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Network-Based Event-Triggered Control for Singular Systems With Quantizations

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***Abstract*—This paper investigates the problem of event-triggered H_∞ control for a networked singular system with both state and input subject to quantizations. First, a discrete event-triggered scheme, which activates only at each sampling instance, is presented. Next, two new sector bound conditions of quantizers are proposed to provide a more intuitive stability analysis and controller design. Then, network conditions, quantizations, and the event-triggered scheme are modeled as a time-delay system. With this model, the criteria are derived for H_∞ performance analysis, and codesigning methods are developed for the event trigger and the quantized state feedback controller. An inverted pendulum controlled through the network is given to demonstrate the effectiveness and potential of the new design techniques.**

***Index Terms*—Event-triggered control, networked singular system, quantization, sector bound condition.**

I. Introduction

EVENT-TRIGGERED schemes, where the sampled signal is transmitted according to an event-triggered condition other than a fixed time interval, have received increasing attention due to its capacity for reducing communication load. Many results have been reported on the problem of event-triggered control or event-based control, such as [1]–[4] and the reference therein. Among them are two types of event-triggered scheme: one with a continuous event-triggered condition [1], [2], and the other with is a discrete event-triggered condition [3], [4]. The continuous event trigger relies on additional hardware to continuously supervise the system state to detect whether the current state exceeds a trigger threshold. Moreover, the continuous event-triggered scheme can only be effective under a given controller, and the controller and the triggered parameters cannot easily be codesigned. In the discrete event-triggered scheme, the triggered condition is detected in discrete sampled instants, and incorporating a codesign algorithm is readily achievable for most practical systems.

In networked control systems (NCSs), the sharing of limited network bandwidth often causes network-induced delays, and data packet dropouts and disorder, which can deteriorate the performance and even destabilize the systems [5]–[11]. In the past decade, many methods have been developed to deal with these network-induced challenging issues, for example, the filtering, identification, and estimation problem in [12]–[15] and the output feedback problem in [16]–[18]. However, most are based on a time-triggered scheme, which can be inefficient in terms of reducing the utilization of limited network bandwidth.

Furthermore, quantization problems inherent in sampled-data systems have been investigated in recent years [19]–[24]. It was shown in [25] that the coarsest quantizer is logarithmic, and the sector bound method is applicable for stabilizing linear single-input–single-output systems with state quantization. The sector bound method in [25] was extended to multiple-input–multiple-output systems in [26] and to guaranteed cost control of continuous systems over networks with state and input quantizations in [27]. In addition, the networked H_∞ control for continuous-time linear systems with state quantization was discussed in [28], and the problem of H_∞ estimation was studied in [29]. The reset quantized state control problem was studied in [30] and [31]. Meanwhile, singular systems are frequently encountered in electronic and economic systems, aerospace, and chemical industries [32]–[36]. Hence, there will be a profound meaning applying quantized control to singular systems. Indeed, the problem of a networked H_∞ filter for singular systems with state quantization was investigated in [6] by the similar method used in [29]. However, when using the sector bound method, the quantization errors have been regarded as a class of uncertainties, which present difficulties in controller design. To the best of the authors' knowledge, although discrete event-triggered control for linear systems has been discussed in [3] and [4], there is no result reported on event-triggered control for networked singular systems that are subject to quantizations. This motivates the research presented in this paper.

The works most pertinent to this paper are [37] and [38]. In fact, this paper stems from the following motivations. First, the quantized control under event-triggered networked systems investigated in [37] is novel but only for regular systems. On

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the other hand, the new sector bound approach used in [38] is under a time-triggered scheme, which has its useful properties, but may lead to the unnecessary usage of limited communication resources. Our aim here is to find a more effective and efficient discrete event-triggered scheme, which only detects the difference between the states sampled in discrete instants regardless of what happens in between updates, and to codesign the event-triggered H_∞ controller for networked singular systems taking into account both communication delays and signal quantizations.

In this paper, the problem of event-trigged H_∞ control for networked singular systems with both state and control input quantizations is investigated. Our contributions are as follows: 1) A new sector bound approach, by which no transformation is needed from system models to uncertain systems, is presented; 2) a discrete event-triggered scheme that only needs supervision of the system state in discrete instants is presented for networked singular systems; and 3) a unified framework, which takes network-induced delays, state and input quantizations, and event triggers into account, is given for codesigning the event detector and the state feedback controller.

The remainder of this paper is organized as follows. Section II formulates the problem. H_∞ performance analysis and quantized state feedback controller design are presented in Section III. Illustrative examples are given in Section IV to demonstrate the effectiveness of the presented method. Finally, this paper is concluded in Section V.

Notations: Throughout this paper, the superscripts "T" and "-1" stand for the transpose of a matrix and the inverse of a matrix; $\mathbb{R}^n$ denotes n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all real matrices with n rows and m columns; $P > 0$ means that P is positive definite; I is the identity matrix with appropriate dimensions; the space of square-integrable vector functions over $[0, \infty)$ is denoted by $\mathcal{L}_2[0, \infty)$, and for $w(t) \in \mathcal{L}_2[0, \infty)$, its norm is given by $\|w(t)\|_2 = \sqrt{\int_0^\infty |w(t)|^2 dt}$; for a symmetric matrix, $*$ denotes the matrix entries implied by symmetry.

II. Problem Formulation

A. Plant Description

The networked singular system, as shown in Fig. 1, comprises a continuous-time-controlled singular system, a set of sensors to provide the state signals, an event detector, two quantizers $f(\cdot)$ and $g(\cdot)$, a zero-order hold (ZOH), actuators, and a data network.

The networked singular system is described as follows:

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \\ z(t) = Cx(t) + Du(t) + Fw(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $w(t) \in \mathbb{R}^p$ is the disturbance input, and $z(t) \in \mathbb{R}^q$ is the controlled output of the plant. The matrices A, B, C, D, E, F, and G are constant matrices with appropriate dimensions, where E may be singular, and we assume that rank $E = r \le n$. For the networked singular system shown in Fig. 1, the following conditions are assumed in this paper.

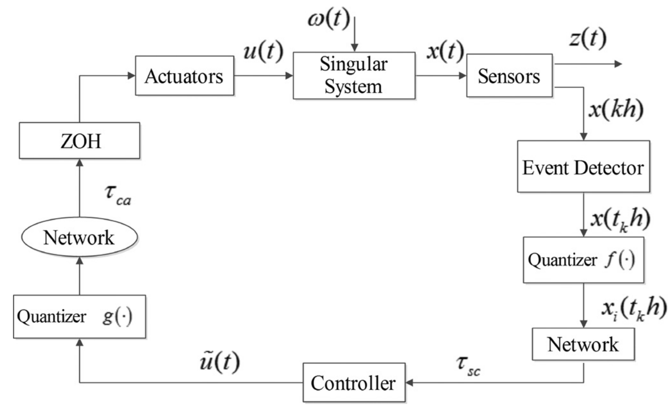


Fig. 1. Block diagram of an event-triggered controlled singular system.

1) The sensors are time triggered with a constant sampling period h. The sampled $x(kh)$ is transmitted to the event detector and is transmitted (or released) at instant $t_k h$ by the event detector, which is located between the sensors and the controller. All state variables of the singular NCS are measurable.
2) The signal in the network is transmitted with a single packet, and the data packet loss does not occur during transmission.

B. Event-Triggered Scheme

To reduce the utilization of the limited network bandwidth, a discrete event-triggered scheme is proposed in this paper to replace the conventional time-triggered mechanism [3], [4]. The event detector uses the following condition to decide on whether the current signal should be transmitted to the controller:

$$t_{k+1}h = t_k h + \min_l \{lh | e^T(i_k h)\Phi e(i_k h) \ge \sigma x^T(t_k h)\Phi x(t_k h)\} \tag{2}$$

where $0 \le \sigma < 1$ is a given scalar parameter, $\Phi > 0$ is a positive matrix to be determined, and $e(i_k h)$ is the error between the two states at the latest transmitted sampling instant and the current sampling instant, i.e., $e(i_k h) = x(t_k h) - x(i_k h)$, where $i_k h = t_k h + lh, l \in \mathbb{N}$.

When the data released at t_k by the event monitor are transmitted to the controller, it incurs a communication delay called the sensor-to-controller delay $\tau_{sc}(t_k)$. Similarly, the controller forwarding the actuation signals at t_k to the actuator incurs another communication delay called the controller-to-actuator delay $\tau_{ca}(t_k)$. These two network-induced delays can be lumped together as the time-varying delay τ_{t_k}, and

$$\tau_{t_k} = \tau_{sc}(t_k) + \tau_{ca}(t_k), 0 \le \tau_m \le \tau_{t_k} \le \tau_M \tag{3}$$

where τ_m and τ_M denote the lower and upper delay bounds, respectively.

C. Event-Triggered Quantized H_∞ Control Problem

The problem of event-triggered H_∞ control with quantizations to be addressed in this paper is to design a state feedback controller, i.e.,

$$u(t) = Kx(t) \tag{4}$$

where K is the controller gain, such that:

1) the resultant closed-loop system with $w(t) = 0$ is regular, impulse free, and stable; and
2) under zero initial conditions, for any nonzero $w(t) \in \mathcal{L}_2[0, \infty)$, the controlled output $z(t)$ satisfies $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$, where γ is a prescribed performance index.

Considering the behavior of the ZOH, the input signal is

$$u(t) = g\left(Kf\left(x(t_k h)\right)\right), t \in \left[t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right). \tag{5}$$

Refer to Fig. 1. We now denote the quantized measurement of $x(t_k h)$ as $\tilde{x}(t_k h)$, the control signal as $\tilde{u}(t)$, and the control input signal as $u(t)$. Then, at the release instant $t_k h$, the following equations can be deduced:

$$\begin{cases} \tilde{x}(t_k h) = f\left(x(t_k h)\right) \\ \tilde{u}\left(t_k h + \tau_{sc}(t_k)\right) = K\tilde{x}(t_k h) \\ u\left(t_k h + \tau_{t_k}\right) = g\left(\tilde{u}\left(t_k h + \tau_{sc}(t_k)\right)\right). \end{cases} \tag{6}$$

The quantizers $f(\cdot) = [f_1(\cdot), f_2(\cdot), \ldots, f_n(\cdot)]^T$ and $g(\cdot) = [g_1(\cdot), g_2(\cdot), \ldots, g_p(\cdot)]^T$ are assumed to be symmetric, that is, $f_j(-v) = -f_j(v)(j = 1, 2, \ldots, n)$ and $g_m(-v) = -g_m(v)(m = 1, 2, \ldots, p)$. Similar to [27], [29], and [37], the quantizers considered in this paper are logarithmic static and time invariant. For each $f(\cdot)$, the set of quantized levels is described as in [26] and [37] by

$$\mathscr{U} = \left\{\pm u_i^{(j)}, u_i^{(j)} = \alpha_j^i u_0^{(j)}, i = \pm 1, \pm 2, \ldots\right\} \cup \left\{\pm u_0^{(j)}\right\} \cup \{0\}, 0 < \alpha_j < 1, u_0^{(j)} > 0. \tag{7}$$

