ON STANDING WAVES AND MODELS OF SHEAR DISPERSION

by

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CONTENTS

SUMMARY

This thesis is concerned with two topics; standing waves on water of finite and infinite depth and models of shear dispersion in channels and pipes.

In the first part standing waves are considered. A stable and accurate numerical method for the calculation of the motion of an interface between two fluids is used to calculate two-dimensional standing waves on deep water. Extremely steep standing waves are determined, significantly steeper than has been previously reported. The peak crest acceleration is used as the determining parameter rather than the wave steepness as the wave steepness is found to have a maximum short of the most extreme wave. Profiles with crest accelerations up to 99% of gravity are calculated and the shape of these extreme standing wave profiles are discussed.

The method is extended to water of finite depth and standing waves are calculated. The effect of the harmonic resonances on the standing waves are investigated and some properties of finite depth standing waves are presented.

The stability of the standing waves on deep water is examined and growth rates of the unstable modes are calculated. It is found that all but very steep standing waves are generally stable to harmonic perturbations. However, standing waves in deep water are typically unstable to subharmonic perturbations via a side-band type instability.

In the second part of this thesis shear dispersion in channels and pipes is considered. High order models of the longitudinal dispersion of a passive contaminant in Poiseuille channel flow and Poiseuille pipe flow are derived and their validity discussed. The derivation is done using centre manifold theory which provides a systematic and consistent approach to calculating each successive approximation. For the case of channel flow, models are also derived when the cross-section, flux and diffusivity all vary; the resultant modifications to the advection velocity and the effective dispersion coefficient are calculated.

In the case of pipe flow, a stable, non-negative finite difference scheme is

formulated which matches the evolution equation to a predetermined order. The limitations imposed by this matching is investigated. The appropriate initial condition to use for the Taylor model of shear dispersion in pipes is derived. It is shown that the commonly used initial condition of simply taking the cross–sectional average is only a first approximation to the correct initial condition. In a similar manner the correct boundary conditions to be used at the inlet and outlet of a finite length pipe are derived. The modifications of the Taylor model to use for a pipe with varying cross–section and varying flow properties is also studied.