” designs are recommended when the form of the metamodel cannot be specified in advance due to lack of insight into the relationship being approximated [30]. Generally, there are two categories of “space filling” designs in construction approaches [31]. Experimental designs are one of these categories constructed by combinational or algebraic methods, such as Latin hypercube design and orthogonal arrays. The other category of experimental designs is constructed by algorithmic optimization based on certain optimal criteria, such as maximum entropy designs and uniform designs.

Uniform design was proposed by Fang [32] to use small experiment runs to arrange physical experiments, in which both factors and levels for each factor are large, when the cost of an experiment run is expensive. A uniform design provides the sample points that uniformly scatter in the design space of experiment. By comparison with other experimental designs in engineering applications, Simpson et al. [26] found that uniform designs are powerful and efficient for building accurate metamodels using different sample sizes. Especially, Yang et al. [33] highlighted the significance of uniform designs in automotive crashworthiness study when only a set of relatively small sample points are available due to the expensive computational cost of simulations. Uniform designs have also been successfully applied in chemistry and chemical engineering [34]. It is shown that uniform designs exhibit many desirable features for a broad variety of applications [35].

Similar to orthogonal array designs, the notation $U_n(q^k)$ is commonly used for uniform design tables, where $n$ is the number of experiments, $q$ is the number of levels for each factor, and $k$ the number of factors. There are a number of experimental tables of uniforms designs available for selection. For instance, some uniform design tables can be obtained at the website: [http://math.hkbu.edu.hk/UniformDesign](http://math.hkbu.edu.hk/UniformDesign). A customized uniform design table for a specific application can be conveniently constructed by minimizing
uniformity measure “$L_2$-discrepancy” using optimization algorithms [31, 36]. Table 5.2 presents a uniform design $U_7(7^3)$, which can arrange 3 inputs with 7-value within 7 experimental scenarios.

Table 5.2
Uniform design table $U_7(7^3)$

<table>
<thead>
<tr>
<th>Sample points No.</th>
<th>Input 1 (Factor 1)</th>
<th>Input 2 (Factor 2)</th>
<th>Input 3 (Factor 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 (L)</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1 (L)</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>3</td>
<td>1 (L)</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>7 (H)</td>
<td>7 (H)</td>
<td>7 (H)</td>
</tr>
</tbody>
</table>

Note:
“L” and “H” represent the lowest level (lower bound) and the highest level (upper bound) of an input, respectively.

The experimental design of sampling strategy for the Procedure 1 in the inner 3-stage optimization is the combination of uniform design $U_k(q^k)$ and $2^k$ factorial design. In other words, the entire sample points for metamodel construction are composed of two sets. One set of sample points obtained by uniform design are evenly spread in the design space of input variables, and the other set of sample points arranged by 2-level factorial design cover the vertices of hypercube defined by the lower and upper bounds for each input. The usage of combinational sampling strategy is based on the following two reasons. Firstly, the optimization outputs obtained by stochastic gradient-free optimization algorithm in Procedure 2 of the inner 3-stage optimization simultaneously presents random errors as in physical experiments and systematic errors as in computer experiments. In light of this, it is necessary to spread sample points over the inside domains as well as the boundaries of the
design space. Secondly, the metamodel constructed in Procedure 3 is then optimized in Procedure 4 of the inner 3-stage optimization. It tends to enhance the confidence to ensure that the global optimum of a metamodel reliably locates at certain vertex of the design space of input variables, if this metamodel is constructed based on the sample points involving the vertices of the design space.

5.3.2 Metamodel construction

There is a variety of alternative metamodels [29] to fit the sampled input-output data acquired according to experimental designs, such as polynomial response surface [37], Kriging model [38], radius basis function (RBF) model [39-41] and artificial neural network (ANN) model [42].

RBF models are constructed by using linear combinations of radially symmetric basis functions in the form of

\[ \tilde{g}(x) = \sum_{i=1}^{N} \beta_i \phi(r_i, c) \]  

(5.5)

where \( x \) is the vector of design variables, \( \phi \) is the radial basis function, \( r_i = \|x - x_i\| \) represents the Euclidean distance between the predicted point \( x \) to the \( i \)-th sample point \( x_i \), \( c \) is a user-defined constant ranging from 0 to 1, \( N \) is the number of sample points, and \( \beta_i \) are the estimated coefficients.

Given a set of \( N \) sample points and corresponding \( N \) output responses at these sampled points, a system of \( N \) linear equations can be solved to obtain the coefficients \( \beta_i \), similar to the linear regression method for the polynomial surface response. The choices for the radial basis function \( \phi \) include [43]:

1. linear \( \phi(r, c) = r + c \)
2. cubic \( \phi(r, c) = (r + c)^3 \)
3. thin plate spline \( \phi(r, c) = r^2 \ln(cr) \)
4. Gaussian \( \phi(r, c) = \exp(-cr^2) \)
multi-quadratic $\phi(r,c) = \sqrt{r^2 + c^2}$

inverse multi-quadratic $\phi(r,c) = \frac{1}{\sqrt{r^2 + c^2}}$

RBF models interpolate the sampled response performance and have a good ability to fit arbitrary functional relationship for both stochastic and deterministic systems [43]. Moreover, it has been reported that RBF model shows better accuracy and robustness for global optimization of both large-scale and small-scale problems, in comparison with the Kriging model and the polynomial surface response [44-46].

After a metamodel is constructed, it is necessary to access the fidelity of the metamodel before the metamodel is used as a ‘surrogate’ model for engineering designs. Mitchell and Morris [47] proposed leave-one-out cross-validation of a metamodel by taking advantage of existing data to fit the metamodel in the validation process. Meckesheimer et al. [48] investigated leave-$k$-out cross-validation method and recommended $k=1$ for cross validation of RBF metamodels.

In the present study, a family of RBF metamodels based on cubic, Gaussian and multi-quadratic $\phi(r,c)$ are alternative forms for metamodel construction in Procedure 3 of the inner 3-stage optimization. Applying leave-one-out cross-validation for all alternative RBF metamodels, the RBF model having the smallest cross-validation error is selected as the “optimal” metamodel used in the optimization of Procedure 4 in the inner 3-stage optimization.

5.4 An efficient finite element model for buckling of AGS composite plates

The proposed methodology for concept design of buckling-resistant AGS composite plates has used an improved finite element (FE) modelling technique for an accurate prediction of the structural response with reasonable computational efficiency which is developed for this purpose. This is used in
the inferior sub-optimization steps of Procedures 2 and 5 of the inner 3-stage optimization process. In this FE model of AGS composite plates, the shell skin is modeled with triangular curved shell elements and the stiffeners are modeled with curved beam elements placed along the edges of the shell elements. The deformation of a beam element is completely defined in terms of the degrees of freedom of the corresponding shell element utilizing compatibility conditions between which helps to eliminate any additional degrees of freedom for the beam element. Moreover, the FE model allows for a parallel (Fig. 5.4(a)) or a perpendicular (Fig. 5.4(b)) stacking scheme of the laminated stiffeners. As the formulation adopted for deriving the beam element overestimates its torsional rigidity, a torsion correction factor is introduced to have a correct torsional stiffness of the beam element.

