Large-Scale-Structure in the Pierre Auger Observatory Data Directions

“The Long Dark Teatime of the Soul”

J. Sorokin
B.Sc. (Honours), Physics

A thesis presented to the University of Adelaide
for admission to the degree of
Doctor of Philosophy.

School of Physical Sciences
Department of Physics
April 2016

1 with thanks for the kind permission of Jane Belson
I wish to thank

my supervisor, Professor Roger Clay for hanging in there
Dr Jonathan Woithe for his kind and obliging help with my various idl problems
Dr Benjamin Whelan for his thoughtful edits
my friend Ramona Adjoran for fixing my hardware problems
Dr Millie Vukovic and Dr Elizabeth Heath for their support
and
my legion of fans
My wonderful mother Yolande
my unexpected husband Michael and my singular brother Adam
fabulous Millie Butler and interesting Cyrus Masters

Thankyou, thankyou,
I love you all
Abstract

This thesis presents a method of analysis of Pierre Auger Observatory Cosmic Ray (CR) directions. I look for evidence of large-scale-structure within these CR directions. I have associated directional events by virtue of the angular proximity of their arrival directions, and within three energy ranges around the Greisen-Zatsepin-Kusmin (GZK) energy limit. I design graph theoretical algorithms to grow minimum spanning trees for these directional events and then 'cut' the trees along certain galactic longitudes and latitudes into ‘branches’, where I expect the galactic magnetic fields or cosmic ray point-sources to exhibit behaviours or patterns, which can be interpreted by branch features. 1,200 simulated CR directions in each energy range provide some statistical context for the Pierre Auger Observatory branch features which may be considered significant. The thesis is a preliminary study of a method of analysis of ‘regions of interest’ which later may be optimized with a full statistical ‘tuning’ analysis.
Statement of Originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library catalogue and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.
Contents

1 An Introduction  1
  1.1 Background  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  3

2 EAS and Two Key Descriptors of Cosmic Rays  7
  2.1 Extensive Air Showers  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  8
    2.1.1 EM Cascades  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  11
    2.1.2 Proton Cascade  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  13
    2.1.3 Mixed Composition  . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  16
  2.2 Composition  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  17
    2.2.1 Before the Knee $\sim (10^6 - 10^{14})$ eV  . . . . . . . . . . . . . . .  18
    2.2.2 Knee to Ankle $\sim (2 \times 10^{15} - 10^{18})$ eV  . . . . . . . . . .  20
    2.2.3 From the Ankle and Beyond $\sim (10^{18} - 10^{21})$ eV  . . . . . . .  26
    2.2.4 GZK Astrophysical Models  . . . . . . . . . . . . . . . . . . . . . . . . .  29
  2.3 Energy Spectrum  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  35
  2.4 EAS Measurements  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  39
  2.5 Pierre Auger Observatory Design  . . . . . . . . . . . . . . . . . . . . . . . . .  41
    2.5.1 Conditions  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  44
    2.5.2 Some SD Details  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  46
    2.5.3 Some FD Details  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  48
    2.5.4 Some Hybrid Event Details  . . . . . . . . . . . . . . . . . . . . . . . . .  51
  2.6 Pierre Auger Observatory Enhancements:  . . . . . . . . . . . . . . . . . . . .  52
2.6.1 AMIGA ................................. 52
2.6.2 HEAT ................................. 52
2.6.3 AERA ................................. 53
2.6.4 Pierre Auger Observatory Prime Upgrade ......................... 53
2.7 The Pierre Auger Observatory Prescription: ......................... 53
   2.7.1 Shuffled Data Sets .......................... 55

3 Galactic Magnetic Field Models 56
   3.1 Lorentz Force .............................. 58
   3.2 Magnetic Field Measuring Techniques .............................. 60
   3.3 Galactic Anisotropies and the Need for a Halo .................. 61
   3.4 Extra-Galactic Sources .......................... 63
       3.4.1 Generic Galactic Magnetic Field Models .................. 68
       3.4.2 Galaxy Volume .............................. 71
       3.4.3 Generalized GMF Model of Jannson and Farrar ............. 77

