This is the peer reviewed author accepted manuscript (post print) version of a published work that appeared in final form in:

Fauza, Gusti, Amer, Yousef, Lee, Sang-Heon & Prasetyo, Hari 2016 'An integrated single-vendor multi-buyer production-inventory policy for food products incorporating quality degradation', *International journal of production economics*, vol. 182, pp. 409-417

This output may not exactly replicate the final published authoritative version for which the publisher owns copyright. It is not the copy of record. This output may be used for non-commercial purposes. This version is licensed under a Creative Commons CC-BY-NC-ND 4.0 license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

The final definitive published version (version of record) is available at:
https://doi.org/10.1016/j.ijpe.2016.09.009

Persistent link to the Research Outputs Repository:
http://researchoutputs.unisa.edu.au/11541.2/122000

General Rights
Copyright and moral rights for the publications made accessible in the Research Outputs Repository are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognize and abide by the legal requirements associated with these rights.

- Users may download and print one copy for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the persistent link identifying the publication in the Research Outputs Repository

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

An integrated single-vendor multi-buyer production-inventory policy for food products incorporating quality degradation

Gusti Fauza[a,b], YousefAmer[c], Sang-HeonLee[c], HariPrasetyo[d]

a School of Engineering, University of South Australia, Mawson Lakes Campus, SA 5095,Australia

b Department of Food Science and Technology, Sebelas Maret University, Jl. Ir. Sutami No.36A, Surakarta, Central Java 57126, Indonesia

c School of Engineering, University of South Australia, Mawson Lakes Campus, SA5095, Australia

d Department of Industrial Engineering, Universitas Muhammadiyah Surakarta, Jl. A. Yani Tromol Pos 1, Pabelan Kartasura, Surakarta, Jawa Tengah 57162, Indonesia

ABSTRACT

Existing studies on models of perishable inventory policy mostly are established according to the base assumption that a fraction of items stocked depletes instantaneously over time due to direct spoilage. For food products, over the storage time the quality does degrade however the quantity of the item remains constant. Therefore, this research proposes an integrated approach for addressing food inventory policy in a supply chain system comprising a single vendor and multiple buyers (SVMB). The kinetic model is applied to represent the quality degradation of the raw material at the vendor (manufacturer), while the shelf-life based price pricing is adopted to characterise the value degradation of finished goods at the buyers (retailers). A mathematical model representing the considered system is established to maximise the joint profit of the whole system. It is achieved by coordinating the manufacturing cycle, the inbound and outbound delivery frequencies of materials over the manufacturing cycle. The numerical test shows that the proposed model returns a better benefit compared to the benchmark model.

Keywords: perishable inventory, quality loss, shelf-life, supply chain, value degradation

1. Introduction

Food security is one of the major problems around the globe. In 2012–2014, the United Nations Food and Agriculture Organisation reported that more than 800 million (15%) of people in the world are still undernourished (FAO, 2014). Despite this fact, roughly

1.3 billion tons of food for human consumption or one-third of food manufactured are wasted per year worldwide across different stages in the supply chain. In industrialised countries particularly, 40% of these losses occur at post processing stages in the form of overproduction due to a lack of coordination between parties involved in the supply chain (Gustavsson et al., 2011). This supply-demand mismatch can be prevented by synchronising the production and inventory policies across the supply chain with the demand.

Coordinating production and inventory policies of food products is complicated as they naturally perish over time. Ghare & Schrader (1963) proposed a mathematical model approach as shown in Equation (1) for representing the inventory profile of perishable items. Over time, the inventory $I(t)$ depletes instantaneously due to demand rate $D(t)$ and deteriorating rate k. Such a deteriorating representation may suit certain perishable items such as gasoline and radioactive materials.

$$\frac{dI(t)}{dt} + kI(t) = -D(t) \tag{1}$$

Since introduced, this model has been adopted as a base model of production-inventory policy for perishable items in literature (to name a few, Park, 1983; Raafat, 1985; Yang & Wee, 2003; Rau et al., 2004; Yu et al. 2012; Taleizadeh et al., 2015). Park (1983) dealt with integrating the production-inventory policy assuming that the raw material decays in storage prior to being used by the production process. Raafat (1985) extended it by incorporating the deterioration of finished goods. Later, Yang & Wee (2003) and Rau et al. (2004) further advanced the model by considering a number of shipment policies to downstream over a production cycle in a multi-stage supply chain. More recently, Yu et al. (2012) and Taleizadeh et al. (2015) studied multi-stage models with different deterioration characteristics for raw materials and finished goods.