![Fig. 5.4. Stiffener ply stacking schemes](image)

As a typical example of the FE model, the stiffener layout and mesh division over the surface of an iso-grid stiffened panel is shown in Fig. 5.5. The longitudinal side of a panel in x-axis is divided into m divisions each of length $\Delta c = a/m$ where $a$ is the length of a panel, $m$ is the number of stiffener blocks in the length direction. The value of $m$ is supplied as a basic input to the computer program developed for this purpose. The length of a stiffener blocks in the y-axis direction may be obtained from simple geometry as $\Delta l = \Delta c \tan \alpha$ where $\alpha$ is the stiffener orientation. The number of stiffener blocks in the width direction are simply obtained as $n = b/\Delta l$ where $b$ is the width of a panel.
It should be noted that the value of $b/\Delta l$ is to be round off to the nearest whole number since $n$ will be an integer. With this value of $n$, the value of $\Delta l$ and $\alpha$ will be adjusted accordingly. A stiffener block may also be subdivided as shown in Fig. 5.5 where a mesh density of $3 \times 3$ within a stiffener block is shown. Once the elastic stiffness matrix and the geometric stiffness matrix of the whole structure (the skin and stiffeners) are derived, the eigenvalue problem for buckling analysis can be solved to obtain the critical buckling load. Refer to Chapter 2 for more details.

**Fig. 5.5.** Stiffener layout and finite element mesh division of an iso-grid stiffened composite panel
5.5 Robust and efficient multimodal PSO using random reflection boundary technique

The proposed methodology for concept design of AGS composite plates subjected to buckling loads has adopted a multimodal PSO algorithm with random reflection boundary developed for this purpose to efficiently perform multimodal maximization in the inferior sub-optimization steps of Procedures 2, 4 and 5 of the inner 3-stage optimization process described in Section 5.2.2.

The basic mechanism of PSO is initially developed in 1995 [49] and inspired by the social and cooperative behavior like bird flocking and fish schooling to locate foods. The PSO system is based on a population (swarm) of particles, which represent potential solutions. These particles explore through the search domain by updating their positions $x^i_k$ according to a specified velocity $v^i_{k-1}$ in search of the optimal solution. Each particle maintains a memory of velocity, which helps it in keeping the track of its previous best position. The previous best positions of the particles are distinguished as the personal best of each individual $p^i_{k-1}$ and the group best of the particle’s neighborhood $p^g_{k-1}$.

The change of the velocity and position of a particle in the original PSO was

$$v^i_k = v^i_{k-1} + c_1 r_1 (p^i_{k-1} - x^i_{k-1}) + c_2 r_2 (p^g_{k-1} - x^i_{k-1})$$  \hspace{1cm} (5.6)

$$x^i_k = x^i_{k-1} + v^i_k$$  \hspace{1cm} (5.7)

where the superscript $i$ denotes the particle and the subscript $k$ denotes the iteration number; $v$ denotes the velocity and $x$ denotes the position; $r_1$ and $r_2$ are uniformly distributed random numbers in the interval $[0, 1]$; $c_1$ and $c_2$ are the two positive acceleration constants controlling the relative velocity toward the personal best and the group best; $p^i_{k-1}$ is the personal best position attained by the particle $i$ in the swarm and $p^g_{k-1}$ is the group best position attained by the particle’s neighborhood so far until iteration $k-1$.

The topology of particle communication is used to determine the group best position of a particle distinguishes PSO as global version and local version. In global version, the group best position of each particle is determined by the
entire swarm of particles, whereas only a sub-swarm of a particle determines this particle’s group best position in local version PSO. In the present methodology of concept design for AGS composite panels with multimodal global optima characteristics, the ring neighborhood topology of particles is adopted to determine the group best position of a particle, because it has shown that the local version PSO based on the ring topology is more robust and efficient for multimodal optimization of composite structures, compared with global version PSO having fully-connected neighborhood topology and other local version PSO algorithms having sparsely-connected neighborhood topology such as species-based PSO (SPSO) and fitness Euclidean-distance ratio based PSO (FER-PSO). Refer to Chapter 3 for more details.

In the original PSO, it is necessary that each component of the speed of a particle should be clamped to a range \([- v_{\text{max}} \ , \ v_{\text{max}}]\). However, the value of \(v_{\text{max}}\) directly and significantly affects the performance of PSO and there is no reasonable rule of thumb for choosing the optimal value of \(v_{\text{max}}\). Motivated by reducing the influence of \(v_{\text{max}}\) for better control of particle search, Shi and Eberhart [50] proposed “inertia weight” to adjust particle velocity change, expressed as

\[
v_i^t = \omega v_i^{t-1} + c_1 r_1 (p_i^{t-1} - x_i^{t-1}) + c_2 r_2 (p^{\text{g}}_i - x_i^{t-1})
\]  

(5.8)

where \(\omega\) is the inertia weight, usually decreasing linearly from about 0.9 to 0.4 in a optimization run. Alternatively, Clerc and Kennedy [51] imposed “constriction factor” to the new velocity, expressed as

\[
v_i^d = \chi (v_i^{d,t} + c_1 r_1 (p_i^{d,t} - x_i^{t-1}) + c_2 r_2 (p^{\text{g}}_i - x_i^{t-1})))
\]  

(5.9)

\[
\chi = \frac{2}{\phi - 2 + \sqrt{\phi^2 - 4\phi}} , \quad \text{where } \phi = c_1 + c_2 > 4
\]  

(5.10)

where \(\chi\) is the constriction factor, commonly taking the value 0.7298, when \(\phi = 4.1\) and \(c_1 = c_2 = 2.05\). 