4 Anisotropies for $E \geq 10^{17}$ eV 87
   4.1 Signal Data Sets .............................. 87
   4.2 Optimizations and Signal Data Sets .............................. 88
   4.3 Anisotropy Results for $E \geq 10^{17}$ eV ....................... 89
       4.3.1 Familiar Methods ............................ 90
       4.3.2 Catalogue Searches ........................... 93
       4.3.3 Energy-Energy-Ordering ........................ 97
       4.3.4 Self Correlation Methods ....................... 109
   4.4 Predicted Anisotropies Around $10^{18}$ eV .................... 111
       4.4.1 Galactic/Extra-Galactic Energy Constraints ............... 113
   4.5 Discussion ..................................... 117

5 Minimum Spanning Tree Theory 118
   5.1 Introduction ................................... 118
## CONTENTS

5.2 Theory of Graphs ................................................................. 120  
  5.2.1 Trees ................................................................. 122  
5.3 Recursion ................................................................. 125  
5.4 Analysis of Algorithms ......................................................... 126  
  5.4.1 Proof of Correctness ......................................................... 127  
  5.4.2 Order of Growth ......................................................... 127  
5.5 Minimum Spanning Trees ......................................................... 131  
5.6 Two Classic Algorithms ......................................................... 135  
  5.6.1 Prim's Algorithm ......................................................... 135  
  5.6.2 Kruskal's Algorithm ......................................................... 138  
5.7 A Fast Minimum Spanning Tree ..................................................... 140  
  5.7.1 Construction ................................................................. 142  
  5.7.2 Our Multifragment MST: The Yggdrasil .................................. 143  

6 Method of Analysis ................................................................. 148  
  6.1 Gaussian Density Contours ......................................................... 148  
    6.1.1 Clusters and Sub-Trees ......................................................... 149  
  6.2 Divisions in the Galactic Plane and Branches ................................... 154  
    6.2.1 Branch Thetas ................................................................. 155  
    6.2.2 Branch Node Sums ................................................................. 160  
  6.3 Shuffled Distribution Types ......................................................... 161  
  6.4 Revision of the Galactic Disk bBand Division of [−15 : 15]° .................................. 164  
  6.5 Generation of Simulated Data-Sets ..................................................... 166  
    6.5.1 Distributions of Simulated Data and Statistics .................................. 168  

7 Tabulated Results ................................................................. 179  
  7.1 The Setting of Extreme Branch Variables ........................................ 182  
  7.2 Allowed Galactic ι and b Branch-Walk-Pair  
    Similarities and Branch Θ Regions of Interest .................................. 184
CONTENTS

7.2.1 Individual Energy Range Tables ........................................ 186
7.2.2 Final Energy Comparison Tables ...................................... 186
7.3 How Effective is the Yggdrasil in Capturing Lines? .................. 188
7.4 Tables With No Galactic Longitude Divisions .......................... 192
  7.4.1 $40 \text{ EeV} < E \leq 50 \text{ EeV}$ ........................................ 192
  7.4.2 $50 \text{ EeV} < E \leq 60 \text{ EeV}$ ........................................ 192
  7.4.3 $E > 60 \text{ EeV}$ .................................................. 192
  7.4.4 Energy Comparison Table 1 .......................................... 196
7.5 Galactic Longitude Filters, both $[-135 : 135]^\circ$ and $[-45 : 45]^\circ$ ........... 200
  7.5.1 $40 \text{ EeV} < E \leq 50 \text{ EeV}$ ........................................ 200
  7.5.2 $50 \text{ EeV} < E \leq 60 \text{ EeV}$ ........................................ 200
  7.5.3 $E > 60 \text{ EeV}$ .................................................. 200
  7.5.4 Energy Comparison Table 2 .......................................... 204
7.6 Galactic Longitude Filters, both $[-135 : -45]^\circ$ and $[45 : 135]^\circ$ ............. 208
  7.6.1 $40 \text{ EeV} < E \leq 50 \text{ EeV}$ ........................................ 208
  7.6.2 $50 \text{ EeV} < E \leq 60 \text{ EeV}$ ........................................ 208
  7.6.3 $E > 60 \text{ EeV}$ .................................................. 208
  7.6.4 Energy Comparison Table 3 .......................................... 212
7.7 Tables with the Galactic Longitude Divisions of $\Delta|l_1 - l_2| > 90^\circ$ ............... 214
  7.7.1 $40 \text{ EeV} < E \leq 50 \text{ EeV}$ ........................................ 214
  7.7.2 $50 \text{ EeV} < E \leq 60 \text{ EeV}$ ........................................ 214
  7.7.3 $E > 60 \text{ EeV}$ .................................................. 215
  7.7.4 Energy Comparison Table 4 .......................................... 219
7.8 Exposure, Positive $\Theta$s and $\Theta$s from Similar Branches ...................... 222