Despite its popularity in literature, the aforementioned studies adopt an instantaneous extinction approach in representing the deterioration characteristics which is hardly suitable

for food products. Therefore, Wu et al. (2006) and Ouyang et al. (2006) introduced a non-instantaneous deterioration model, i.e. only after certain period in storage does the inventory level starts deteriorating, with stock-dependent demand. This approach might be applicable for fresh foods, such as fruits and vegetables. Later, Chang et al. (2010) and Geetha & Uthayakumar (2010) extended Wu et al.'s model. Chang et al. (2010) changed the objective to total profit maximization while Geetha & Uthayakumar (2010) allowed shortages with partial backlogged. More recent studies accommodating the non-instantaneous depletion approach can be found in Maihami & Nakhai Kamalabadi (2012), Shah et al. (2013), Soni (2013), Chung et al. (2014) and Wu et al. (2014).

It is common for food products that the deterioration over the planning horizon affects the quality or value of the products; however since it is still within the acceptable consumption or production standards, the quantity of products is unaffected. Blackburn & Scudder (2009) investigated the value change of melon and sweet corns through a supply chain. They proposed an approach of controlling the picking rate and the size of delivery batch from the harvesting area to the cooling facility to minimise the value loss across the supply chain. Rong et al. (2011) applied a kinetic model to represent the quality loss of peppers through a supply chain. Temperature, one of main aspects for maintaining food quality, is incorporated into the production-distribution policy. Note that these studies consider one single form of food products across a supply chain.

Food products flow through different stages across the supply chain over which their quality degrades with different deterioration behaviours depending on their forms (Heldman, 2011). For instance, the quality of wheat flour as raw material, in terms of the gluten content, declines over its storage time (Miœ, 2003). Therefore, adjustments to the production process are needed to maintain the quality of finished goods, e.g. noodles. Then, at retailers' shelves, customers typically perceive the value of noodles from their remaining shelf-life. Tsiros &

Heilman (2005) reported that the customer's willingness to pay (WTP) for a product starts decreasing once it is getting closer to its expiration date. During this degradation period, without any action, e.g. marking down the price, the demand rate would decrease leading to a profit reduction due to an increased number of outdated items. Despite such circumstances are very common in food industries, to the best of knowledge, no research has addressed these issues simultaneously. Therefore, this research proposes a new model of an integrated production-inventory approach for managing the flow of a food product incorporating different quality characteristics both as a raw material and finished goods as discussed previously. In addition, a single-vendor and multi-buyer supply chain system considered in this research would further leverage the contribution of this paper as in terms of supply chain configurations the majority of existing studies focus on either single-level supply chains or two-level supply chains with a single actor at each level.

The remaining sections of this paper are arranged into the following structures. Problem description, model assumptions and notation are described in Section 2, while Section 3 presents the development phase of the proposed model. Section 4 contains numerical tests for measuring the performance of the model and the findings are also discussed. The last section summarises this paper.

2. Problem description, assumptions and notation

Consider a supply chain network where a vendor (manufacturer) produces a single product with a printed expiration date from a single main raw material to meet a constant demand (d_{ret_j}) from N buyers (retailers), where $j = 1, 2, \ldots, N$. The quality of raw material stored at the warehouse deteriorates over time with a constant rate of k following the principle of the kinetic model. Consequently, the production process should be adjusted for meeting the predetermined quality of the finished product with an extra cost (cost of quality loss, c_{loss}). Meanwhile, a shelf-life based pricing is applied for representing the value

degradation of the finished product at the retailers. Numerous types of food products fitting with this configuration include noodles, bakeries, and yogurt.