In the present methodology, the random reflection boundary technique is used for the ring topology based PSO to deal with the problem induced from the use of hard bound of velocity \(v_{\text{max}}\). The random reflection boundary technique
randomly reinitializes the position components of a particle in the half or full feasible range in those dimensions, where the particle is moving away the search space. The implement of the half-range of random reflection boundary is expressed as follows:

\[
x_k^d = \text{random}\left(\frac{x_{\min}^d + x_{\max}^d}{2}, x_{\max}^d\right), \quad \text{if } x_k^d > x_{\max}^d
\]

and

\[
x_k^d = \text{random}\left(x_{\min}^d, \frac{x_{\min}^d + x_{\max}^d}{2}\right), \quad \text{if } x_k^d < x_{\min}^d
\]

where \(\text{random}(A,B)\) denotes a random value drawn from a uniform distribution on the interval \([A,B]\). Similarly, the full-range of random reflection boundary is expressed as follows:

\[
x_k^d = \text{random}\left(x_{\min}^d, x_{\max}^d\right), \quad \text{if } x_k^d < x_{\min}^d \quad \text{or} \quad x_k^d > x_{\max}^d
\]

Table 5.3 shows the guidelines for the usage of the random reflection boundary technique and the particle speed limit \(v_{\max}\) in the ring topology based PSO. It has been shown that the random reflection boundary technique is able to highly enhance the robustness and global searching ability of PSO by simply eliminating \(v_{\max}\) or effectively reducing the sensitivity of \(v_{\max}\) for better robustness and efficiency of optimization. Refer to Chapter 4 for more details.
Table 5.3
Guidelines for the ring topology based PSO using the random reflection boundary

<table>
<thead>
<tr>
<th>Low dimension problems</th>
<th>High dimension problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia weight $h^*$</td>
<td>Inertia weight $f^*$</td>
</tr>
<tr>
<td>Constriction factor $h^*$</td>
<td>Constriction factor $f^*$</td>
</tr>
</tbody>
</table>

Note:
(1) Letter “h” and “f” represent half-range and full-range of random reflection boundary, respectively; subscript “*” refers to no usage of velocity limit $v_{\text{max}}$; subscript “-” refers to the combination with $v_{\text{max}}$.
(2) 10 and 30 dimensions are typical low and high dimensional problems, respectively.

5.6 Application of the new methodology

The proposed methodology for the concept design of AGS laminated composite panels is applied to a case study of a buckling-resistant design of AGS composite plates. This numerical example is used to demonstrate the procedures of the outer 3-step optimization process and the inner 3-stage optimization process of this novel methodology in details.

5.6.1 Problem description, sample points selection and PSO setting

For the simply supported AGS composite plate as shown in Fig. 5.6 (length $a = 1.732$ m; width $b = 1$ m; skin thickness $t = 8.128$ mm), there are 4 options for the layouts of the stiffeners which gives four different type of the stiffening-grid (ortho-grid, x-grid, bi-grid and iso-grid). The orientation of the inclined stiffeners for x-grid and bi-grid stiffening schemes is 45 degree ($\alpha \approx 45^\circ$ as in Fig. 5.5) while that is 30 degree ($\alpha = 30^\circ$) for the iso-grid stiffening arrangement. The skin and stiffeners are made of layers having
symmetric stacking sequence which can have 2-layer stacking with 5 possible discrete orientations: 0°, ±30°, ±45°, ±60° and 90°. The number of layers of the skin and each stiffener are pre-prescribed and kept unchanged, whereas the cross-section sizes and the spacing of stiffeners can change. The number of ply of the skin and each stiffener is taken as 64 and the layers of each stiffener have same stacking sequence and perpendicular orientation with respect to the skin surface (Fig. 5.4(b)) in the present case study. The graphite/epoxy composite layers used have the following properties: $E_1 = 138$ GPa, $E_2 = 10.3$ GPa, $G_{12} = G_{13} = 6.6$ GPa, $G_{23} =$2.6 GPa, $\nu_{12} = 0.21$. The uniform in-plane compressive loads are applied along the longitudinal direction (x axis) of the panels. The parametric study on the buckling capacity of these AGS composite plates with 4 stiffener blocks in the x direction (Fig. 5.6) has been conducted in Section 2.4, Chapter 2.

Fig. 5.6. A simply supported grid stiffened composite plate with different stiffener layouts
(Number of stiffener blocks in the x direction: $m = 4$)
The problem is to conduct the buckling-resistant design of AGS composite plates by investigating the performance of each feasible concept design in terms of their unit-stiffening-ratio $\mu$ as defined in Eq. (5.1). For the sake of a simple illustration of the proposed methodology, the following six design variables are accounted: the stiffener-grid type ($T$), the ratio of stiffener width to the skin thickness ($n_b$), the ratio of stiffener height to the skin thickness ($n_d$), the number of stiffener blocks in the length direction ($m$), the ply orientation of the skin ($\theta_s$) and the ply orientation of stiffeners ($\theta_d$).

For the first step of the outer 3-step optimization of this example, there is only one critical discrete design variable which is the stiffener-grid type $T$ that can have four options (i.e. ortho-grid, x-grid, bi- and iso-grid schemes). Therefore there are four sub-optimizations in the second step of the outer 3-step optimization. For the inner 3-stage optimization, the sub-vector related to ply-orientation design for the inferior sub-optimizations is $x_1=(\theta_s, \theta_d)$ and the sub-vector related to other design for the inferior sub-optimizations is $x_2=(n_b, n_d, m)$.

A 64-layer un-stiffened bare composite plate (1,732 mm $\times$ 1,000 mm $\times$ 8.128 mm) having same symmetric stacking sequence and material (graphite/epoxy laminated composite) is used as the reference to calculate the unit-stiffening-ratio $\mu$ for each stiffener-grid arrangement. For each stiffener-grid scheme, four values of each design variable of other type (i.e. $n_b$, $n_d$ and $m$) are considered in the sampling strategy of the experimental design which is listed in Table 5.4. There are 20 sample points which are selected in this case study to construct the metamodel in the sub-space of design variables $S(x_2)=S(n_b, n_d, m)$. The first 12 sampling points are arranged by uniform design $U_{12}(4^3)$ and the rest eight are arranged by $2^3$ full-factorial design as shown in Table 5.5.

According to the guidelines of applying the random reflection boundary as listed in Table 5.3, the ring topology based PSO with the constriction factor
and the full-range of random reflection boundary is used for the inferior sub-optimization for the design variables related to ply-orientations in Procedure 2 of the inner 3-stage optimization. The ring topology based PSO with inertial weight and the half-range of random reflection boundary are used for the inferior sub-optimization of other type of design variables in Procedure 4 of the inner 3-stage optimization for this case study.