The associated quantizer $f_j(\cdot)$ is defined as

$$f_j(v) = \begin{cases} u_i^{(j)}, & \text{if } \frac{1}{1+\sigma_j} u_i^{(j)} < v \leq \frac{1}{1-\sigma_j} u_i^{(j)}, v > 0 \\ 0, & \text{if } v = 0 \\ -f_j(-v), & \text{if } v < 0 \end{cases}$$

where $\sigma_j = (1 - \alpha_j)/(1 + \alpha_j)$, and α_j is also called the quantization density of quantizer $f_j(\cdot)$. Similarly, the quantizer $g_j(\cdot)(j = 1, 2, \ldots, p)$ is of quantization densities ρ_j and denote $\pi_j = (1 - \rho_j)/(1 + \rho_j)$. For a given logarithmic quantizer $f_j(\cdot)$, a sector bound condition was proposed as follows:

$$f_j(v) = (I + \Delta_f)v \tag{8}$$

where $\Delta_f = \text{diag}\{\Delta_{f_1}, \Delta_{f_2}, \ldots, \Delta_{f_n}\}$, and $\Delta_{f_n} \in [-\sigma_j, \sigma_j]$. For the quantizer on the controller side, the same definition can be applied. It follows that

$$g_j(v) = (I + \Delta_g)v \tag{9}$$

where $\Delta_g = \text{diag}\{\Delta_{g_1}, \Delta_{g_2}, \ldots, \Delta_{g_p}\}$, and $\Delta_{g_p} \in [-\pi_j, \pi_j]$. Combining with (6)–(9), we have

$$u\left(t_k h + \tau_{t_k}\right) = (I + \Delta_g)K(I + \Delta_f)x(t_k h)$$
$$t \in \left[t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right). \tag{10}$$

Then, the system can be transferred to linear systems with norm-bounded uncertainty, which was employed in [29] and [37]. However, due to the uncertainties on both sides of controller gain matrix K, the controller is difficult to design.

In the following, two new sector bound conditions of quantizers are proposed. We first denote

$$\Lambda = \text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\}, \ \Lambda_0 = I - \Lambda, \Lambda_1 = I + \Lambda$$
$$\Pi = \text{diag}\{\pi_1, \pi_2, \ldots, \pi_p\}, \ \Pi_0 = I - \Pi, \Pi_1 = I + \Pi.$$

Then, for any diagonal matrices $S > 0$ and $H > 0$, the following inequalities hold:

$$\left[f(x(t_k h)) - \Lambda_0 x(t_k h)\right]^T S \left[f(x(t_k h)) - \Lambda_1 x(t_k h)\right] \leq 0 \tag{11}$$

$$\left[g\left(Kf\left(x(t_k h)\right)\right) - \Pi_0 Kf\left(x(t_k h)\right)\right]^T H \times \left[g\left(Kf\left(x(t_k h)\right)\right) - \Pi_1 Kf\left(x(t_k h)\right)\right] \leq 0. \tag{12}$$

Remark 1: It should be mentioned that the sector bound conditions are much simpler and more applicable. Unlike some existing works (for example, [27], [29], and [37]), the difficulty associated with stability analysis and H_∞ controller design can be effectively overcome by using these conditions.

Substituting (5) into (1) yields the following closed-loop system:

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bg\left(Kf\left(x(t_k h)\right)\right) + Gw(t) \\ z(t) = Cx(t) + Dg\left(Kf\left(x(t_k h)\right)\right) + Fw(t). \end{cases} \tag{13}$$

D. Time-Delay Modeling

Next, using the same technique as in [37], we convert the event-triggered NCSs (13) into a new time-delay system, which can be analyzed by the well-developed theory on time-delay systems. First, suppose there exists a finite positive integer m such that $t_{k+1} = t_k + m + 1$. Then, the interval $[t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$ can be decomposed into the following subintervals:

$$\left[t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right) = \bigcup_{l=0}^{m} T_l \tag{14}$$

where $T_l = [i_k h + \tau_{i_k}, i_k h + h + \tau_{i_k+1}), i_k h = t_k h + lh, l = 0, 1, \ldots, m$. Moreover, $x(t_k h)$ and $x(t_k h + lh)$ satisfy the event-triggered sampling scheme (2).

For convenience, we denote

$$\tau(t) = t - i_k h \tag{15}$$

where $t \in T_l$, and we have

$$0 < \tau_m \leq \tau(t) \leq \tau_M + h \equiv \bar{\tau}. \tag{16}$$

Based on the above analysis, the closed-loop system (13) can be rewritten as

$$\begin{cases} E\dot{x}(t)=Ax(t)+Bg\left(Kf\left(x\left(t-\tau(t)\right)+e(i_kh)\right)\right)+Gw(t) \\ z(t)=Cx(t)+Dg\left(Kf\left(x\left(t-\tau(t)\right)+e(i_kh)\right)\right) \\ \quad +Fw(t), t\in\left[t_kh+\tau_{t_k}, t_{k+1}h+\tau_{t_{k+1}}\right) \\ x(t)=\phi(t), t\in[-\bar{\tau},0) \end{cases} \tag{17}$$

where $\phi(t)$ is the initial function of $x(t)$.

Remark 2: The problem formulated above differs from some existing works concerned with quantized feedback control, for example, [6] and [38], in which only the effect of quantization was considered. In this work, we consider not only the effect of quantization but also the event-triggered scheme, which is used to save the limited communication resources, for networked singular systems. Moreover, the event-triggered condition (2) only supervises the difference between the states sampled in discrete instants, and it needs no extra hardware to continuously monitor the state of the plant.

We end this section by recalling the following lemma, which will be used in the sequel.

Lemma 1: [39] For any vectors $X, Y \in R^n$ and positive-definite matrix $Q \in R^{n\times n}$, the following inequality holds:

$$2X^TY \le X^TQX + Y^TQ^{-1}Y.$$

III. Main Results

Here, we consider the quantized H_∞ control of the networked singular system (17) under the event-triggered scheme based on (2). We first give sufficient conditions for the closed-loop system (17) to be regular, impulse free, and stable with an H_∞ performance index γ. Then, we propose a design method for the quantized state feedback controller.

A. H_∞ Performance Analysis

Based on the new sector bound conditions (11) and (12), we present the following H_∞ performance analysis result.

Theorem 1: Given scalars $\gamma > 0$, $0 \le \sigma < 1$, τ_m, $\bar{\tau}$, and the controller gain matrix K, the closed-loop system (17) is regular, impulse free, and stable with H_∞ performance index γ under the event-triggering scheme (2), if there exist matrices $Q_1 = Q_1^T > 0, Q_2 = Q_2^T > 0, Z_i = Z_i^T > 0\ (i = 1, 2, 3)$, $\Phi > 0, P, N, M$, and any diagonal matrices $S > 0$ and $H > 0$ with appropriate dimensions such that

$$E^TP = P^TE \ge 0 \tag{18}$$

$$\begin{bmatrix} \Psi_1 & \Psi_2 \\ * & \Psi_3 \end{bmatrix} < 0 \tag{19}$$

where

$$\Psi_1 = \begin{bmatrix} \varphi+\Gamma+\Gamma^T & \sqrt{\bar{\tau}}N & \sqrt{\bar{\tau}}M \\ * & -Z_3 & 0 \\ * & * & -Z_3 \end{bmatrix}$$

$$\Psi_2 = \begin{bmatrix} T_2 & T_1 & \mathscr{A}Z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \Psi_3 = \begin{bmatrix} -\gamma^2 I & F^T & G^TZ \\ * & -I & 0 \\ * & * & -Z \end{bmatrix}$$

$$T_1 = \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 & D \end{bmatrix}^T$$

$$T_2 = \begin{bmatrix} G^TP & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathscr{A} = \begin{bmatrix} A & 0 & 0 & 0 & 0 & 0 & B \end{bmatrix}^T$$

$$Z = \tau_m Z_1 + (\bar{\tau}-\tau_m)Z_2 + 2\bar{\tau}Z_3$$

$$\Gamma = \begin{bmatrix} N & -N+M & 0 & M & 0 & 0 & 0 \end{bmatrix} E$$

$$\varphi = \begin{bmatrix} \varphi_{11} & 0 & \varphi_{13} & 0 & 0 & 0 & P^TB \\ * & \varphi_{22} & 0 & 0 & \varphi_{25} & 2S & 0 \\ * & * & \varphi_{33} & \varphi_{34} & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 2S & 0 \\ * & * & * & * & * & \varphi_{66} & \varphi_{67} \\ * & * & * & * & * & * & -2H \end{bmatrix}$$

$$\varphi_{11} = P^TA + A^TP + Q_1 + Q_2 - (1/\tau_m)E^T(Z_1+Z_3)E$$

$$\varphi_{13} = (1/\tau_m)E^T(Z_1+Z_3)E$$

$$\varphi_{22} = \sigma\Phi - 2\Lambda_1 S\Lambda_0, \ \varphi_{25} = -2\Lambda_1 S\Lambda_0$$

$$\varphi_{33} = -Q_1 - \left[1/\tau_m + 1/(\bar{\tau}-\tau_m)\right]E^T(Z_1+Z_3)E$$

$$\varphi_{34} = \left[1/(\tau-\tau_m)\right]E^T(Z_2+Z_3)E$$

$$\varphi_{44} = -Q_2 - \left[1/(\tau-\tau_m)\right]E^T(Z_2+Z_3)E$$

$$\varphi_{55} = -\Phi - 2\Lambda_1 S\Lambda_0$$

$$\varphi_{66} = -2S - 2K^T\Pi_0 H\Pi_1 K, \varphi_{67} = 2K^TH.$$

Proof: We first show that the networked singular system (17) is regular and impulse free. Since rank $E = r \le n$, there must exist two invertible matrices $\tilde{G}$ and $\tilde{H} \in \mathbb{R}^{n\times n}$ such that

$$\tilde{E} = \tilde{G}E\tilde{H} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \tilde{G}A\tilde{H} = \begin{bmatrix} A_{1,11} & A_{1,12} \\ A_{1,21} & A_{1,22} \end{bmatrix}.$$

Similar to the method used in [35], we know that $A_{1,22}$ is nonsingular, which implies that the pair of (E, A) is regular and impulse free, it follows that the networked singular system (17) is regular and impulse free. In the following, we will show that the networked singular system (17) is stable under the event-triggering scheme (2).