8 Conclusion ............................................................................. 225

Appendices ............................................................................. 232
# CONTENTS

A  South and North $40\text{ EeV} < E \leq 50 \text{ EeV:}$  
   A.1 Yggdrasil Equatorial ($\text{RA,dec}$) Co-ordinates  
   A.2 Yggdrasil Galactic ($l,b$) Co-ordinates.  
   A.3 bBands  
   A.4 Galactic Longitude Quadrants and bBands  
   A.5 bBands where $\Delta |l_1 - l_2| > 90^\circ$  

B  South and North $50\text{ EeV} < E \leq 60 \text{ EeV:}$  
   B.1 Yggdrasil Equatorial ($\text{RA,dec}$) Co-ordinates  
   B.2 Yggdrasil Galactic ($l,b$) Co-ordinates.  
   B.3 bBands  
   B.4 Galactic Longitude Quadrants and bBands  
   B.5 bBands where $\Delta |l_1 - l_2| > 90^\circ$  

C  South and North $E > 60 \text{ EeV:}$  
   C.1 Yggdrasil Equatorial ($\text{RA,dec}$) Co-ordinates  
   C.2 Yggdrasil Galactic ($l,b$) Co-ordinates.  
   C.3 bBands  
   C.4 Galactic Longitude Quadrants and bBands  
   C.5 bBands where $\Delta |l_1 - l_2| > 90^\circ$  

References
## List of Figures

1.1 energy spectrum of CRs ................................................. 5  
2.1 Longitudinal EAS development for proton and iron ................. 10  
2.2 \( \langle X_{\text{max}} \rangle \) vs Energy ........................................... 15  
2.3 refractory nuclides .................................................. 20  
2.4 elemental and all-particle energy density flux ......................... 23  
2.5 frequency distribution of \( \log N_e \) vs \( \log N_{\mu}^{tr} \) ............. 24  
2.6 KASCADE-Grande all particle energy spectrum ....................... 25  
2.7 proton energy loss lengths .......................................... 31  
2.8 energy loss lengths for oxygen and iron ............................ 32  
2.9 O and Fe spectra and EGMF ....................................... 33  
2.10 Spectrum and \( \langle X_{\text{max}} \rangle \) eV for mixed nuclei, \( E = 10^{20.5} \) eV .......... 34  
2.11 cosmological evolution scenarios with respect to Energy spectrum .... 35  
2.12 all particle energy density spectrum ................................ 36  
2.13 exposures of UHECR arrays ........................................ 38  
2.14 Pierre Auger Observatory exposure .................................. 41  
2.15 view of Pierre Auger Observatory in 2009 ............................ 45  
2.16 SD example .......................................................... 48  
2.17 FD telescope ......................................................... 49  
3.1 proton sites at \( E > 10^{20} \) eV ......................................... 65  
3.2 proton sites at \( E \sim 10^{20} \) eV incorporating geometrical and radiative losses. ...... 69
LIST OF FIGURES

3.3 iron sites at $E \sim 10^{20}$ eV .............................................. 70
3.4 Disk and halo components of GMF ........................................ 72
3.5 The X-field ................................................................. 77
3.6 GMF as seen in x-y slices .................................................. 80
3.7 The predicted field strength of the optimized GMF model .............. 81
3.8 The X-field ................................................................. 82
3.9 Predicted 60 EeV proton deflections colour bar ............................ 85
3.10 S-PASS, linearly polarized intensity, $P_I$ at 2.3 GHz ................. 86

4.1 upper limits on the anisotropy amplitude taken in the first harmonic .... 92
4.2 Pierre Auger Observatory arrival directions for $E \geq 55$ EeV ............ 96
4.3 Arrival directions of iron nuclei from the Virgo cluster .................... 102
4.4 hough transform of CR directions on an arc .............................. 104
4.5 Great circle counting bins .................................................. 105
4.6 magnetic spectrometer geometry ........................................... 106
4.7 proton and mixed composition flux ....................................... 107
4.8 MSA $\zeta_{i,j}$ skymap of 69 arrival directions. ............................ 109
4.9 Predicted Amplitudes for turbulent field dipole profile 1 and profile 2 114
4.10 Predicted dipole Amplitudes versus turbulent galactic field strength .... 116