Table 5.4
The discrete values of $n_b$, $n_d$ and $m$ for sample points

<table>
<thead>
<tr>
<th>Value level No.</th>
<th>$n_b$</th>
<th>$n_d$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (lower bound)</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>4 (upper bound)</td>
<td>4</td>
<td>12</td>
<td>32</td>
</tr>
</tbody>
</table>
Table 5.5
Sample points in the sub-space $S(x_2) = S(n_b, n_d, m)$ for metamodel construction

<table>
<thead>
<tr>
<th>Sample points No.</th>
<th>$n_b$</th>
<th>$n_d$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 (1)</td>
<td>6 (2)</td>
<td>32 (4)</td>
</tr>
<tr>
<td>2</td>
<td>1 (1)</td>
<td>12 (4)</td>
<td>16 (3)</td>
</tr>
<tr>
<td>3</td>
<td>1 (1)</td>
<td>6 (2)</td>
<td>8 (2)</td>
</tr>
<tr>
<td>4</td>
<td>2 (2)</td>
<td>12 (4)</td>
<td>4 (1)</td>
</tr>
<tr>
<td>5</td>
<td>2 (2)</td>
<td>6 (2)</td>
<td>32 (4)</td>
</tr>
<tr>
<td>6</td>
<td>2 (2)</td>
<td>12 (4)</td>
<td>16 (3)</td>
</tr>
<tr>
<td>7</td>
<td>3 (3)</td>
<td>3 (1)</td>
<td>8 (2)</td>
</tr>
<tr>
<td>8</td>
<td>3 (3)</td>
<td>9 (3)</td>
<td>4 (1)</td>
</tr>
<tr>
<td>9</td>
<td>3 (3)</td>
<td>3 (1)</td>
<td>32 (4)</td>
</tr>
<tr>
<td>10</td>
<td>4 (4)</td>
<td>9 (3)</td>
<td>16 (3)</td>
</tr>
<tr>
<td>11</td>
<td>4 (4)</td>
<td>3 (1)</td>
<td>8 (2)</td>
</tr>
<tr>
<td>12</td>
<td>4 (4)</td>
<td>9 (3)</td>
<td>4 (1)</td>
</tr>
<tr>
<td>13</td>
<td>1 (1)</td>
<td>3 (1)</td>
<td>4 (1)</td>
</tr>
<tr>
<td>14</td>
<td>1 (1)</td>
<td>3 (1)</td>
<td>32 (4)</td>
</tr>
<tr>
<td>15</td>
<td>1 (1)</td>
<td>12 (4)</td>
<td>4 (1)</td>
</tr>
<tr>
<td>16</td>
<td>1 (1)</td>
<td>12 (4)</td>
<td>32 (4)</td>
</tr>
<tr>
<td>17</td>
<td>4 (4)</td>
<td>3 (1)</td>
<td>4 (1)</td>
</tr>
<tr>
<td>18</td>
<td>4 (4)</td>
<td>3 (1)</td>
<td>32 (4)</td>
</tr>
<tr>
<td>19</td>
<td>4 (4)</td>
<td>12 (4)</td>
<td>4 (1)</td>
</tr>
<tr>
<td>20</td>
<td>4 (4)</td>
<td>12 (4)</td>
<td>32 (4)</td>
</tr>
</tbody>
</table>

Note:
(1) The number in the bracket represents the corresponding discrete value level of an input defined in Table 5.4.
(2) The first 12 out of 20 sample points are arranged by uniform design $U_{12}(4^3)$ and the last remaining 8 sample points are arranged by $2^3$ full factorial design.
5.6.2 Results and discussion

The buckling load of the reference bare composite plate is first optimized and used to calculate the unit-stiffening-ratio of the AGS composite plate with certain stiffener-grid arrangement. Table 5.6 has listed 14 different solutions for the optimal stacking sequence obtained from the ring topology based PSO algorithm for multimodal optimization where the best optimal solutions for the critical in-plane load are 132.87 kN/m based on a tolerance of 0.5%.

Table 5.6
Optimal stacking sequence of the reference un-stiffened plate with symmetrical 64 plies

<table>
<thead>
<tr>
<th>Design No.</th>
<th>Stacking sequence</th>
<th>Buckling load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>($\pm 45_3/\pm 30/\pm 45_9/\pm 30/\pm 45/\pm 30)_S$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>($\pm 45_3/\pm 30/\pm 45_9/\pm 30/\pm 45_2)_S$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>($\pm 45_3/\pm 30/\pm 45_{10}/\pm 30_2)_S$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>($\pm 45_3/\pm 30/\pm 45_{10}/\pm 30/\pm 45)_S$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>($\pm 45_3/\pm 30/\pm 45_{12})_S$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>($\pm 45_2/\pm 30/\pm 45_5/\pm 30/\pm 45_5/\pm 60)_S$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>($\pm 45_6/\pm 30/\pm 45_2/\pm 30/\pm 45_2/\pm 30/\pm 45/\pm 60)_S$</td>
<td>132.87 kN/m</td>
</tr>
<tr>
<td>8</td>
<td>($\pm 45_6/\pm 30/\pm 45_5/\pm 60/\pm 30/\pm 45/\pm 60)_S$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>($\pm 45_{13}/\pm 30/\pm 45/\pm 0_2)_S$</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>($\pm 45_{13}/\pm 30/\pm 45_2)_S$</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>($\pm 45_{13}/\pm 30/\pm 45//\pm 60)_S$</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>($\pm 45_{15}/\pm 30)_S$</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>($\pm 45_{16})_S$</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>($\pm 45_{15}/\pm 60)_S$</td>
<td></td>
</tr>
</tbody>
</table>

For each sample point of the sub-space of other type of design variables (non-ply-orientation design variables) defined as sub-vector $S(x_2) = S(n_b, n_d, m)$ as listed in Table 5.5, the optimal buckling load of the AGS composite plate with certain type of stiffener-grid arrangement is optimized by varying the ply
orientations of the skin and the stiffeners in the sub-space of ply-orientation related sub-vector \( S(x_i) = S(\theta_s, \theta_u) \), i.e., Max \( \mu = g_1(x_i) = g_1(\theta_s, \theta_u) \). Based on the optimal buckling loads of stiffened composite plates and the reference un-stiffened plate, the unit-stiffening-ratio \( \mu \) for every grid stiffener type at each sample point are calculated using Eq. (5.1) and shown in Table 5.7.