Consider when the system is free from external disturbances, with $w(t) = 0$. We define the following functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{20}$$

where

$$V_1(t) = x^T(t)E^T Px(t)$$

$$V_2(t) = \int_{t-\tau_m}^{t} x^T(s)Q_1x(s)ds + \int_{t-\bar{\tau}}^{t} x^T(s)Q_2x(s)ds$$

$$V_3(t) = \int_{-\tau_m}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^T Z_1E\dot{x}(s)d\theta ds + \int_{-\bar{\tau}}^{-\tau_m}\int_{t+\theta}^{t} \dot{x}^T(s)E^T Z_2E\dot{x}(s)d\theta ds + 2\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^T Z_3E\dot{x}(s)d\theta ds.$$

Taking the derivative of $V(t)$ for $t \in [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$, we introduce the free weighting matrices, i.e.,

$$\ell_1 = 2\xi^T(t)N\left[Ex(t) - Ex(t-\tau(t)) - \int_{t-\tau(t)}^{t} E\dot{x}(s)ds\right] = 0$$

$$\ell_2 = 2\xi^T(t)M\left[Ex(t-\tau(t)) - Ex(t-\bar{\tau}) - \int_{t-\bar{\tau}}^{t-\tau(t)} E\dot{x}(s)ds\right] = 0 \tag{21}$$

where $\xi^T(t) = [\eta^T(t)\ e^T(i_kh)]$, with

$$\eta^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & x^T(t-\tau_m) & x^T(t-\bar{\tau}) \end{bmatrix}$$

and N and M are matrices with appropriate dimensions. According to Lemma 1 and combining the sector bound conditions (11) and (12) with the event-triggered scheme (2), we have

$$\dot{V}(t) \le \xi^T(t)\Xi\xi(t) \tag{22}$$

where $\Xi = \varphi + \Gamma + \Gamma^T + \bar{\tau}NZ_3^{-1}N^T + \bar{\tau}MZ_3^{-1}M^T + \mathscr{A}Z\mathscr{A}^T$, with $Z = \tau_mZ_1 + (\bar{\tau} - \tau_m)Z_2 + 2\bar{\tau}Z_3$. According to Schur complement, from (19), we have

$$\begin{bmatrix} \varphi + \Gamma + \Gamma^T & \sqrt{\bar{\tau}}N & \sqrt{\bar{\tau}}M & \mathscr{A}Z \\ * & -Z_3 & 0 & 0 \\ * & * & -Z_3 & 0 \\ * & * & * & -Z \end{bmatrix} < 0 \tag{23}$$

which means $\dot{V}(t) < 0$. Therefore, system (17) is stable.

Now, we address the H_∞ performance of the networked singular system (17). Consider when the system is subject to external disturbances, with $w(t) \ne 0$. We use the following performance index:

$$\wp(t) = \int_0^\infty \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right] dt.$$

Under zero initial conditions, we have

$$\wp(t) = \int_0^\infty \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t)\right] dt - V(\infty)$$

$$\le \varsigma^T(t)\begin{bmatrix} \Xi + T_1 + T_1^T & T_2 \\ * & -\gamma^2I + F^TF + G^TZG \end{bmatrix}\varsigma(t)$$

with $\varsigma^T(t) = [\xi^T(t)\ \ w^T(t)]$. By Schur complement, from (19), we have

$$\begin{bmatrix} \Xi + T_1 + T_1^T & T_2 \\ * & -\gamma^2I + F^TF + G^TZG \end{bmatrix} < 0$$

which means $\wp(t) < 0$. That is, under zero initial conditions, for any nonzero $w(t) \in \mathcal{L}_2[0,\infty)$, the control output $z(t)$ satisfies $\|z(t)\|_2 \le \gamma\|w(t)\|_2$. This completes the proof. ■

B. Quantized State Feedback Controller Design

Based on Theorem 1, we present the codesign algorithm for the networked singular system (17) as follows.

Theorem 2: For given scalars $\gamma > 0$, $0 \le \sigma < 1$, τ_m, $\bar{\tau}$, and ρ_i $(i = 1, 2, \dots, 5)$, the singular NCS (17) is regular, impulse free, and stable with an H_∞ performance index γ under the event-triggering scheme (2), if there exist matrices $\widetilde{Q}_1 = \widetilde{Q}_1^T > 0$, $\widetilde{Q}_2 = \widetilde{Q}_2^T > 0$, $\widetilde{Z}_i = \widetilde{Z}_i^T > 0$ $(i = 1, 2, 3)$, $\widetilde{\Phi} > 0$, $\widetilde{N}$, $\widetilde{M}$, Y, nonsingular $\widetilde{P}$ and any diagonal matrices $\widetilde{S} > 0$, $S > 0$, $H > 0$ with appropriate dimensions such that

$$\widetilde{P}^TE^T = E\widetilde{P} \ge 0 \tag{24}$$

$$\begin{bmatrix} \tilde{\Psi}_1 & \tilde{\Psi}_2 \\ * & \tilde{\Psi}_3 \end{bmatrix} < 0 \tag{25}$$

where

$$\tilde{\Psi}_1 = \begin{bmatrix} \widetilde{\varphi} + \widetilde{\Gamma} + \widetilde{\Gamma}^T & \sqrt{\bar{\tau}}\widetilde{N} & \sqrt{\bar{\tau}}\widetilde{M} & \widetilde{T}_2 & \widetilde{T}_1 \\ * & -\widetilde{Z}_3 & 0 & 0 & 0 \\ * & * & -\widetilde{Z}_3 & 0 & 0 \\ * & * & * & -\gamma^2I & F^T \\ * & * & * & * & -I \end{bmatrix}$$

$$\tilde{\Psi}_2 = \begin{bmatrix} \widetilde{\mathscr{A}} & \widetilde{\mathscr{A}} & \widetilde{\mathscr{A}} & \widetilde{T}_3 & \widetilde{T}_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ G^T & G^T & G^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Psi}_3 = \mathrm{diag}\{\tilde{\varrho}_1, \tilde{\varrho}_2, \tilde{\varrho}_3, \tilde{\varrho}_4, \tilde{\varrho}_5\}$$

$$\tilde{\varrho}_1 = \frac{1}{\tau_m\rho_1^2}(\widetilde{Z}_1 - 2\rho_1E\widetilde{P}),\ \tilde{\varrho}_2 = \frac{1}{(\bar{\tau} - \tau_m)\rho_2^2}(\widetilde{Z}_2 - 2\rho_2E\widetilde{P})$$

$$\tilde{\varrho}_3 = \frac{1}{2\bar{\tau}\rho_3^2}(\widetilde{Z}_3 - 2\rho_3E\widetilde{P}),\ \tilde{\varrho}_4 = \frac{1}{2}(\rho_4^2S - 2\rho_4I)$$

$$\tilde{\varrho}_5 = \frac{1}{2}\left(\rho_5^2H - 2\rho_5I\right)$$

$$\widetilde{T}_1 = \begin{bmatrix} C\widetilde{P} & DY & 0 & 0 & DY & DY & D \end{bmatrix}^T$$

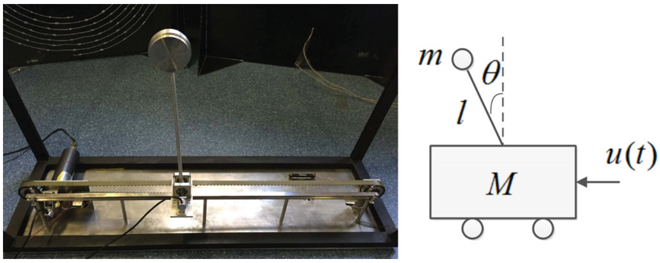
$$\widetilde{T}_2 = \begin{bmatrix} G^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\widetilde{T}_3 = \begin{bmatrix} 0 & \Lambda\widetilde{P}^T & 0 & 0 & \Lambda\widetilde{P}^T & 0 & 0 \end{bmatrix}^T$$

$$\widetilde{T}_4 = \begin{bmatrix} 0 & \Pi Y & 0 & 0 & \Pi Y & \Pi Y & 0 \end{bmatrix}^T$$

$$\widetilde{\mathscr{A}} = \begin{bmatrix} A\widetilde{P} & BY & 0 & 0 & BY & BY & B \end{bmatrix}^T$$

$$\widetilde{\Gamma} = \begin{bmatrix} \widetilde{N} & -\widetilde{N} + \widetilde{M} & 0 & \widetilde{M} & 0 & 0 & 0 \end{bmatrix}$$



(a) (b)

Fig. 2. Networked inverted pendulum. (a) Inverted pendulum in the laboratory at The University of Adelaide. (b) Schematic of the inverted pendulum.

$$\widetilde{\varphi} = \begin{bmatrix} \widetilde{\varphi}_{11} & BY & \widetilde{\varphi}_{13} & 0 & BY & BY & B \\ * & \sigma\widetilde{\Phi} & 0 & 0 & 0 & 0 & 0 \\ * & * & \widetilde{\varphi}_{33} & \widetilde{\varphi}_{34} & 0 & 0 & 0 \\ * & * & * & \widetilde{\varphi}_{44} & 0 & 0 & 0 \\ * & * & * & * & -\widetilde{\Phi} & 0 & 0 \\ * & * & * & * & * & -2\widetilde{S} & 0 \\ * & * & * & * & * & * & -2H \end{bmatrix}$$

$$\widetilde{\varphi}_{11} = A\widetilde{P} + \widetilde{P}A^T + \widetilde{Q}_1 + \widetilde{Q}_2 - (1/\tau_m)(\widetilde{Z}_1 + \widetilde{Z}_3)$$
$$\widetilde{\varphi}_{13} = (1/\tau_m)(\widetilde{Z}_1 + \widetilde{Z}_3),$$
$$\widetilde{\varphi}_{33} = -\widetilde{Q}_1 - (1/\tau_m), (\widetilde{Z}_1 + \widetilde{Z}_3) - [1/(\bar{\tau} - \tau_m)]\,(\widetilde{Z}_2 + \widetilde{Z}_3)$$
$$\widetilde{\varphi}_{34} = [1/(\bar{\tau} - \tau_m)]\,(\widetilde{Z}_2 + \widetilde{Z}_3),$$
$$\widetilde{\varphi}_{44} = -\widetilde{Q}_2 - [1/(\bar{\tau} - \tau_m)]\,(\widetilde{Z}_2 + \widetilde{Z}_3).$$

Furthermore, a desired state feedback controller gain is

$$K = Y\widetilde{P}^{-1}. \tag{26}$$

Proof: Similar to the method used in [4, Th. 2], Theorem 2 can be proved.

IV. Examples

We use two examples to demonstrate the effectiveness of the proposed method. The first example is a networked regular system to show less conservatism of our results, whereas the second example is a networked singular system to show the effectiveness in reducing the network usage of the proposed method.

Example 1: Consider an inverted pendulum on a cart controlled over a network. The schematic of an inverted pendulum is shown in Fig. 2, and the linearized plant model (1) is characterized by the following parameters [3] and [37]:

$$E = I, \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml} \end{bmatrix}$$

where $M = 10$ kg is the cart mass, $m = 1$ kg is the mass of the pendulum bob, $l = 3$ m is the length of the pendulum arm, and $g = 10$ m/s^2 is the gravitational acceleration.

Since the eigenvalues of A are $\{0, 0, 1.8257, -1.8257\}$, the system is unstable without a controller. The state variables x_i $(i = 1, 2, 3, 4)$ are the cart position, the cart velocity, the pendulum bob angle, and the pendulum bob angular velocity. The initial state vector is set as $x_0(t) = [1.5 \;\; -0.5 \;\; 0.8 \;\; -1]^T$.

We consider two cases with different parameters.