5.1 acyclic, non acyclic graph example ........................................ 121
5.2 a MST example .............................................................. 123
5.3 Cycle and cut properties of MSTs ....................................... 133
5.4 illustrated proof by contradiction of cycle property ........................ 134
5.5 illustrated proof by contradiction of cut property .......................... 134
5.6 Prim’s MST ................................................................. 137

6.1 Density contours in RA vs dec for all 952 events $20$ EeV $< E \leq 30$ EeV (current to 2/8/2010) ......................... 152
6.2 Density contours of captured events in RA vs dec for 20 EeV < E ≤ 30 EeV (current to 2/8/2010)  

6.3 Sub-trees of RA vs dec in 20 EeV < E ≤ 30 EeV (current to 2/8/2010).  

6.4 Ygg of RA vs dec for events in 20 EeV < E ≤ 30 EeV (current to 2/8/2010).  

6.5 Ygg of galactic l vs galactic b for 20 EeV in < E ≤ 30 EeV (current to 2/8/2010).  

6.6 Skymap of original Pierre Auger Observatory CR directions for 27 events with E > 57 EeV.  

6.7 Test horizontal line in South Ygg (current to 8/11/2012).  

6.8 Flat skymap of Pierre Auger Observatory branch Θ ROIs. (current to 8/11/2012).  

6.9 Pierre Auger Observatory data branches of the Ygg in galactic l vs b for events E > 60 EeV (current to 2/8/2010).  

6.10 2012 extension for E > 60 EeV to 2010 data and Figure 6.9.  

6.11 Example of southern galactic shuffle density contour of RA vs dec for 131 events in 30 EeV < E ≤ 40 EeV (current to 8/11/2012).  

6.12 Type 1 distribution of sets of shuffled branch Θ’s vs branch Θ numbers for 40 EeV < E ≤ 50 EeV (current to 8/11/2012).  

6.13 Type 2 distribution of sets of branch event numbers vs branch event number frequency for 40 EeV < E ≤ 50 EeV (current to 8/11/2012).  

6.14 Type 3 shuffled distribution of shuffles vs shuffled branch sums for 40 EeV < E ≤ 50 EeV (current to 8/11/2012).  

7.1 Example 1: Good capture of six linear events 10° apart.  

7.2 Example 2: Poor capture of six linear events 10° apart.  

7.3 Lines in North Ygg (current to 8/11/2012).  

7.4 Lines in South Ygg (current to 8/11/2012).  

7.5 Flat skymap of Pierre Auger Observatory data, for CR events 12 EeV < E ≤ 15 EeV (current to 8/11/2012).  

7.6 Flat skymap of Pierre Auger Observatory selected BBand branches of interest. For 40 EeV < E ≤ 50 EeV (current to 8/11/2012).
LIST OF FIGURES

7.7 Flat skymap of Pierre Auger Observatory data BBand branches of interest. For 
50 EeV < E ≤ 60 EeV (current to 8/11/2012). 223
7.8 Flat skymap of Pierre Auger Observatory selected branches of interest. For E > 
60 EeV (current to 8/11/2012). 224
8.1 Flat skymap of Pierre Auger Observatory branch Θ with a positive result. (current to 
8/11/2012). 227
A.1 South Yggdrasil of all events in RA vs dec for 40 EeV < E ≤ 50 EeV (current to 
8/11/2012). 233
A.2 North Yggdrasil of all events in RA vs dec for 40 EeV < E ≤ 50 EeV (current to 
8/11/2012). 234
A.3 South Yggdrasil of all events in l vs b for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 235
A.4 North Yggdrasil of all events in l vs b for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 236
A.5 South bBand branches for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 237
A.6 North bBand branches for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 238
A.7 bBand shuffle Θ node vs Θ for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 239
A.8 bBand shuffle node frequency vs nodes for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 240
A.9 bBand shuffle nodes vs shuffles for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 241
A.10 bBand shuffle Θ nodes vs shuffled Θ where Gal l ∈ ([−45 : 45], [−135 : 135])° for 40 EeV 
< E ≤ 50 EeV (current to 8/11/2012). 242
A.11 bBand shuffle node frequency vs shuffled node number where Gal l ∈ ([−45 : 45], [−135 : 135])° for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 243
A.12 bBand shuffle nodes vs shuffles where Gal l ∈ ([−45 : 45], [−135 : 135])° for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 244
A.13 bBand shuffle Θ nodes vs shuffled Θ where Gal l ∈ ([−45 : −135], [45 : 135])° for 40 EeV < E ≤ 50 EeV (current to 8/11/2013). 245
A.14 bBand shuffle node frequency vs shuffled node number where Gal l ∈ ([−45 : 
−135], [45 : 135])° for 40 EeV < E ≤ 50 EeV (current to 8/11/2012). 246
A.15 *b*Band shuffle nodes vs shuffles where Gal $l \in ([−45 : −135], [45 : 135])^\circ$ for 40 EeV < $E$ ≤ 50 EeV (current to 8/11/2012). ................................. 247