### Table 5.7
The unit-stiffening-ratio \( \mu \) for every stiffener-grid type at each sample point

<table>
<thead>
<tr>
<th>Sampling No.</th>
<th>Ortho-grid</th>
<th>X-grid</th>
<th>Bi-grid</th>
<th>Iso-grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.0</td>
<td>38.9</td>
<td>35.4</td>
<td>36.3</td>
</tr>
<tr>
<td>2</td>
<td>68.9</td>
<td>22.9</td>
<td>91.7</td>
<td>89.5</td>
</tr>
<tr>
<td>3</td>
<td>19.6</td>
<td>9.0</td>
<td>30.0</td>
<td>20.9</td>
</tr>
<tr>
<td>4</td>
<td>2.9</td>
<td>2.1</td>
<td>4.8</td>
<td>4.7</td>
</tr>
<tr>
<td>5</td>
<td>29.3</td>
<td>35.8</td>
<td>31.8</td>
<td>33.1</td>
</tr>
<tr>
<td>6</td>
<td>42.9</td>
<td>14.9</td>
<td>88.6</td>
<td>79.5</td>
</tr>
<tr>
<td>7</td>
<td>8.8</td>
<td>7.9</td>
<td>11.9</td>
<td>11.4</td>
</tr>
<tr>
<td>8</td>
<td>2.8</td>
<td>2.0</td>
<td>4.5</td>
<td>4.3</td>
</tr>
<tr>
<td>9</td>
<td>10.1</td>
<td>13.9</td>
<td>10.8</td>
<td>11.3</td>
</tr>
<tr>
<td>10</td>
<td>31.1</td>
<td>11.0</td>
<td>56.9</td>
<td>55.0</td>
</tr>
<tr>
<td>11</td>
<td>9.6</td>
<td>7.1</td>
<td>12.1</td>
<td>11.7</td>
</tr>
<tr>
<td>12</td>
<td>2.5</td>
<td>1.8</td>
<td>3.8</td>
<td>3.6</td>
</tr>
<tr>
<td>13</td>
<td>3.0</td>
<td>2.6</td>
<td>4.9</td>
<td>5.0</td>
</tr>
<tr>
<td>14</td>
<td>9.4</td>
<td>12.5</td>
<td>12.4</td>
<td>12.7</td>
</tr>
<tr>
<td>15</td>
<td>3.5</td>
<td>2.6</td>
<td>6.5</td>
<td>6.3</td>
</tr>
<tr>
<td>16</td>
<td>63.7</td>
<td>35.7</td>
<td>88.5</td>
<td>88.8</td>
</tr>
<tr>
<td>17</td>
<td>3.5</td>
<td>2.4</td>
<td>6.5</td>
<td>6.1</td>
</tr>
<tr>
<td>18</td>
<td>9.6</td>
<td>13.5</td>
<td>10.0</td>
<td>10.5</td>
</tr>
<tr>
<td>19</td>
<td>2.1</td>
<td>1.5</td>
<td>3.1</td>
<td>3.0</td>
</tr>
<tr>
<td>20</td>
<td>19.2</td>
<td>11.8</td>
<td>72.7</td>
<td>34.5</td>
</tr>
</tbody>
</table>
Using the data listed in Tables 5.5 and 5.7, a group of RBF metamodels based on cubic Gaussian and multi-quadratic basis functions \( \hat{\mu} = g_2(x_2) = g_2(n_b, n_d, m) \) are constructed where the input vector is a sub-vector \( x_2 = (n_b, n_d, m) \) related to non-ply-orientation design variables and the output is the estimated unit-stiffening-ratio \( \hat{\mu} \). In terms of the selection of the “optimal” metamodel based on leave-one-out cross-validation for all alternative RBF metamodels, it is observed from Table 5.8 that the cubic RBF model has the smallest cross-validation error for the cases of x-grid and bi-grid stiffener arrangements and the multi-quadratic RBF model is the best for the cases of ortho-grid and iso-grid stiffener arrangements.

After the optimal metamodel result is obtained, the optimization on this selected metamodel is conducted to find the estimated best unit-stiffening-ratio \( \hat{\mu} \) at the corresponding optimal non-ply-orientation sub-vector \( x_2^* = (n_b^*, n_d^*, m^*) \) for each stiffener-grid type in the subspace of \( S(x_2) = S(n_b, n_d, m) \). For estimation error analysis, the actual (or un-estimated) optimal unit-stiffening-ratio \( \mu^* \) at \( x_2 = (n_b, n_d, m) \) is obtained by varying the ply orientations of the skin and the stiffeners in the subspace \( S(x_1) = S(\theta_s, \theta_{st}) \), which is based on FE model analysis rather than RBF metamodel approximation. As shown in Table 5.8, the relative difference between the estimated optimal unit-stiffening-ratio \( \hat{\mu} \) and the real optimal unit-stiffening-ratio \( \mu^* \) at \( x_2^* = (n_b^*, n_d^*, m^*) \) varied from -5.7% to -22.0%, and this shows that the accuracy of the RBF metamodel on optimization is satisfactory. Moreover, the real optimal unit-stiffening-ratio \( \mu^* \) for \( x_2^* = (n_b^*, n_d^*, m^*) \) is 21.3% to 82.0% greater than the estimated maximum unit-stiffening-ratio \( \hat{\mu} \) obtained from optimization on the RBF metamodels, as listed in Table 5.8. It helps to further confirm that the real unit-stiffening-ratio \( \mu^* \) is an optimal one, which is based on coupling the inferior sub-optimizations between the subspace of non-ply-orientation sub-vector \( S(x_2) = S(n_b, n_d, m) \) at the Stage 2 and the subspace of ply-orientation sub-
vector \( S(x_i) = S(\theta_s, \theta_{st}) \) at the Stages 1 and 3 in the inner 3-stage optimization.

**Table 5.8**

Optimization results and error analysis on the RBF metamodels

<table>
<thead>
<tr>
<th>Stiffener-grid type</th>
<th>Optimal RBF metamodel</th>
<th>( \mu^*_s )</th>
<th>( n^*_b )</th>
<th>( n^*_d )</th>
<th>( m^* )</th>
<th>( \hat{\mu}^* )</th>
<th>( \mu^* )</th>
<th>( \frac{\hat{\mu}^* - \mu^<em>}{\mu^</em>} )</th>
<th>( \frac{\mu^* - \mu^<em>_s}{\mu^</em>} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ortho-grid</td>
<td>multi-quadratic</td>
<td>68.9</td>
<td>1.0</td>
<td>12.0</td>
<td>22</td>
<td>78.6</td>
<td>100.7</td>
<td>-22.0</td>
<td>46.2</td>
</tr>
<tr>
<td>X-grid</td>
<td>cubic</td>
<td>38.9</td>
<td>1.0</td>
<td>8.4</td>
<td>27</td>
<td>62.8</td>
<td>70.8</td>
<td>-11.2</td>
<td>82.0</td>
</tr>
<tr>
<td>Bi-grid</td>
<td>cubic</td>
<td>91.7</td>
<td>1.0</td>
<td>12.0</td>
<td>24</td>
<td>104.9</td>
<td>111.2</td>
<td>-5.7</td>
<td>21.3</td>
</tr>
<tr>
<td>Iso-grid</td>
<td>multi-quadratic</td>
<td>89.5</td>
<td>1.0</td>
<td>12.0</td>
<td>23</td>
<td>104.4</td>
<td>115.9</td>
<td>-10.0</td>
<td>34.9</td>
</tr>
</tbody>
</table>