Case 1—H_∞ Control Without Quantizations:

$$C = G^T = F^T = [1 \quad 1 \quad 1 \quad 1], \quad D = 0.1, \quad \Lambda = \Pi = 0$$

$$w(t) = \begin{cases} \mathrm{sgn}\,(\sin(t)), & \text{if } t \in [0, 10] \\ 0, & \text{others.} \end{cases}$$

Case 2—H_∞ Control With Quantizations:

$$C = G^T = F^T = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \quad D = 0.1$$

$$w(t) = \begin{cases} \mathrm{sgn}\,(\sin(t)), & \text{if } t \in [0, 10] \\ 0, & \text{others} \end{cases}$$

and the parameters for the quantizer $f(\cdot)$ are assumed to be $\alpha_1 = \alpha_3 = 0.9$ and $\alpha_2 = \alpha_4 = 0.8$, that is

$$\Lambda = \begin{bmatrix} 0.0526 & 0 & 0 & 0 \\ 0 & 0.1111 & 0 & 0 \\ 0 & 0 & 0.0526 & 0 \\ 0 & 0 & 0 & 0.1111 \end{bmatrix}$$

whereas the quantized density of $g(\cdot)$ is assumed to be $\alpha_1 = 0.9$, that is, $\Pi = 0.0526$.

In Case 1, under the conditions of $h = 0.01$, $\sigma = 0.1$, $\bar{\tau} = 0.16$, $\rho_1 = \rho_2 = \rho_3 = 0.46$, and $\rho_4 = \rho_5 = 0.23$, the H_∞ performance index in [3] is $\gamma = 200$. In our scheme, according to Theorem 2 and setting $\tau_m = 0.01$, the minimum of H_∞ performance index $\gamma_{\min} = 85$. The correspondent feedback gain K_2 and the event-triggering matrix Φ_5 are

$$K_2 = \begin{bmatrix} 5.8955 & 16.2858 & 334.4121 & 186.8863 \end{bmatrix}$$

$$\Phi_5 = \begin{bmatrix} 4.2235 & -4.6241 & -18.8239 & 33.11503 \\ -4.6241 & 12.5586 & 44.1248 & -78.4754 \\ -18.8239 & 44.1248 & 170.8534 & -302.9154 \\ 33.1150 & -78.4754 & -302.9154 & 537.1290 \end{bmatrix}.$$

The state responses $x(t)$ and release instants are shown in Fig. 3 for this setting. The number of triggers is 86 times.

In Case 2, the effect of two quantizers is considered. We set $\bar{\tau} = 0.24$, $\tau_m = 0.01$, $\rho_1 = \rho_2 = \rho_3 = 0.44$, and $\rho_4 = \rho_5 = 0.21$, and Table I gives the different results for different triggered parameter values of σ. It shows that the larger the parameter

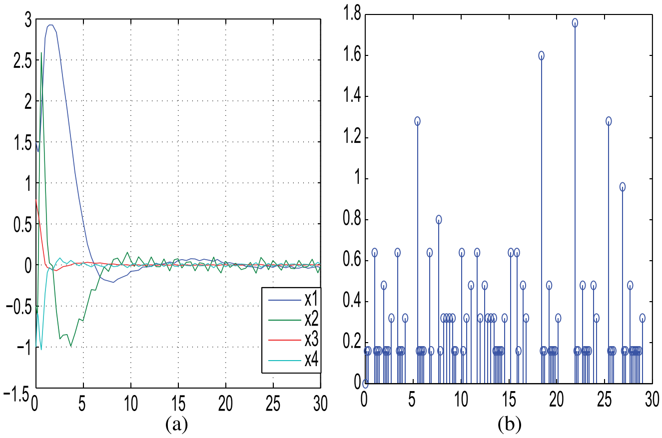


Fig. 3. State response $x(t)$ and release instants under K_2 and Φ_5. (a) State response $x(t)$. (b) Release instants.

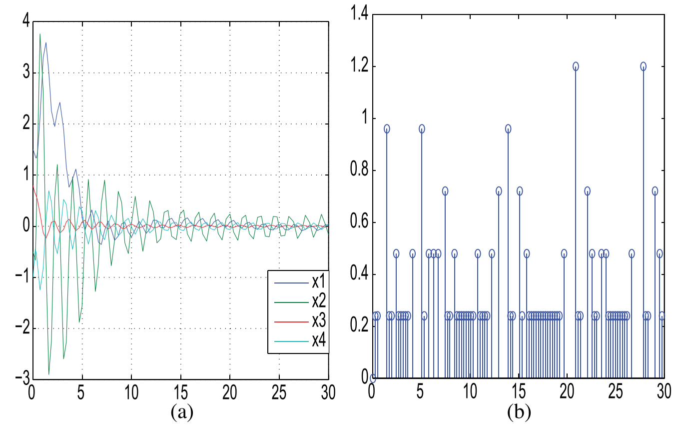


Fig. 4. State response $x(t)$ and release instants under $K_{3,4}$. (a) State response $x(t)$. (b) Release instants.

TABLE I
$\gamma_{\min}$, K_3, AND Φ_6 FOR DIFFERENT σ VALUES

σ	0	0.01	0.02	0.03
γ_{min}	65	68	72	75
K_3	$K_{3,1}$	$K_{3,2}$	$K_{3,3}$	$K_{3,4}$
Φ_6	$\Phi_{6,1}$	$\Phi_{6,2}$	$\Phi_{6,3}$	$\Phi_{6,4}$

σ, the larger the minimum value of γ. Other parameters and values in Table I are

$$K_{3,1} = \begin{bmatrix}4.9257 & 13.6401 & 283.3992 & 158.1448\end{bmatrix}$$
$$K_{3,2} = \begin{bmatrix}5.3323 & 14.8106 & 302.2490 & 168.8918\end{bmatrix}$$
$$K_{3,3} = \begin{bmatrix}5.0948 & 14.5901 & 304.2014 & 169.9699\end{bmatrix}$$
$$K_{3,4} = \begin{bmatrix}4.7482 & 14.0801 & 301.9530 & 168.6491\end{bmatrix}$$
$$\Phi_{6,1} = 10^2 \times \begin{bmatrix}2.5368 & -0.0000 & -0.0000 & 0.0000\\ -0.0000 & 2.5368 & 0.0000 & -0.0000\\ -0.0000 & 0.0000 & 2.5368 & -0.0000\\ 0.0000 & -0.0000 & -0.0000 & 2.5368\end{bmatrix}$$
$$\Phi_{6,2} = \begin{bmatrix}17.0000 & -8.8000 & -36.6000 & 63.7000\\ -8.8000 & 35.9000 & 117.1000 & -209.4000\\ -36.6000 & 117.1000 & 522.9000 & -927.5000\\ 63.7000 & -209.4000 & -927.5000 & 1645.3000\end{bmatrix}$$
$$\Phi_{6,3} = \begin{bmatrix}7.8968 & -4.4337 & -21.8893 & 38.1572\\ -4.4337 & 16.3175 & 54.8386 & -97.9276\\ -21.8893 & 54.8386 & 253.9125 & -449.5681\\ 38.1572 & -97.9276 & -449.5681 & 796.1969\end{bmatrix}$$
$$\Phi_{6,4} = \begin{bmatrix}4.7845 & -2.7674 & -14.0746 & 24.5370\\ -2.7674 & 10.0585 & 33.4759 & -59.7880\\ -14.0746 & 33.4759 & 155.5644 & -275.2709\\ 24.5370 & -59.7880 & -275.2709 & 487.2295\end{bmatrix}.$$

Fig. 4 shows the state responses $x(t)$ and release instants. Over the simulation period, there are 82 triggers. We remark that if a time-triggered scheme is used instead, the number of triggers will be 3000 times. The result is a clear indication that our event-triggered approach is efficient in terms of utilizing the network bandwidth resource.

Example 2: This example illustrates the quantized H_∞ control on a singular NCS. Consider the singular NCS (1). The associated parameters are

$$E = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},\ A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0.3 & 0\end{bmatrix},\ B = \begin{bmatrix}0\\ 0.1\\ 0\\ 0.033\end{bmatrix}$$
$$C = G^T = F^T = \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}, D = 0.1$$
$$w(t) = \begin{cases}\operatorname{sgn}(\sin(t)), & \text{if } t \in [0, 10]\\ 0, & \text{others.}\end{cases}$$

The parameters for the quantizer $f(\cdot)$ are taken as $\alpha_1 = \alpha_3 = 0.9$ and $\alpha_2 = \alpha_4 = 0.8$. We set $\bar{\tau} = 0.24$, $\tau_m = 0.01$, $\rho_1 = \rho_2 = \rho_3 = 0.44$, $\rho_4 = \rho_5 = 0.21$, and $\sigma = 0.02$, and according to Theorem 2, the minimum H_∞ performance index $\gamma_{\min} = 51.36$. The corresponding feedback gain and the event-triggered matrix are

$$K_5 = \begin{bmatrix}2.5480 & 9.3207 & 237.6465 & 161.8992\end{bmatrix}$$
$$\Phi_8 = \begin{bmatrix}2.0941 & -0.0477 & -0.0049 & -0.2502\\ -0.0477 & 1.7747 & -0.2384 & -0.1521\\ -0.0049 & -0.2384 & 0.0713 & -0.0431\\ -0.2502 & -0.1521 & -0.0431 & 0.1580\end{bmatrix}.$$

Furthermore, the initial state is $x_0(t) = [1.5\ -0.5\ 0.8\ -1]^T$, and the state responses for $x(t)$ and the release instants are shown in Fig. 5. We observe that the number of triggers is 627, which is much lower than 15 000 triggers when using the time-triggered scheme. The result again demonstrates the capability of the event-triggered approach in reducing the network bandwidth usage.

V. CONCLUSION

Aiming to reduce the load of network communication, the problem of event-triggered H_∞ control for networked singular

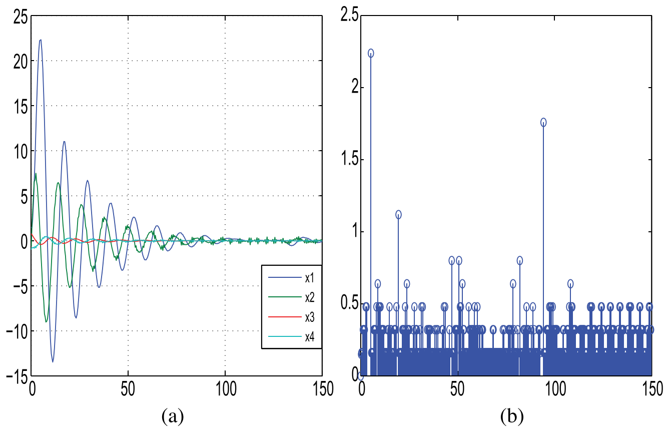


Fig. 5. State response $x(t)$ and release instants under K_5. (a) State response $x(t)$. (b) Release instants.

systems with quantizations in both the measured states and the generated control inputs has been studied in this paper. By considering the characteristics of event-triggered schemes and taking the quantizations into account, we presented a new time-delay model. Based on this model, we derived a new H_∞ performance criterion that guarantees that the closed-loop system of the singular networked system is regular, impulse free, and stable with a prescribed H_∞ performance index γ. The codesign of the event-triggered condition and the state feedback controller has also been derived based on a free-weighting-matrix approach. Two examples have been given to show the effectiveness of the theoretical results obtained.