A.16 *b*Band shuffle $\Theta$ nodes vs $\Theta$ where $\Delta|l_1 - l_2| > 90^\circ$ for 40 EeV < $E$ ≤ 50 EeV: (current to 8/11/2012). .................................................. 248

A.17 *b*Band freq shuffle nodes vs shuffle node number where $\Delta|l_1 - l_2| > 90^\circ$ for 40 EeV < $E$ ≤ 50 EeV (current to 8/11/2012). .................................................. 249

A.18 *b*Band shuffle nodes vs shuffles where $\Delta|l_1 - l_2| > 90^\circ$ for 40 EeV < $E$ ≤ 50 EeV (current to 8/11/2012). .................................................. 250

B.1 South Yggdrasil of all events in RA vs dec for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2012). .................................................. 251

B.2 North Yggdrasil of all events in RA vs dec for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2012). .................................................. 252

B.3 South Yggdrasil of all events in $l$ vs $b$ for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2013). 253

B.4 North Yggdrasil of all events in $l$ vs $b$ for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2012). 254

B.5 South *b*Band branches for 60 EeV ≥ $E$ > 50 EeV (current to 8/11/2012). ................................. 255

B.6 North *b*Band branches for 60 EeV ≥ $E$ > 50 EeV (current to 8/11/2012). ................................. 256

B.7 *b*Band shuffle $\Theta$ node vs $\Theta$ for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2012). ................................. 257

B.8 *b*Band shuffle node frequency vs nodes for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2012). ................................. 258

B.9 *b*Band shuffle nodes vs shuffles for 50 EeV > $E$ ≤ 60 EeV (current to 8/11/2012). ................................. 259

B.10 *b*Band shuffle $\Theta$ nodes vs $\Theta$ where Gal $l \in ([−45 : 45], [−135 : 135])^\circ$ for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2012). ................................. 260

B.11 *b*Band shuffle node frequency vs shuffled node number where Gal $l \in ([−45 : 45], [−135 : 135])^\circ$ for 60 EeV ≥ $E$ > 50 EeV (current to 8/11/2012). ................................. 261

B.12 *b*Band shuffle nodes vs shuffles where Gal $l \in ([−45 : 45], [−135 : 135])^\circ$ for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2012). ................................. 262

B.13 *b*Band shuffle $\Theta$ nodes vs shuffled $\Theta$ where Gal $l \in ([−45 : −135], [45 : 135])^\circ$ for 50 EeV < $E$ ≤ 60 EeV (current to 8/11/2012). ................................. 263
LIST OF FIGURES

B.14 bBand shuffle node frequency vs shuffled node number where Gal $l \in ([−45 : 45], [−135 : 135])^o$ for 50 EeV < E ≤ 60 EeV (current to 8/11/2012). 

B.15 bBand shuffle nodes vs shuffles where Gal $l \in ([−45 : −135], [45 : 135])^o$ for 50 EeV < E ≤ 60 EeV (current to 8/11/2012). 

B.16 bBand shuffle Θ nodes vs Θ where Δ|$l_1$ − $l_2$| > 90° for 50 EeV < E ≤ 60 EeV (current to 8/11/2012). 

B.17 bBand freq shuffle nodes vs shuffle node number where Δ|$l_1$ − $l_2$| > 90° for 50 EeV < E ≤ 60 EeV (current to 8/11/2012). 

B.18 bBand shuffle nodes vs shuffles where Δ|$l_1$ − $l_2$| > 90° for 50 EeV < E ≤ 60 EeV (current to 8/11/2012). 

C.1 South Yggdrasil of all events in RA vs dec for E > 60 EeV (current to 8/11/2012) . 

C.2 North Yggdrasil of all events in RA vs dec for E > 60 EeV (current to 8/11/2012) . 

C.3 South Yggdrasil of all events in l vs b for E > 60 EeV current to 8/11/2012 . 

