



**THERMALLY INDUCED VIBRATIONS OF VISCOELASTIC PLATES
AND SHALLOW SHELLS OF ARBITRARY SHAPE**

by

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SUMMARY

The work presented in this thesis involves an analysis of the thermally induced vibration of viscoelastic plates and shallow shells of arbitrary shape. The viscoelastic stress-strain law is expressed in a linear operator form and a variety of different models are considered. The analysis is based on the method of contour lines. In all cases, solutions are found by separation into two sets of simultaneous ordinary differential equations, one set in the contour line variable and the other in a time variable.

In Chapter 1, a brief introduction to the study of thermally induced vibrations of plates and shallow shells is given and the purpose and scope of the thesis presented.

In Chapter 2, the heat conduction problem for plates and shallow shells of arbitrary shape is detailed. A method for solving this problem for plates and shallow shells which have linear temperature variation through their thickness is presented and demonstrated by example. A conformal mapping approach to the problem of finding contour line functions is also developed.

In Chapter 3, the viscoelastic stress-strain laws are discussed and a method for studying the thermally induced small amplitude vibration of viscoelastic plates of arbitrary shape is then developed. As an illustrative example, the thermally induced vibration of a rectangular plate of Kelvin material is studied.

The theories of linear elasticity are quite often inadequate in describing the behaviour of many modern day materials. For this reason, the methods used for the thermally induced small amplitude vibration are extended in Chapter 4 to enable a study of the thermally induced large amplitude vibration of viscoelastic plates. The Galerkin method is employed to find approximate solutions to a nonlinear

ordinary differential equation involving the contour line variable. The thermally induced vibration of a cardioidal shaped plate of material which can be represented by the Standard Linear Solid model, is studied as an example.

In Chapter 5, the method of Chapter 3 is extended to study the thermally induced small amplitude vibration of viscoelastic shallow shells. For illustration, the thermally induced vibration of an elliptical shallow shell of Maxwell material is studied.

Finally, in Chapter 6, the thermally induced large amplitude vibration of viscoelastic shallow shells is studied. In particular, the thermally induced vibration of a circular shallow shell of Kelvin material is examined.

Some concluding remarks are made in Chapter 7 and also some suggestions for further research in this area are given.

SIGNED STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University.

To the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

I consent to this thesis being made available for photocopying and loan.

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