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Network-Based Event-Triggered Control for Singular Systems With Quantizations

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***Abstract*—This paper investigates the problem of event-triggered H_∞ control for a networked singular system with both state and input subject to quantizations. First, a discrete event-triggered scheme, which activates only at each sampling instance, is presented. Next, two new sector bound conditions of quantizers are proposed to provide a more intuitive stability analysis and controller design. Then, network conditions, quantizations, and the event-triggered scheme are modeled as a time-delay system. With this model, the criteria are derived for H_∞ performance analysis, and codesigning methods are developed for the event trigger and the quantized state feedback controller. An inverted pendulum controlled through the network is given to demonstrate the effectiveness and potential of the new design techniques.**

***Index Terms*—Event-triggered control, networked singular system, quantization, sector bound condition.**

I. Introduction

EVENT-TRIGGERED schemes, where the sampled signal is transmitted according to an event-triggered condition other than a fixed time interval, have received increasing attention due to its capacity for reducing communication load. Many results have been reported on the problem of event-triggered control or event-based control, such as [1]–[4] and the reference therein. Among them are two types of event-triggered scheme: one with a continuous event-triggered condition [1], [2], and the other with is a discrete event-triggered condition [3], [4]. The continuous event trigger relies on additional hardware to continuously supervise the system state to detect whether the current state exceeds a trigger threshold. Moreover, the continuous event-triggered scheme can only be effective under a given controller, and the controller and the triggered parameters cannot easily be codesigned. In the discrete event-triggered scheme, the triggered condition is detected in discrete sampled instants, and incorporating a codesign algorithm is readily achievable for most practical systems.

In networked control systems (NCSs), the sharing of limited network bandwidth often causes network-induced delays, and data packet dropouts and disorder, which can deteriorate the performance and even destabilize the systems [5]–[11]. In the past decade, many methods have been developed to deal with these network-induced challenging issues, for example, the filtering, identification, and estimation problem in [12]–[15] and the output feedback problem in [16]–[18]. However, most are based on a time-triggered scheme, which can be inefficient in terms of reducing the utilization of limited network bandwidth.

Furthermore, quantization problems inherent in sampled-data systems have been investigated in recent years [19]–[24]. It was shown in [25] that the coarsest quantizer is logarithmic, and the sector bound method is applicable for stabilizing linear single-input–single-output systems with state quantization. The sector bound method in [25] was extended to multiple-input–multiple-output systems in [26] and to guaranteed cost control of continuous systems over networks with state and input quantizations in [27]. In addition, the networked H_∞ control for continuous-time linear systems with state quantization was discussed in [28], and the problem of H_∞ estimation was studied in [29]. The reset quantized state control problem was studied in [30] and [31]. Meanwhile, singular systems are frequently encountered in electronic and economic systems, aerospace, and chemical industries [32]–[36]. Hence, there will be a profound meaning applying quantized control to singular systems. Indeed, the problem of a networked H_∞ filter for singular systems with state quantization was investigated in [6] by the similar method used in [29]. However, when using the sector bound method, the quantization errors have been regarded as a class of uncertainties, which present difficulties in controller design. To the best of the authors' knowledge, although discrete event-triggered control for linear systems has been discussed in [3] and [4], there is no result reported on event-triggered control for networked singular systems that are subject to quantizations. This motivates the research presented in this paper.

The works most pertinent to this paper are [37] and [38]. In fact, this paper stems from the following motivations. First, the quantized control under event-triggered networked systems investigated in [37] is novel but only for regular systems. On

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the other hand, the new sector bound approach used in [38] is under a time-triggered scheme, which has its useful properties, but may lead to the unnecessary usage of limited communication resources. Our aim here is to find a more effective and efficient discrete event-triggered scheme, which only detects the difference between the states sampled in discrete instants regardless of what happens in between updates, and to codesign the event-triggered H_∞ controller for networked singular systems taking into account both communication delays and signal quantizations.

In this paper, the problem of event-trigged H_∞ control for networked singular systems with both state and control input quantizations is investigated. Our contributions are as follows: 1) A new sector bound approach, by which no transformation is needed from system models to uncertain systems, is presented; 2) a discrete event-triggered scheme that only needs supervision of the system state in discrete instants is presented for networked singular systems; and 3) a unified framework, which takes network-induced delays, state and input quantizations, and event triggers into account, is given for codesigning the event detector and the state feedback controller.

The remainder of this paper is organized as follows. Section II formulates the problem. H_∞ performance analysis and quantized state feedback controller design are presented in Section III. Illustrative examples are given in Section IV to demonstrate the effectiveness of the presented method. Finally, this paper is concluded in Section V.

Notations: Throughout this paper, the superscripts "T" and "-1" stand for the transpose of a matrix and the inverse of a matrix; $\mathbb{R}^n$ denotes n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all real matrices with n rows and m columns; $P > 0$ means that P is positive definite; I is the identity matrix with appropriate dimensions; the space of square-integrable vector functions over $[0, \infty)$ is denoted by $\mathcal{L}_2[0, \infty)$, and for $w(t) \in \mathcal{L}_2[0, \infty)$, its norm is given by $\|w(t)\|_2 = \sqrt{\int_0^\infty |w(t)|^2 dt}$; for a symmetric matrix, $*$ denotes the matrix entries implied by symmetry.

II. Problem Formulation

A. Plant Description

The networked singular system, as shown in Fig. 1, comprises a continuous-time-controlled singular system, a set of sensors to provide the state signals, an event detector, two quantizers $f(\cdot)$ and $g(\cdot)$, a zero-order hold (ZOH), actuators, and a data network.

The networked singular system is described as follows:

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \\ z(t) = Cx(t) + Du(t) + Fw(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $w(t) \in \mathbb{R}^p$ is the disturbance input, and $z(t) \in \mathbb{R}^q$ is the controlled output of the plant. The matrices A, B, C, D, E, F, and G are constant matrices with appropriate dimensions, where E may be singular, and we assume that rank $E = r \leq n$. For the networked singular system shown in Fig. 1, the following conditions are assumed in this paper.

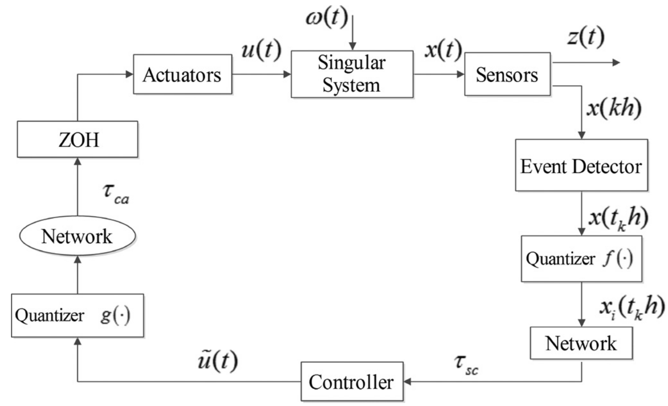


Fig. 1. Block diagram of an event-triggered controlled singular system.

1) The sensors are time triggered with a constant sampling period h. The sampled $x(kh)$ is transmitted to the event detector and is transmitted (or released) at instant $t_k h$ by the event detector, which is located between the sensors and the controller. All state variables of the singular NCS are measurable.
2) The signal in the network is transmitted with a single packet, and the data packet loss does not occur during transmission.

B. Event-Triggered Scheme

To reduce the utilization of the limited network bandwidth, a discrete event-triggered scheme is proposed in this paper to replace the conventional time-triggered mechanism [3], [4]. The event detector uses the following condition to decide on whether the current signal should be transmitted to the controller:

$$t_{k+1}h = t_k h + \min_l \{ lh | e^T(i_k h)\Phi e(i_k h) \geq \sigma x^T(t_k h)\Phi x(t_k h) \} \tag{2}$$

where $0 \leq \sigma < 1$ is a given scalar parameter, $\Phi > 0$ is a positive matrix to be determined, and $e(i_k h)$ is the error between the two states at the latest transmitted sampling instant and the current sampling instant, i.e., $e(i_k h) = x(t_k h) - x(i_k h)$, where $i_k h = t_k h + lh, l \in \mathbb{N}$.

When the data released at t_k by the event monitor are transmitted to the controller, it incurs a communication delay called the sensor-to-controller delay $\tau_{sc}(t_k)$. Similarly, the controller forwarding the actuation signals at t_k to the actuator incurs another communication delay called the controller-to-actuator delay $\tau_{ca}(t_k)$. These two network-induced delays can be lumped together as the time-varying delay τ_{t_k}, and

$$\tau_{t_k} = \tau_{sc}(t_k) + \tau_{ca}(t_k), 0 \leq \tau_m \leq \tau_{t_k} \leq \tau_M \tag{3}$$

where τ_m and τ_M denote the lower and upper delay bounds, respectively.

C. Event-Triggered Quantized H_∞ Control Problem

The problem of event-triggered H_∞ control with quantizations to be addressed in this paper is to design a state feedback controller, i.e.,

$$u(t) = Kx(t) \tag{4}$$

where K is the controller gain, such that:

1) the resultant closed-loop system with $w(t) = 0$ is regular, impulse free, and stable; and
2) under zero initial conditions, for any nonzero $w(t) \in \mathcal{L}_2[0,\infty)$, the controlled output $z(t)$ satisfies $\|z(t)\|_2 \leq \gamma\|w(t)\|_2$, where γ is a prescribed performance index.

Considering the behavior of the ZOH, the input signal is

$$u(t) = g\left(Kf\left(x(t_kh)\right)\right), t \in \left[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right). \tag{5}$$

Refer to Fig. 1. We now denote the quantized measurement of $x(t_kh)$ as $\tilde{x}(t_kh)$, the control signal as $\tilde{u}(t)$, and the control input signal as $u(t)$. Then, at the release instant t_kh, the following equations can be deduced:

$$\begin{cases} \tilde{x}(t_kh) = f\left(x(t_kh)\right) \\ \tilde{u}\left(t_kh + \tau_{sc}(t_k)\right) = K\tilde{x}(t_kh) \\ u\left(t_kh + \tau_{t_k}\right) = g\left(\tilde{u}\left(t_kh + \tau_{sc}(t_k)\right)\right). \end{cases} \tag{6}$$

The quantizers $f(\cdot) = [f_1(\cdot), f_2(\cdot), \ldots, f_n(\cdot)]^T$ and $g(\cdot) = [g_1(\cdot), g_2(\cdot), \ldots, g_p(\cdot)]^T$ are assumed to be symmetric, that is, $f_j(-v) = -f_j(v)(j = 1, 2, \ldots, n)$ and $g_m(-v) = -g_m(v)(m = 1, 2, \ldots, p)$. Similar to [27], [29], and [37], the quantizers considered in this paper are logarithmic static and time invariant. For each $f(\cdot)$, the set of quantized levels is described as in [26] and [37] by

$$\mathscr{U} = \left\{\pm u_i^{(j)}, u_i^{(j)} = \alpha_j^i u_0^{(j)}, i = \pm 1, \pm 2, \ldots\right\} \cup \left\{\pm u_0^{(j)}\right\} \cup \{0\}, 0 < \alpha_j < 1, u_0^{(j)} > 0. \tag{7}$$