C.4 North Yggdrasil of all events in l vs b for E > 60 EeV (current to 8/11/2012) . 

C.5 South bBand branches for E > 60 EeV (current to 8/11/2012) . 

C.6 North bBand branches for E > 60 EeV (current to 8/11/2012) . 

C.7 bBand shuffle Θ node vs Θ for E > 60 EeV (current to 8/11/2012) . 

C.8 bBand shuffle node frequency vs nodes for E > 60 EeV (current to 8/11/2012) . 

C.9 bBand shuffle nodes vs shuffles for E > 60 EeV (current to 8/11/2012) . 

C.10 bBand shuffle Θ nodes vs Θ where Gal $l \in ([−135 : 135]), ([−45 : 45])^o$ for E > 60 EeV . 

C.11 bBand shuffle node frequency vs shuffled node number where Gal $l \in ([−45 : 45], [−135 : 135])^o$ for E > 60 EeV (current to 8/11/2012). 

C.12 bBand shuffle nodes vs shuffles where Gal $l \in ([−45 : 45], [−135 : 135])^o$ for E > 60 EeV (current to 8/11/2012). 

C.13 bBand shuffle Θ nodes vs shuffled Θ where Gal $l \in ([−45 : −135], [45 : 135])^o$ for E > 60 EeV (current to 8/11/2012).
C.14 Band shuffle node frequency vs shuffled node number where Gal l ∈ ([−45 : 45], [−135 : 135])° for E > 60 EeV (current to 8/11/2012) .................................................. 282

C.15 Band shuffle nodes vs shuffles where Gal l ∈ ([−45 : −135], [45 : 135])° for E > 60 EeV (current to 8/11/2012) .................................................. 283

C.16 Band shuffle Θ nodes vs Θ where ∆|l₁ - l₂| > 90° for E > 60 EeV (current to 8/11/2012) .................................................. 284

C.17 Band freq shuffle nodes vs shuffle node number where ∆|l₁ - l₂| > 90° for E > 60 EeV (current to 8/11/2012) .................................................. 285

C.18 Band shuffle nodes vs shuffles where ∆|l₁ - l₂| > 90° for E > 60 EeV (current to 8/11/2012) .................................................. 286
List of Tables

2.1 Final fits to all experiments across energy features. ......................... 39
3.1 Optimization Table of GMF ........................................... 83
5.1 Basic Asymptopic Efficiency Classes ....................................... 131
7.1 Conditions For Extreme Branch Variables ................................. 183
7.2 Tags and Their Conditions .................................................. 187
7.3 40 EeV < E ≤ 50 EeV: No Filters. ........................................ 193
7.4 50 EeV < E ≤ 60 EeV: No Filters. ........................................ 194
7.5 E > 60 EeV: No Filters. .................................................... 195
7.6 Energy Comparison Table 1. No Filters. .................................... 199
7.7 40 EeV < E ≤ 50 EeV: Inter-Spiral Arms. ................................. 201
7.8 50 EeV < E ≤ 60 EeV: Inter-Spiral Arms. ................................. 202
7.9 E > 60 EeV: Inter-Spiral Arms. ............................................. 203
7.10 Energy Comparison Table 2. Inter-Spiral Arms. ........................... 207
7.11 40 EeV < E ≤ 50 EeV: Spiral Arms. ..................................... 209
7.12 50 EeV < E ≤ 60 EeV: Spiral Arms. ..................................... 210
7.13 E > 60 EeV: Spiral Arms. .................................................. 211
7.14 Energy Comparison Table 3. Spiral Arms. .................................. 213
7.15 40 EeV < E ≤ 50 EeV: ∆|l_1 - l_2| > 90° ................................. 216
7.16 50 EeV < E ≤ 60 EeV: ∆|l_1 - l_2| > 90°. ................................. 217
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.17</td>
<td>E &gt; 60 EeV: $</td>
<td>\Delta l_1 - l_2</td>
</tr>
<tr>
<td>7.18</td>
<td>Energy Comparison Table 4. $</td>
<td>\Delta l_1 - l_2</td>
</tr>
<tr>
<td>8.1</td>
<td>Cen A ~ [−50.5, 19.4]$^\circ$ Branches</td>
<td>230</td>
</tr>
<tr>
<td>8.2</td>
<td>Positive Result Branches and their Interesting Similar Branches</td>
<td>231</td>
</tr>
</tbody>
</table>