As summarized in Table 5.9 for the alternative stiffener grid types at initial design phrase, the optimal unit-stiffening-ratio \( \mu^*_s \) and the corresponding optimal design variables \( x^* = (x^*_2, x^*_i) = (n^*_b, n^*_d, m^*, \theta^*_s, \theta^*_{st}) \) identified by the Procedure 6 of the inner 3-stage optimization are regarded as the approximation of the global optima in the entire design space \( S(x) = S(n_b, n_d, m, \theta_s, \theta_{st}) \). By using ring topology based PSO algorithm for multimodal optimization, it should be noted that the optimal \( \theta^*_s \) and \( \theta^*_{st} \) are multimodal solution pairs for iso-grid and bi-grid stiffener arrangements, as shown in Table 5.9, if the acceptable difference tolerance for optimal unit-stiffening-ratio is 0.5%. The AGS composite plate with iso-grid stiffener scheme has the best of the optimal unit-stiffening-ratio 115.9 while the AGS composite plate with x-grid stiffener scheme has the worst of the optimal unit-
stiffening-ratio 70.8. Between the best and the worst, those AGS composite plate with bi-grid and ortho-grid stiffener scheme have the optimal unit-stiffening-ratio 111.2 and 100.7, respectively. Therefore, the recommendations of the stiffener-grid type for the refined optimization of construction design could be the iso-grid, bi-grid and ortho-grid type in the descending order in this case study, if the optimal unit-stiffening-ratio of the AGS composite plate is required to be greater than 100.

Table 5.9
The summary of selection rank for the optional stiffener-grid types and the corresponding design variables in terms of unit-stiffening-ratio

<table>
<thead>
<tr>
<th>Rank</th>
<th>Stiffener-grid type</th>
<th>$\mu^*$</th>
<th>$n_b^*$</th>
<th>$n_d^*$</th>
<th>$m^*$</th>
<th>$\theta_s^*$</th>
<th>$\theta_{st}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Iso-grid</td>
<td>115.9</td>
<td>1.0</td>
<td>12.0</td>
<td>23</td>
<td>(0_2/±30/±60/±30/±45/±30</td>
<td>(0_2/±30/±45/0_2/±30/0_4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>/0_2/±60/±45/±60/±45/±60</td>
<td>/90_2/±60/±30/0_2/±60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>/±30/0_2)S</td>
<td>/±30/0_2/±30)S</td>
</tr>
<tr>
<td>2</td>
<td>Bi-grid</td>
<td>111.2</td>
<td>1.0</td>
<td>12.0</td>
<td>24</td>
<td>(±45/±60/±30/±45/0_2/±45_2</td>
<td>(±45/±30/0_2/±30/0_2/±60/±45</td>
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<td>/60/90_2/±30/±45/±60/±30</td>
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<td>/90_2/0_2/±30/±60/0_2)S</td>
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<tr>
<td>3</td>
<td>Ortho-grid</td>
<td>100.7</td>
<td>1.0</td>
<td>12.0</td>
<td>22</td>
<td>(±30/±60/±30/±45/0_2/±45_2</td>
<td>(±45/±30/0_2/±30/0_2/±60/±45</td>
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<tr>
<td>4</td>
<td>X-grid</td>
<td>70.8</td>
<td>1.0</td>
<td>8.4</td>
<td>27</td>
<td>(±45/±30/±45/0_2/±60/±45</td>
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<td>/0_2/±30/±45/0_2/±60/0_2)S</td>
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It is observed that the optimal \( n^*_b \) has the lower bound of \( n_b \) for each optimal design of stiffener grid configuration, and this means that increasing the widths of stiffeners is not helpful to get better unit-stiffening-ratio. Additionally, the RBF metamodel can provide detail insight into understanding the influence of \( x_2 = (n_b, n_d, m) \) on the unit-stiffening-ratio \( \mu \).

Figs. 5.7-5.10 present the relationship between \( \mu \) and design vector \( x_2 = (n_b, n_d, m) \) at the point \( x_2^* = (n^*_b, n^*_d, m^*) \) for ortho-grid, x-grid, bi-grid and iso-grid type, respectively. Using bi-grid type as an example, it is shown that the stiffer width ratio \( n_b \) within the design space has apparently lower influence to the unit-stiffening-ratio than stiffer height ratio \( n_d \) (see Fig. 5.9(a)). It is also shown that the unit-stiffening-ratio first increases until a peak and then decreases with the increase of stiffener block number in x-axis \( m \) (or the decrease of stiffener spacing) (see Figs. 5.9(b) and 5.9(c)).
(a) $\mu$ versus $n_p$ and $n_d$ when $m = m^* = 22$

(b) $\mu$ versus $n_b$ and $m$ when $n_d = n_d^* = 12.0$

(c) $\mu$ versus $n_b$ and $m$ when $n_b = n_b^* = 1.0$

Fig. 5.7. Ortho-grid type: $\mu$ versus design vector $x_2 = (n_b, n_d, m)$

based on RBF metamodel
Chapter 5

![Graphs showing (a) $\mu$ versus $n_b$ and $n_d$ when $m = m^* = 27$, (b) $\mu$ versus $n_b$ and $m$ when $n_d = n_d^* = 8.4$, and (c) $\mu$ versus $n_d$ and $m$ when $n_b = n_b^* = 1.0$.]

Fig. 5.8. X-grid type: $\mu$ versus design vector $x_2 = (n_b, n_d, m)$

based on RBF metamodel
Fig. 5.9. Bi-grid type: $\mu$ versus design vector $x = (n_b, n_d, m)$ based on RBF metamodel.
Fig. 5.10. Iso-grid type: $\mu$ versus design vector $x_2 = (n_b, n_d, m)$ based on RBF metamodel
5.7 Conclusions

A novel multi-level methodology consists of an inner 3-stage optimization process nested within an outer 3-step optimization process is proposed for the optimal design of AGS composite panels at their concept design phase. The proposed methodology decomposed the complex optimization problem into sub-optimizations and inferior sub-optimizations in order to have reduced subspaces of design variables. This helps to get an effective and efficient approach to identify the global optimum of AGS composite panels for the further refinement during detailed construction design phase in practical applications.

For the first step of the outer 3-step optimization process, the critical discrete design variables, which significantly determine the mechanical performance of AGS composite panels, are identified to decompose the design variable space. In the second step of the outer 3-step optimization, the sub-optimizations are then done in the reduced design variable space which excludes the critical discrete design variables according to the combination of the critical discrete design variables. The optimal results of these sub-optimizations are compared in the third step of the outer 3-step optimization to find out the most appropriate preliminary configurations for topology, geometry and material orientation of AGS composite panels for the detailed refinement.