The inventory policy at the retailers is concerned with the determination of ordering cycle or delivery interval (τ_Δ) of finished goods to meet all demand considering the product value degradation at their shelves. A longer τ_Δ may be beneficial to reduce the ordering cost, however, it would increase the holding cost and the risk of having more outdated items. At the manufacturer, the production and inventory policies are related to specifying the production cycle time (T) and the raw material procurement cycle while considering the quality loss of raw material during the storage time. A longer production cycle time may reduce the setup cost but the holding cost would elevate. The manufacturer bears a lower ordering cost when the procurement cycle is longer, however, it would raise the quality loss of raw material due to the longer storage. Dealing with this circumstance, the production-inventory policy across the supply chain is synchronised by setting the delivery interval of finished goods to be an integer fraction of the manufacturer's production cycle, i.e. over one production cycle T the retailers receive their orders in n deliveries, where n is an integer equal to T/τ_Δ, and setting the frequency of raw material shipments over T to be an integer m. Thus, the problem of production-inventory policy for the entire supply chain can be defined as how to determine the number of inbound material deliveries over the production cycle (m), the frequency of outbound material shipments over the production cycle (n), and the manufacturer's production cycle (T) such that the total profit of the whole supply chain is maximised.

The model is developed based on these following main assumptions:

a. The production rate and the demand rate are constant.

b. The retailers pay transportation and order handling costs.

c. The raw material arrives at the manufacturer at its maximum value (Q_{max}) which declines at a constant rate.

d. Customer's willingness to pay (WTP) for a product decreases linearly once the product reaches its time to deteriorate (τ_{start}).

e. Lead time is ignored.

f. No shortage is allowed.

The following notation is used throughout this paper.

Parameters

For the manufacturer

P	:	production rate (unit product/unit time)
D	:	total demand rate from all retailers (unit product /unit time)
α	:	the ratio of the total demand to production rate
k	:	deterioration rate (unit quality/ unit time)
η	:	power factor or the reaction order in the kinetic model
Q_{max}	:	maximum quality level of raw material (unit quality)
Q_{min}	:	minimum quality level of raw material (unit quality)
$Q(t)$	:	remaining quality level of raw material at time t (unit quality)
τ_{max}	:	maximum duration of the raw material stored at the warehouse (unit time)
c_{raw}	:	purchasing cost of raw material ($/unit product)
A_{raw}	:	delivering cost of raw material ($/shipment)
q_{raw}	:	ordering size of raw material (unit product)
H_{raw}	:	holding cost of raw material ($/unit product/ unit time)
c_{loss}	:	cost of quality loss ($/unit quality/unit time)

$L(m,T)$: total cost of quality loss ($/unit time)

$TC_{raw}(m,T)$: total cost of raw material system ($/unit time)

c_{mfc} : manufacturing cost ($/unit product)

A_{mfc} : production setup cost ($/setup)

H_{mfc} : holding cost of finished goods ($/unit product/unit time)

$\bar{I}_{mfc}$: average inventory of finished goods at the manufacturer (unit product)

τ_{run} : production run time where $\tau_{run} = D.T/P$ (unit time)

$TC_{mfc}(m,T)$: total cost of finished goods system ($/unit time)

$TP_{mfc}(m,n,T)$: total profit of the manufacturer ($/unit time)

For the retailers

c_{ret} : product selling price from manufacturer to retailers ($/unit product)

d_{ret_j} : demand rate of retailer j (unit product/ unit time)

H_{ret_j} : holding cost of finished goods at retailer j ($/unit product/ unit time)

A_{ret_j} : transportation cost for delivering product to retailer j ($/shipment)

τ_{sl} : product shelf-life (unit time)

τ_{start} : time when customer's WTP starts decreasing (unit time)

p_{max} : maximum product price ($/unit product)

p_{min} : minimum product price ($/unit product)

$p(t)$: price of product at age t ($/unit product)

E_{ij} : age of batch i when entering retailer j (unit time)

q_{ret_j} : delivery size for retailer j (unit product)

τ_{Δ} : delivery interval of finished goods (unit time)

$TC_{ret}(n,T)$: total cost at retailers' system ($/unit time)

$R_{ij}(n,T)$: revenue of batch i at retailer j ($/unit time)

$TP_{ret}(n,T)$: total profit of the retailers ($/unit time)

$TP(m,n,T)$: total profit of the integrated supply chain system ($/unit time)