The associated quantizer $f_j(\cdot)$ is defined as

$$f_j(v) = \begin{cases} u_i^{(j)}, & \text{if } \frac{1}{1+\sigma_j}u_i^{(j)} < v \leq \frac{1}{1-\sigma_j}u_i^{(j)}, v > 0 \\ 0, & \text{if } v = 0 \\ -f_j(-v), & \text{if } v < 0 \end{cases}$$

where $\sigma_j = (1-\alpha_j)/(1+\alpha_j)$, and α_j is also called the quantization density of quantizer $f_j(\cdot)$. Similarly, the quantizer $g_j(\cdot)(j = 1, 2, \ldots, p)$ is of quantization densities ρ_j and denote $\pi_j = (1-\rho_j)/(1+\rho_j)$. For a given logarithmic quantizer $f_j(\cdot)$, a sector bound condition was proposed as follows:

$$f_j(v) = (I + \Delta_f)v \tag{8}$$

where $\Delta_f = \operatorname{diag}\{\Delta_{f_1}, \Delta_{f_2}, \ldots, \Delta_{f_n}\}$, and $\Delta_{f_n} \in [-\sigma_j, \sigma_j]$. For the quantizer on the controller side, the same definition can be applied. It follows that

$$g_j(v) = (I + \Delta_g)v \tag{9}$$

where $\Delta_g = \operatorname{diag}\{\Delta_{g_1}, \Delta_{g_2}, \ldots, \Delta_{g_p}\}$, and $\Delta_{g_p} \in [-\pi_j, \pi_j]$. Combining with (6)–(9), we have

$$\begin{aligned} u\left(t_kh + \tau_{t_k}\right) &= (I + \Delta_g)K(I + \Delta_f)x(t_kh) \\ & \quad t \in \left[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right). \end{aligned} \tag{10}$$

Then, the system can be transferred to linear systems with norm-bounded uncertainty, which was employed in [29] and [37]. However, due to the uncertainties on both sides of controller gain matrix K, the controller is difficult to design.

In the following, two new sector bound conditions of quantizers are proposed. We first denote

$$\begin{aligned} \Lambda &= \operatorname{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\},\ \Lambda_0 = I - \Lambda, \Lambda_1 = I + \Lambda \\ \Pi &= \operatorname{diag}\{\pi_1, \pi_2, \ldots, \pi_p\},\ \Pi_0 = I - \Pi, \Pi_1 = I + \Pi. \end{aligned}$$

Then, for any diagonal matrices $S > 0$ and $H > 0$, the following inequalities hold:

$$\left[f(x(t_kh)) - \Lambda_0 x(t_kh)\right]^T S \left[f(x(t_kh)) - \Lambda_1 x(t_kh)\right] \leq 0 \tag{11}$$

$$\begin{aligned} &\left[g\left(Kf\left(x(t_kh)\right)\right) - \Pi_0 Kf\left(x(t_kh)\right)\right]^T H \\ &\times \left[g\left(Kf\left(x(t_kh)\right)\right) - \Pi_1 Kf\left(x(t_kh)\right)\right] \leq 0. \end{aligned} \tag{12}$$

Remark 1: It should be mentioned that the sector bound conditions are much simpler and more applicable. Unlike some existing works (for example, [27], [29], and [37]), the difficulty associated with stability analysis and H_∞ controller design can be effectively overcome by using these conditions.

Substituting (5) into (1) yields the following closed-loop system:

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bg\left(Kf\left(x(t_kh)\right)\right) + Gw(t) \\ z(t) = Cx(t) + Dg\left(Kf\left(x(t_kh)\right)\right) + Fw(t). \end{cases} \tag{13}$$

D. Time-Delay Modeling

Next, using the same technique as in [37], we convert the event-triggered NCSs (13) into a new time-delay system, which can be analyzed by the well-developed theory on time-delay systems. First, suppose there exists a finite positive integer m such that $t_{k+1} = t_k + m + 1$. Then, the interval $[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$ can be decomposed into the following subintervals:

$$\left[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right) = \bigcup_{l=0}^{m} T_l \tag{14}$$

where $T_l = [i_kh + \tau_{i_k}, i_kh + h + \tau_{i_k+1}), i_kh = t_kh + lh, l = 0, 1, \ldots, m$. Moreover, $x(t_kh)$ and $x(t_kh + lh)$ satisfy the event-triggered sampling scheme (2).

For convenience, we denote

$$\tau(t) = t - i_kh \tag{15}$$

where $t \in T_l$, and we have

$$0 < \tau_m \leq \tau(t) \leq \tau_M + h \equiv \bar{\tau}. \tag{16}$$

Based on the above analysis, the closed-loop system (13) can be rewritten as

$$\begin{cases} E\dot{x}(t)=Ax(t)+Bg\left(Kf\left(x\left(t-\tau(t)\right)+e(i_kh)\right)\right)+Gw(t) \\ z(t)=Cx(t)+Dg\left(Kf\left(x\left(t-\tau(t)\right)+e(i_kh)\right)\right) \\ \quad +Fw(t), t\in\left[t_kh+\tau_{t_k}, t_{k+1}h+\tau_{t_{k+1}}\right) \\ x(t)=\phi(t), t\in[-\bar{\tau},0) \end{cases} \tag{17}$$

where $\phi(t)$ is the initial function of $x(t)$.

Remark 2: The problem formulated above differs from some existing works concerned with quantized feedback control, for example, [6] and [38], in which only the effect of quantization was considered. In this work, we consider not only the effect of quantization but also the event-triggered scheme, which is used to save the limited communication resources, for networked singular systems. Moreover, the event-triggered condition (2) only supervises the difference between the states sampled in discrete instants, and it needs no extra hardware to continuously monitor the state of the plant.

We end this section by recalling the following lemma, which will be used in the sequel.

Lemma 1: [39] For any vectors $X, Y \in R^n$ and positive-definite matrix $Q \in R^{n\times n}$, the following inequality holds:

$$2X^TY \le X^TQX + Y^TQ^{-1}Y.$$

III. Main Results

Here, we consider the quantized H_∞ control of the networked singular system (17) under the event-triggered scheme based on (2). We first give sufficient conditions for the closed-loop system (17) to be regular, impulse free, and stable with an H_∞ performance index γ. Then, we propose a design method for the quantized state feedback controller.

A. H_∞ Performance Analysis

Based on the new sector bound conditions (11) and (12), we present the following H_∞ performance analysis result.

Theorem 1: Given scalars $\gamma > 0$, $0 \le \sigma < 1$, τ_m, $\bar{\tau}$, and the controller gain matrix K, the closed-loop system (17) is regular, impulse free, and stable with H_∞ performance index γ under the event-triggering scheme (2), if there exist matrices $Q_1 = Q_1^T > 0, Q_2 = Q_2^T > 0$, $Z_i = Z_i^T > 0$ $(i = 1, 2, 3)$, $\Phi > 0, P, N, M$, and any diagonal matrices $S > 0$ and $H > 0$ with appropriate dimensions such that

$$E^TP = P^TE \ge 0 \tag{18}$$

$$\begin{bmatrix} \Psi_1 & \Psi_2 \\ * & \Psi_3 \end{bmatrix} < 0 \tag{19}$$

where

$$\Psi_1 = \begin{bmatrix} \varphi+\Gamma+\Gamma^T & \sqrt{\bar{\tau}}N & \sqrt{\bar{\tau}}M \\ * & -Z_3 & 0 \\ * & * & -Z_3 \end{bmatrix}$$

$$\Psi_2 = \begin{bmatrix} T_2 & T_1 & \mathscr{A}Z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \Psi_3 = \begin{bmatrix} -\gamma^2 I & F^T & G^TZ \\ * & -I & 0 \\ * & * & -Z \end{bmatrix}$$

$$T_1 = \begin{bmatrix} C & 0 & 0 & 0 & 0 & 0 & D \end{bmatrix}^T$$

$$T_2 = \begin{bmatrix} G^TP & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathscr{A} = \begin{bmatrix} A & 0 & 0 & 0 & 0 & 0 & B \end{bmatrix}^T$$

$$Z = \tau_m Z_1 + (\bar{\tau} - \tau_m)Z_2 + 2\bar{\tau}Z_3$$

$$\Gamma = \begin{bmatrix} N & -N+M & 0 & M & 0 & 0 & 0 \end{bmatrix} E$$

$$\varphi = \begin{bmatrix} \varphi_{11} & 0 & \varphi_{13} & 0 & 0 & 0 & P^TB \\ * & \varphi_{22} & 0 & 0 & \varphi_{25} & 2S & 0 \\ * & * & \varphi_{33} & \varphi_{34} & 0 & 0 & 0 \\ * & * & * & \varphi_{44} & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 2S & 0 \\ * & * & * & * & * & \varphi_{66} & \varphi_{67} \\ * & * & * & * & * & * & -2H \end{bmatrix}$$

$$\varphi_{11} = P^TA + A^TP + Q_1 + Q_2 - (1/\tau_m)E^T(Z_1 + Z_3)E$$

$$\varphi_{13} = (1/\tau_m)E^T(Z_1 + Z_3)E$$

$$\varphi_{22} = \sigma\Phi - 2\Lambda_1S\Lambda_0, \ \varphi_{25} = -2\Lambda_1S\Lambda_0$$

$$\varphi_{33} = -Q_1 - \left[1/\tau_m + 1/(\bar{\tau} - \tau_m)\right]E^T(Z_1 + Z_3)E$$

$$\varphi_{34} = \left[1/(\tau - \tau_m)\right]E^T(Z_2 + Z_3)E$$

$$\varphi_{44} = -Q_2 - \left[1/(\tau - \tau_m)\right]E^T(Z_2 + Z_3)E$$

$$\varphi_{55} = -\Phi - 2\Lambda_1S\Lambda_0$$

$$\varphi_{66} = -2S - 2K^T\Pi_0H\Pi_1K, \varphi_{67} = 2K^TH.$$

Proof: We first show that the networked singular system (17) is regular and impulse free. Since rank $E = r \le n$, there must exist two invertible matrices $\tilde{G}$ and $\tilde{H} \in \mathbb{R}^{n\times n}$ such that

$$\tilde{E} = \tilde{G}E\tilde{H} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \tilde{G}A\tilde{H} = \begin{bmatrix} A_{1,11} & A_{1,12} \\ A_{1,21} & A_{1,22} \end{bmatrix}.$$

Similar to the method used in [35], we know that $A_{1,22}$ is nonsingular, which implies that the pair of (E, A) is regular and impulse free, it follows that the networked singular system (17) is regular and impulse free. In the following, we will show that the networked singular system (17) is stable under the event-triggering scheme (2).