Each sub-optimization process without varying the critical discrete design variables of an AGS composite panel in the second step of the outer 3-step optimization are solved by the inner 3-stage optimization process. At the first stage of the inner 3-stage optimization, the sub-optimizations in the outer 3-step optimization are conducted in the reduced subspace of the ply-orientation design variables at the sample points evenly scattered in the reduced subspace of the other type of design variables. Based on the outcomes of the first stage of the inner 3-stage optimization, a metamodel is constructed to represent the intermediate optimization results which incorporated the optimal effect of the ply-orientation design variables in the subspace of the other type of design.
variables. This constructed metamodel is then validated and optimized to determine the optimal results for the non-ply-orientation design variables. At the third stage of the inner 3-stage optimization, the global optimal design of AGS composite panel with certain values of the critical discrete design variables is found according to the optimal results for the non-ply-orientation design variables obtained in the second stage of the inner 3-stage optimization process. The optimal design found by the inner 3-stage optimization is the global approximation of the optimal design of the sub-optimization in the outer 3-step optimization due to the fact that the full interactions of all design variables in the inferior sub-optimization are considered conveniently by means of a metamodel to represent the intermediate optimization effects.

A case study has been presented to show the application of the developed methodology for an AGS composite plate having ortho-grid, x-grid, bi-grid and iso-grid schemes in the preliminary buckling-resistant design. The ring topology based PSO using the random reflection boundary for efficient multimodal optimization and the FE tool for a computationally efficient buckling analysis of AGS composite plates are incorporated to study this application problem. Following the proposed methodology, the sub-optimizations in the outer 3-step optimization process are used to identify the discrete design variable of stiffener-grid type, and the optimal design of the AGS composite plate for certain stiffener-grid scheme is obtained through the inner 3-stage optimization. In this case, the iso-grid scheme is found to be the most attractive stiffener-grid type and the optimal width of stiffener for each stiffener grid scheme has the lower bound of the feasible stiffener width range in terms of the unit-stiffening-ratio. The effects of other type of design variables (non-ply-orientation design variables) on the unit-stiffening-ratio are presented and discussed based on the constructed metamodel.

The results has shown that the proposed methodology is suitable for the buckling-resistant design of AGS composite panels, because the outer 3-step optimization process can deal with a large number of critical discrete design variables simultaneously for complex optimal design problems. In addition the inner 3-stage optimization process using the metamodeling techniques has
strongly enhanced the efficiency and effectiveness in solving the sub-optimizations in the outer 3-step optimization process. It should be noted that the frameworks of the proposed methodology using multi-level optimization approach are suitable for the implementation in a parallel computation platform to increase the computation efficiency further. The basic thoughts of the proposed methodology of the outer 3-step optimization scheme nested with the inner 3-stage optimization scheme are flexible and suitable for extension to a general optimization design of engineering problems having various system configurations, many design variables and complex system responses.
References


Chapter 6

Conclusions

Since the Second World War fiber reinforced polymer (FRP) composites have become more attractive as a structural material in a variety of engineering practices, such as infrastructure construction, automobile industry and aerospace engineering, due to its high efficiency of material utility and broad freedom of structural optimization design in comparison with conventional materials, such as metals. Stiffened composite panel structures are used in many applications, such as bridge deck, car body and aircraft wing. Composite stiffeners are used to enhance the bending stiffness of thin-walled panels and increase their resistant to prevent buckling without much material weight addition. Recently, with the development of manufacturing techniques, advanced grid stiffened (AGS) composite panels have increasingly emerged and gained more attention as these grid-stiffening configurations help to enhance the structural efficiency in a more effective way in complex loading conditions compared with conventional unidirectionally-stiffened composite panels. These grid-stiffening configurations provide more available options to select outstanding concept designs of AGS composite panels against buckling for the final appropriate design development at the final construction stage. However, the concept design of AGS composite panels against buckling has to simultaneously deal with a great number of the continuous-discrete mixed design variables, such as pattern of stiffening-grid (topology design variables) and size and spacing of stiffeners (geometry design variables) as well as thickness and orientations of skin and stiffener layers (material design variables). Moreover, the optimization design of AGS composite structures is usually a typical multimodal optimization problem, in which there exist multiple global optimal solutions with identical/close optima of structural
Chapter 6

performance. In addition, if accurate finite element (FE) model and global multimodal optimization algorithm are used, this conceptual optimization design problem for AGS composite panels subjected to buckling loads require greatly expensive even unaffordable computation cost for practical applications, due to model, analysis and optimization complexity in this structural topology optimization. Therefore, it is necessary and urgent to develop an effective and efficient methodology to systematically solve this significant and challenging optimization problem.

The specific contributions of this research are summarised in Section 6.1. The limitations of this research and the recommendations for future work are also presented in Section 6.2.

6.1 Research contributions

This research focuses on establishing a practical methodology for concept design of advanced grid stiffened composite panels against buckling. This effective and efficient methodology of multi-level optimization is based on an inner 3-stage optimization process nested within an outer 3-step optimization process as well as metamodeling techniques. Moreover, the performance of this multi-level optimization methodology can be further consolidated for better efficiency by incorporating the improved structural modelling component for buckling analysis and the developed particle swarm optimization (PSO) techniques for multimodal optimization. The main contributions of the present study are concluded as follows:

1. An improved FE model has been proposed to conduct fast and accurate prediction of buckling capacity of AGS composite plates. There are several features for this model. First, 3D degenerated beam elements developed to model composite stiffeners by corresponding modifications and improvements have completely compatible degrees of freedom of 3D degenerated shell elements adopted to represent bare skins. Second, putting the stiffener elements along the edge of the shell elements makes no require of increasing additional nodes for the
stiffener elements, which benefits fast finite element modelling and analysis. Third, using the developed beam elements allows for parallel and vertical stacking schemes of composite stiffeners and convenient introduction of correction factor of stiffener torsion rigidity. Base on this improved model, a buckling analysis program for AGS composite plates has been coded in FORTRAN and this program has been validated by a number of test examples including ortho-grid, x-grid, bi-grid and iso-grid stiffened composite plates. It indicates that the modified FE model can efficiently and correctly cover a wild range of stiffened composite plates for buckling analysis. Therefore, with sufficient accuracy and reliable confidence, this proposed FE model is suitable for the concept design of AGS composite plates against buckling where efficient structural analysis is required for global optimization using modern heuristic algorithms.