Consider when the system is free from external disturbances, with $w(t) = 0$. We define the following functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{20}$$

where

$$V_1(t) = x^T(t)E^T P x(t)$$

$$V_2(t) = \int_{t-\tau_m}^{t} x^T(s)Q_1 x(s)ds + \int_{t-\bar{\tau}}^{t} x^T(s)Q_2 x(s)ds$$

$$V_3(t) = \int_{-\tau_m}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^T Z_1 E\dot{x}(s)d\theta ds + \int_{-\bar{\tau}}^{-\tau_m}\int_{t+\theta}^{t} \dot{x}^T(s)E^T Z_2 E\dot{x}(s)d\theta ds + 2\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^T Z_3 E\dot{x}(s)d\theta ds.$$

Taking the derivative of $V(t)$ for $t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$, we introduce the free weighting matrices, i.e.,

$$\ell_1 = 2\xi^T(t)N\left[Ex(t) - Ex(t-\tau(t)) - \int_{t-\tau(t)}^{t} E\dot{x}(s)ds\right] = 0$$

$$\ell_2 = 2\xi^T(t)M\left[Ex(t-\tau(t)) - Ex(t-\bar{\tau}) - \int_{t-\bar{\tau}}^{t-\tau(t)} E\dot{x}(s)ds\right] = 0 \tag{21}$$

where $\xi^T(t) = [\eta^T(t)\ e^T(i_k h)]$, with

$$\eta^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & x^T(t-\tau_m) & x^T(t-\bar{\tau}) \end{bmatrix}$$

and N and M are matrices with appropriate dimensions. According to Lemma 1 and combining the sector bound conditions (11) and (12) with the event-triggered scheme (2), we have

$$\dot{V}(t) \le \xi^T(t)\Xi\xi(t) \tag{22}$$

where $\Xi = \varphi + \Gamma + \Gamma^T + \bar{\tau}NZ_3^{-1}N^T + \bar{\tau}MZ_3^{-1}M^T + \mathscr{A}Z\mathscr{A}^T$, with $Z = \tau_m Z_1 + (\bar{\tau} - \tau_m)Z_2 + 2\bar{\tau}Z_3$. According to Schur complement, from (19), we have

$$\begin{bmatrix} \varphi + \Gamma + \Gamma^T & \sqrt{\bar{\tau}}N & \sqrt{\bar{\tau}}M & \mathscr{A}Z \\ * & -Z_3 & 0 & 0 \\ * & * & -Z_3 & 0 \\ * & * & * & -Z \end{bmatrix} < 0 \tag{23}$$

which means $\dot{V}(t) < 0$. Therefore, system (17) is stable.

Now, we address the H_∞ performance of the networked singular system (17). Consider when the system is subject to external disturbances, with $w(t) \neq 0$. We use the following performance index:

$$\wp(t) = \int_0^\infty \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right] dt.$$

Under zero initial conditions, we have

$$\wp(t) = \int_0^\infty \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t)\right] dt - V(\infty)$$
$$\le \varsigma^T(t)\begin{bmatrix} \Xi + T_1 + T_1^T & T_2 \\ * & -\gamma^2 I + F^T F + G^T Z G \end{bmatrix}\varsigma(t)$$

with $\varsigma^T(t) = [\xi^T(t)\ \ w^T(t)]$. By Schur complement, from (19), we have

$$\begin{bmatrix} \Xi + T_1 + T_1^T & T_2 \\ * & -\gamma^2 I + F^T F + G^T Z G \end{bmatrix} < 0$$

which means $\wp(t) < 0$. That is, under zero initial conditions, for any nonzero $w(t) \in \mathcal{L}_2[0,\infty)$, the control output $z(t)$ satisfies $\|z(t)\|_2 \le \gamma\|w(t)\|_2$. This completes the proof. ■

B. Quantized State Feedback Controller Design

Based on Theorem 1, we present the codesign algorithm for the networked singular system (17) as follows.

Theorem 2: For given scalars $\gamma > 0$, $0 \le \sigma < 1$, τ_m, $\bar{\tau}$, and ρ_i $(i = 1, 2, \ldots, 5)$, the singular NCS (17) is regular, impulse free, and stable with an H_∞ performance index γ under the event-triggering scheme (2), if there exist matrices $\widetilde{Q}_1 = \widetilde{Q}_1^T > 0$, $\widetilde{Q}_2 = \widetilde{Q}_2^T > 0$, $\widetilde{Z}_i = \widetilde{Z}_i^T > 0$ $(i = 1, 2, 3)$, $\widetilde{\Phi} > 0$, $\widetilde{N}$, $\widetilde{M}$, Y, nonsingular $\widetilde{P}$ and any diagonal matrices $\widetilde{S} > 0$, $S > 0$, $H > 0$ with appropriate dimensions such that

$$\widetilde{P}^T E^T = E\widetilde{P} \ge 0 \tag{24}$$

$$\begin{bmatrix} \tilde{\Psi}_1 & \tilde{\Psi}_2 \\ * & \tilde{\Psi}_3 \end{bmatrix} < 0 \tag{25}$$

where

$$\tilde{\Psi}_1 = \begin{bmatrix} \widetilde{\varphi} + \widetilde{\Gamma} + \widetilde{\Gamma}^T & \sqrt{\bar{\tau}}\widetilde{N} & \sqrt{\bar{\tau}}\widetilde{M} & \widetilde{T}_2 & \widetilde{T}_1 \\ * & -\widetilde{Z}_3 & 0 & 0 & 0 \\ * & * & -\widetilde{Z}_3 & 0 & 0 \\ * & * & * & -\gamma^2 I & F^T \\ * & * & * & * & -I \end{bmatrix}$$

$$\tilde{\Psi}_2 = \begin{bmatrix} \widetilde{\mathscr{A}} & \widetilde{\mathscr{A}} & \widetilde{\mathscr{A}} & \widetilde{T}_3 & \widetilde{T}_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ G^T & G^T & G^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Psi}_3 = \mathrm{diag}\{\widetilde{\varrho}_1, \widetilde{\varrho}_2, \widetilde{\varrho}_3, \widetilde{\varrho}_4, \widetilde{\varrho}_5\}$$

$$\widetilde{\varrho}_1 = \frac{1}{\tau_m\rho_1^2}(\widetilde{Z}_1 - 2\rho_1 E\widetilde{P}),\ \widetilde{\varrho}_2 = \frac{1}{(\bar{\tau}-\tau_m)\rho_2^2}(\widetilde{Z}_2 - 2\rho_2 E\widetilde{P})$$

$$\widetilde{\varrho}_3 = \frac{1}{2\bar{\tau}\rho_3^2}(\widetilde{Z}_3 - 2\rho_3 E\widetilde{P}),\ \widetilde{\varrho}_4 = \frac{1}{2}(\rho_4^2 S - 2\rho_4 I)$$

$$\widetilde{\varrho}_5 = \frac{1}{2}\left(\rho_5^2 H - 2\rho_5 I\right)$$

$$\widetilde{T}_1 = \begin{bmatrix} C\widetilde{P} & DY & 0 & 0 & DY & DY & D \end{bmatrix}^T$$

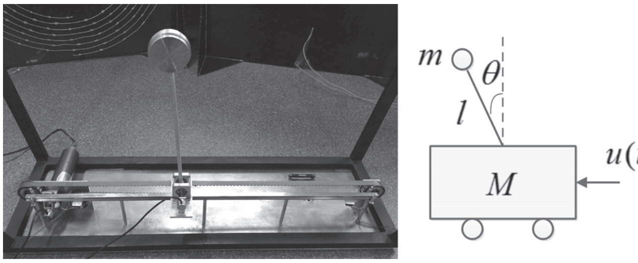
$$\widetilde{T}_2 = \begin{bmatrix} G^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\widetilde{T}_3 = \begin{bmatrix} 0 & \Lambda\widetilde{P}^T & 0 & 0 & \Lambda\widetilde{P}^T & 0 & 0 \end{bmatrix}^T$$

$$\widetilde{T}_4 = \begin{bmatrix} 0 & \Pi Y & 0 & 0 & \Pi Y & \Pi Y & 0 \end{bmatrix}^T$$

$$\widetilde{\mathscr{A}} = \begin{bmatrix} A\widetilde{P} & BY & 0 & 0 & BY & BY & B \end{bmatrix}^T$$

$$\widetilde{\Gamma} = \begin{bmatrix} \widetilde{N} & -\widetilde{N} + \widetilde{M} & 0 & \widetilde{M} & 0 & 0 & 0 \end{bmatrix}$$



(a) (b)

Fig. 2. Networked inverted pendulum. (a) Inverted pendulum in the laboratory at The University of Adelaide. (b) Schematic of the inverted pendulum.

$$\widetilde{\varphi} = \begin{bmatrix} \widetilde{\varphi}_{11} & BY & \widetilde{\varphi}_{13} & 0 & BY & BY & B \\ * & \sigma\widetilde{\Phi} & 0 & 0 & 0 & 0 & 0 \\ * & * & \widetilde{\varphi}_{33} & \widetilde{\varphi}_{34} & 0 & 0 & 0 \\ * & * & * & \widetilde{\varphi}_{44} & 0 & 0 & 0 \\ * & * & * & * & -\widetilde{\Phi} & 0 & 0 \\ * & * & * & * & * & -2\widetilde{S} & 0 \\ * & * & * & * & * & * & -2H \end{bmatrix}$$

$$\widetilde{\varphi}_{11} = A\widetilde{P} + \widetilde{P}A^T + \widetilde{Q}_1 + \widetilde{Q}_2 - (1/\tau_m)(\widetilde{Z}_1 + \widetilde{Z}_3)$$

$$\widetilde{\varphi}_{13} = (1/\tau_m)(\widetilde{Z}_1 + \widetilde{Z}_3),$$

$$\widetilde{\varphi}_{33} = -\widetilde{Q}_1 - (1/\tau_m), (\widetilde{Z}_1 + \widetilde{Z}_3) - [1/(\bar{\tau} - \tau_m)]\,(\widetilde{Z}_2 + \widetilde{Z}_3)$$

$$\widetilde{\varphi}_{34} = [1/(\bar{\tau} - \tau_m)]\,(\widetilde{Z}_2 + \widetilde{Z}_3),$$

$$\widetilde{\varphi}_{44} = -\widetilde{Q}_2 - [1/(\bar{\tau} - \tau_m)]\,(\widetilde{Z}_2 + \widetilde{Z}_3).$$

Furthermore, a desired state feedback controller gain is

$$K = Y\widetilde{P}^{-1}. \tag{26}$$

Proof: Similar to the method used in [4, Th. 2], Theorem 2 can be proved.

IV. Examples

We use two examples to demonstrate the effectiveness of the proposed method. The first example is a networked regular system to show less conservatism of our results, whereas the second example is a networked singular system to show the effectiveness in reducing the network usage of the proposed method.

Example 1: Consider an inverted pendulum on a cart controlled over a network. The schematic of an inverted pendulum is shown in Fig. 2, and the linearized plant model (1) is characterized by the following parameters [3] and [37]:

$$E = I, \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml} \end{bmatrix}$$

where $M = 10$ kg is the cart mass, $m = 1$ kg is the mass of the pendulum bob, $l = 3$ m is the length of the pendulum arm, and $g = 10$ m/s^2 is the gravitational acceleration.

Since the eigenvalues of A are $\{0, 0, 1.8257, -1.8257\}$, the system is unstable without a controller. The state variables x_i $(i = 1, 2, 3, 4)$ are the cart position, the cart velocity, the pendulum bob angle, and the pendulum bob angular velocity. The initial state vector is set as $x_0(t) = [1.5\ \ -0.5\ 0.8\ \ -1]^T$.

We consider two cases with different parameters.