2. The application of multimodal optimization of PSO using niching techniques has been explored in order to extend conventional unimodal optimization to challenging multimodal optimization of composite structures, because optimal design of composite structures is a typical multimodal optimization problem. A niching PSO algorithm with niching parameters (i.e. the species-based PSO) and two niching PSO algorithms without niching parameters (i.e. the fitness Euclidean-distance ratio based PSO and the ring topology based PSO) have been investigated for a maximum buckling capacity design problem of a simply supported composite laminate which has been well studied before. For this maximum buckling problem, it has been reported in literature that the maximum critical buckling load factor of the first-best-fitness designs is 3973.014, and that there are 7 global optima. Fortunately, 14 global sub-optimal solutions with second-best-fitness buckling load factor of 3972.996 have been simultaneously discovered in a single optimization process for the first time by all of the 3 niching PSO algorithms under investigation, when the numbers of swarm particles and evolution generations are sufficient large. Even though the relative difference of maximum
critical buckling load factor between the first-best-fitness solutions and
the second-best-fitness designs is $4.53 \times 10^{-6}$, 11 of 14 second-best-
fitness designs are independent to 7 first-best-fitness designs, because
they are mathematically far away from the neighborhood of any first-
best-fitness design in the search space of design variables. This
optimal buckling-resistant design problem with 7 first-best-fitness
solutions and 14 newly-discovered second-best-fitness solutions can
become a benchmark used to test the performance of multimodal/unimodal optimization algorithms for optimization of
composite structures. The performances of the 3 niching PSO
algorithms to find the first-best-fitness and second-best-fitness
solutions in a single run have been statistically compared in terms of
swarm particle numbers and evolution generations. It indicates that the
performance of the species-based PSO strongly relies on the value of
the niching radius parameter. It also indicated that the ring topology
PSO algorithm without requiring any additional user-selected niching
parameter has reliable performance. Thus the ring topology PSO
algorithms including r3psos and r2psos are recommended for solving
multimodal optimization design of composite structures, especially for
the engineers with no/limited experiences of structural optimization.

3. The new technique of random reflection boundary has been proposed
to replace the method of fixed absorption boundary in conventional
PSO in order to eliminate/reduce the influence of particles’ velocity
limit $v_{\text{max}}$ for better algorithmic performance, because the parameter
particles’ velocity limit $v_{\text{max}}$ is indispensable and sensitive but
empirically user-predefined for conventional PSO algorithms. Five
unimodal and five multimodal testing functions for optimization in
both 10-dimension and 30-dimension search space are used to test the
effectiveness of half-range and full-range random reflection boundary
for various PSO variants having different PSO topology (global and
local version) and PSO convergence control approach (inertia weight
method and constriction factor method). It has shown that PSO
algorithms using the proposed random reflection boundary have better solution accuracy and success reliability in a single run than those using the traditional fixed absorption boundary according to the statistical analysis of these numerical experiments on the benchmark functions. The empirical guidelines for effective application of the half-range and full-range random reflection boundary technique have been also established on the basis of these numerical experiments of the testing functions. The empirical guidelines for the random reflection boundary technique have been further applied in a practical engineering problem on optimal buckling design of a composite panel. In this practical engineering problem, it has demonstrated that PSO using the random reflection boundary technique make it possible to eliminate the sensitive parameter of particles’ velocity limit $v_{\text{max}}$ and achieve satisfactory reliability and accuracy of optimization performance. Therefore, by successfully eliminating/reducing the influence of particles’ velocity limit $v_{\text{max}}$, the random reflection boundary technique is able to strongly enhance robustness and applicability of PSO for various complicated real-world practices such as optimal design of composite structures against buckling.

4. A novel multi-level methodology based on an inner 3-stage optimization process nested within an outer 3-step optimization process has been established in order to effectively and efficiently conduct concept design of AGS composite panels against buckling. According to the outer 3-step optimization process, the whole optimization problem for concept design is first decomposed into limited number of sub-optimization problems in terms of the identified critical discrete design variables. Then each of these sub-optimization problems with reduced searching space is individually solved by the inner 3-stage optimization process. Finally, the optimal design solutions obtained from each sub-optimization are compared to identify the best solution of the concept design of AGS composite panels against buckling. In the inner 3-stage optimization process, the
design variables excluding the identified critical discrete design variables of the outer 3-step optimization process are divided into the ply-orientation variable group and the other (non-ply-orientation) variable group. Thus each sub-optimization of the outer 3-step optimization process can be further decomposed into two inferior sub-optimization problems with more reduced searching space and each inferior sub-optimization problem is only relevant to the ply-orientation design variables or the other design variables. Metamodelling techniques based on designs of experiments (i.e. uniform experimental design and full-factorial experimental design), RBF metamodels and metamodel cross-validation method are adopted to incorporate and represent the intermediate optimization effects of the ply-orientation design variables in the inferior sub-optimization stage just related to the other (non-ply-orientation) design variables. Thus the inner 3-stage optimization process incorporating with metamodel techniques permits to “simultaneously” deal with large design variables for optimization of AGS composite panels, by taking full interaction and coupling between the ply-orientation and the other (non-ply-orientation) design variables in an effective and efficient way. In this multi-level methodology, the improved FE model (mentioned in the above contribution 1) can be incorporated to carry on fast buckling analysis of AGS composite plates, and the ring topology PSO algorithms (mentioned in the above contribution 2) that use the random reflection boundary technique (mentioned in the above contribution 3) can serve as the global multimodal optimization algorithm to solve each inferior sub-optimization problem in the inner 3-stage optimization process. The proposed methodology has been applied to an AGS composite plate having ortho-grid, x-grid, bi-grid and iso-grid stiffener pattern options, and the outstanding concept designs have been successfully identified by maximizing the defined unit-stiffening-efficiency ratio. In this application problem, it has also found that the iso-grid scheme is the most attractive stiffener grid configuration, and that the optimal width of stiffener for each stiffening-grid scheme has the lower bound of the feasible stiffener
width range. More importantly, the essential thoughts of the novel multi-level optimization strategy and the 3-stage optimization process nested in the 3-step optimization process can be generally and conveniently extended to other optimal design of engineering systems where a large number of design variables and complex systematic responses are involved.

6.2 Research limitations and future work

It should be noted that there are some limitations in the current research and these limitations identify further research directions and are addressed as below.

1. The present research is limited to the structure type of AGS composite plates, however, stiffened composite shells or curved panels are similar structure types widely used in engineering applications. Therefore, it is clear that the improved finite element model and the proposed multi-level methodology for design optimization should be extended to the concept design of AGS composite shell structures subjected to buckling loads.

2. The present research only considers buckling resistance in the concept design of AGS composite panels, however, strength capacity, first ply failure and vibration characteristic are other critical design requirements for AGS composite panels for real applications. So it is necessary to take into account more engineering demands other than buckling requirement for the optimization of AGS composite panels. These additional engineering demands may serve as either constraint conditions or independent objectives in the design optimization problem of AGS composite panels. In other words, the present research for concept design of AGS composite panels should be extended to constrained optimization problem from unconstrained optimization and to multi-objective optimization problem from single-objective optimization.
3. The uncertainty of design variables of AGS composite panels is not considered in present research, however, the material manufacturing deviations, construction errors of design variables and randomness of applied loads are not evitable in practical engineering applications. Therefore, robust design and reliability based optimization design are very important for practical design of AGS composite panels in the future work.