Case 1—H_∞ Control Without Quantizations:

$$C = G^T = F^T = [1 \quad 1 \quad 1 \quad 1], \quad D = 0.1, \quad \Lambda = \Pi = 0$$

$$w(t) = \begin{cases} \operatorname{sgn}(\sin(t)), & \text{if } t \in [0, 10] \\ 0, & \text{others.} \end{cases}$$

Case 2—H_∞ Control With Quantizations:

$$C = G^T = F^T = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \quad D = 0.1$$

$$w(t) = \begin{cases} \operatorname{sgn}(\sin(t)), & \text{if } t \in [0, 10] \\ 0, & \text{others} \end{cases}$$

and the parameters for the quantizer $f(\cdot)$ are assumed to be $\alpha_1 = \alpha_3 = 0.9$ and $\alpha_2 = \alpha_4 = 0.8$, that is

$$\Lambda = \begin{bmatrix} 0.0526 & 0 & 0 & 0 \\ 0 & 0.1111 & 0 & 0 \\ 0 & 0 & 0.0526 & 0 \\ 0 & 0 & 0 & 0.1111 \end{bmatrix}$$

whereas the quantized density of $g(\cdot)$ is assumed to be $\alpha_1 = 0.9$, that is, $\Pi = 0.0526$.

In Case 1, under the conditions of $h = 0.01$, $\sigma = 0.1$, $\bar{\tau} = 0.16$, $\rho_1 = \rho_2 = \rho_3 = 0.46$, and $\rho_4 = \rho_5 = 0.23$, the H_∞ performance index in [3] is $\gamma = 200$. In our scheme, according to Theorem 2 and setting $\tau_m = 0.01$, the minimum of H_∞ performance index $\gamma_{\min} = 85$. The correspondent feedback gain K_2 and the event-triggering matrix Φ_5 are

$$K_2 = \begin{bmatrix} 5.8955 & 16.2858 & 334.4121 & 186.8863 \end{bmatrix}$$

$$\Phi_5 = \begin{bmatrix} 4.2235 & -4.6241 & -18.8239 & 33.11503 \\ -4.6241 & 12.5586 & 44.1248 & -78.4754 \\ -18.8239 & 44.1248 & 170.8534 & -302.9154 \\ 33.1150 & -78.4754 & -302.9154 & 537.1290 \end{bmatrix}.$$

The state responses $x(t)$ and release instants are shown in Fig. 3 for this setting. The number of triggers is 86 times.

In Case 2, the effect of two quantizers is considered. We set $\bar{\tau} = 0.24$, $\tau_m = 0.01$, $\rho_1 = \rho_2 = \rho_3 = 0.44$, and $\rho_4 = \rho_5 = 0.21$, and Table I gives the different results for different triggered parameter values of σ. It shows that the larger the parameter

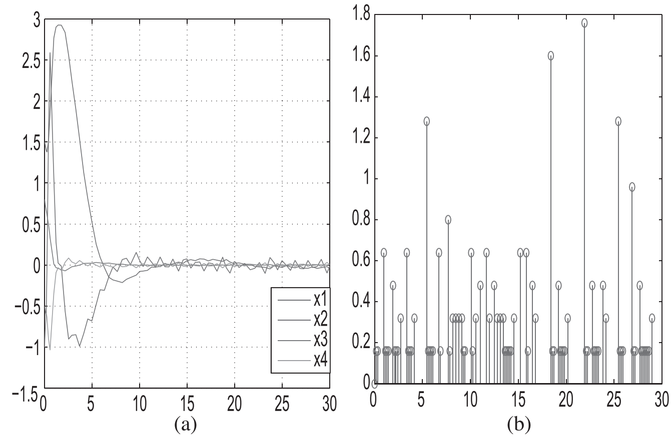


Fig. 3. State response $x(t)$ and release instants under K_2 and Φ_5. (a) State response $x(t)$. (b) Release instants.

TABLE I
$\gamma_{\min}$, K_3, AND Φ_6 FOR DIFFERENT σ VALUES

σ	0	0.01	0.02	0.03
γ_{min}	65	68	72	75
K_3	$K_{3,1}$	$K_{3,2}$	$K_{3,3}$	$K_{3,4}$
Φ_6	$\Phi_{6,1}$	$\Phi_{6,2}$	$\Phi_{6,3}$	$\Phi_{6,4}$

σ, the larger the minimum value of γ. Other parameters and values in Table I are

$$K_{3,1} = \begin{bmatrix} 4.9257 & 13.6401 & 283.3992 & 158.1448 \end{bmatrix}$$

$$K_{3,2} = \begin{bmatrix} 5.3323 & 14.8106 & 302.2490 & 168.8918 \end{bmatrix}$$

$$K_{3,3} = \begin{bmatrix} 5.0948 & 14.5901 & 304.2014 & 169.9699 \end{bmatrix}$$

$$K_{3,4} = \begin{bmatrix} 4.7482 & 14.0801 & 301.9530 & 168.6491 \end{bmatrix}$$

$$\Phi_{6,1} = 10^2 \times \begin{bmatrix} 2.5368 & -0.0000 & -0.0000 & 0.0000 \\ -0.0000 & 2.5368 & 0.0000 & -0.0000 \\ -0.0000 & 0.0000 & 2.5368 & -0.0000 \\ 0.0000 & -0.0000 & -0.0000 & 2.5368 \end{bmatrix}$$

$$\Phi_{6,2} = \begin{bmatrix} 17.0000 & -8.8000 & -36.6000 & 63.7000 \\ -8.8000 & 35.9000 & 117.1000 & -209.4000 \\ -36.6000 & 117.1000 & 522.9000 & -927.5000 \\ 63.7000 & -209.4000 & -927.5000 & 1645.3000 \end{bmatrix}$$

$$\Phi_{6,3} = \begin{bmatrix} 7.8968 & -4.4337 & -21.8893 & 38.1572 \\ -4.4337 & 16.3175 & 54.8386 & -97.9276 \\ -21.8893 & 54.8386 & 253.9125 & -449.5681 \\ 38.1572 & -97.9276 & -449.5681 & 796.1969 \end{bmatrix}$$

$$\Phi_{6,4} = \begin{bmatrix} 4.7845 & -2.7674 & -14.0746 & 24.5370 \\ -2.7674 & 10.0585 & 33.4759 & -59.7880 \\ -14.0746 & 33.4759 & 155.5644 & -275.2709 \\ 24.5370 & -59.7880 & -275.2709 & 487.2295 \end{bmatrix}.$$

Fig. 4 shows the state responses $x(t)$ and release instants. Over the simulation period, there are 82 triggers. We remark that if a time-triggered scheme is used instead, the number of triggers will be 3000 times. The result is a clear indication that our event-triggered approach is efficient in terms of utilizing the network bandwidth resource.

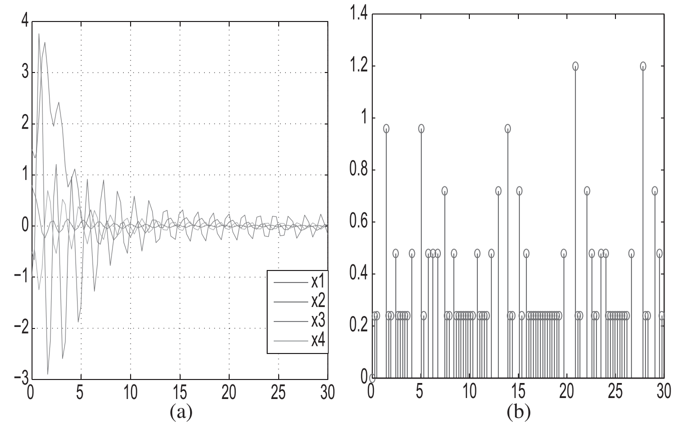


Fig. 4. State response $x(t)$ and release instants under $K_{3,4}$. (a) State response $x(t)$. (b) Release instants.

Example 2: This example illustrates the quantized H_∞ control on a singular NCS. Consider the singular NCS (1). The associated parameters are

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.3 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ 0.033 \end{bmatrix}$$

$$C = G^T = F^T = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, D = 0.1$$

$$w(t) = \begin{cases} \operatorname{sgn}(\sin(t)), & \text{if } t \in [0, 10] \\ 0, & \text{others.} \end{cases}$$

The parameters for the quantizer $f(\cdot)$ are taken as $\alpha_1 = \alpha_3 = 0.9$ and $\alpha_2 = \alpha_4 = 0.8$. We set $\bar{\tau} = 0.24$, $\tau_m = 0.01$, $\rho_1 = \rho_2 = \rho_3 = 0.44$, $\rho_4 = \rho_5 = 0.21$, and $\sigma = 0.02$, and according to Theorem 2, the minimum H_∞ performance index $\gamma_{\min} = 51.36$. The corresponding feedback gain and the event-triggered matrix are

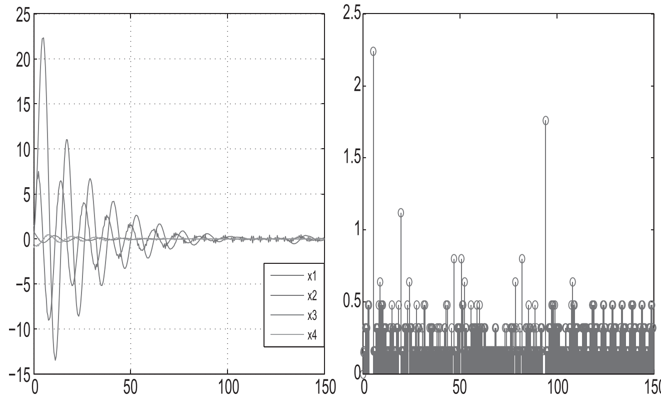
$$K_5 = \begin{bmatrix} 2.5480 & 9.3207 & 237.6465 & 161.8992 \end{bmatrix}$$

$$\Phi_8 = \begin{bmatrix} 2.0941 & -0.0477 & -0.0049 & -0.2502 \\ -0.0477 & 1.7747 & -0.2384 & -0.1521 \\ -0.0049 & -0.2384 & 0.0713 & -0.0431 \\ -0.2502 & -0.1521 & -0.0431 & 0.1580 \end{bmatrix}.$$

Furthermore, the initial state is $x_0(t) = [1.5 \; -0.5 \; 0.8 \; -1]^T$, and the state responses for $x(t)$ and the release instants are shown in Fig. 5. We observe that the number of triggers is 627, which is much lower than 15 000 triggers when using the time-triggered scheme. The result again demonstrates the capability of the event-triggered approach in reducing the network bandwidth usage.

V. CONCLUSION

Aiming to reduce the load of network communication, the problem of event-triggered H_∞ control for networked singular



(a) (b)

Fig. 5. State response $x(t)$ and release instants under K_5. (a) State response $x(t)$. (b) Release instants.

systems with quantizations in both the measured states and the generated control inputs has been studied in this paper. By considering the characteristics of event-triggered schemes and taking the quantizations into account, we presented a new time-delay model. Based on this model, we derived a new H_∞ performance criterion that guarantees that the closed-loop system of the singular networked system is regular, impulse free, and stable with a prescribed H_∞ performance index γ. The codesign of the event-triggered condition and the state feedback controller has also been derived based on a free-weighting-matrix approach. Two examples have been given to show the effectiveness of the theoretical results obtained.

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