Indexing

i : the i^{th} batch delivered to retailer (i = 1, 2, ..., n)

j : the j^{th} retailer (j = 1, 2, ..., N)

Decision variables

m : number of raw material arrivals for one production cycle

n : number of finished product deliveries for one production cycle

T : manufacturer's production cycle time (unit time)

3. Model development

In line with the majority studies discussed in this paper, the formulation of the production-inventory policy model considers a finite production rate with continuous planning horizon, i.e. the inventory builds up during the production time and is continuously depleted by demand over the planning horizon. The inventory must be able to meet all demand over the planning horizon. Before establishing the production-inventory model, the approaches for formulating the quality loss in both raw material and finished goods are discussed in the following subsections.

3.1 Quality loss of the raw material

The rate of quality loss of raw materials in the warehouse is influenced by the storage conditions such as temperature and humidity. This research adopts a kinetic model approach to represent the rate of quality loss of the raw material as shown in Equation (2),

$$\frac{dQ}{dt} = kQ^{\eta} \qquad (2)$$

where Q represents the quality performance such as gluten content, t and k denote time and deterioration rate respectively, while η expresses the power factor or the reaction order (Labuza, 1982). The quality of raw material is assumed to be at its maximum value (Q_{max}) when it enters the manufacturer's site and then its quality declines over time following Equation (2). Assuming that $\eta = 0$ or the raw material has a constant declining rate; the quality state over time can therefore be expressed by Equations (3).

$$Q(t) = Q_{max} - kt \tag{3}$$

Assuming that a single unit end product requires a single unit of raw material, then the raw material is consumed at rate P during the production run time (τ_{run}), where $\tau_{run} = D.T/P$. Thus, it arrives at the manufacturer's warehouse in intervals of τ_{run}/m or $D.T/(m.P)$ and its quality starts degrading during the cycle time at rate k. Figure 1 shows the inventory and quality profiles over a shipment interval (τ_{run}/m).

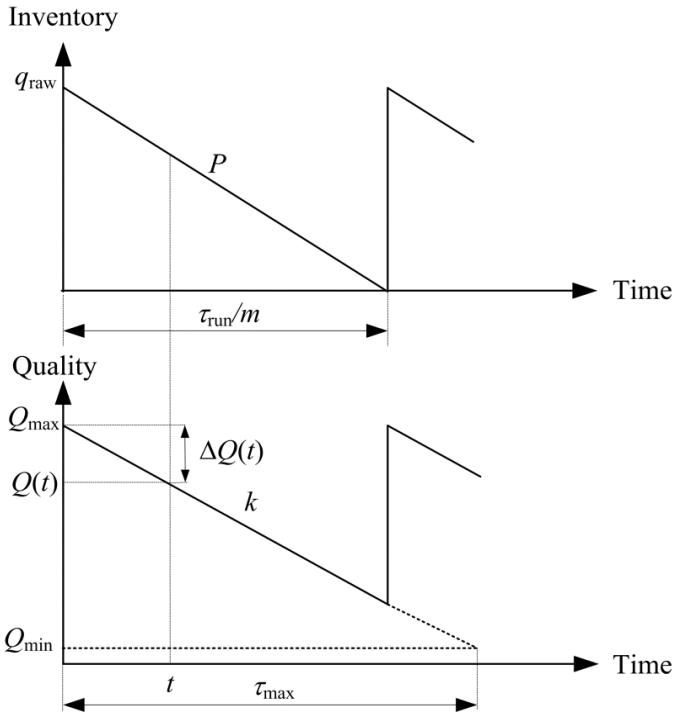


Figure 1. Relation between inventory and quality loss

As seen in Figure 1, for the raw material consumed for production at time t, where t is the time indicating the duration of this raw material stored in the warehouse, it will incur a

quality lost or ΔQ by $Q_{\max} - Q(t)$ with a cost of c_{loss}. Hence, the total cost of quality loss per unit time $L(m,T)$ of the raw material for all batches over one production cycle can be represented by Equation (4).

$$L(m,T) = \frac{c_{\text{loss}} mP}{T} \int_0^{\frac{DT}{mP}} \Delta Q(t) dt \tag{4}$$

Note that in Figure 1, $\tau_{\max}$ denotes the maximum duration of the raw material stored at the warehouse before it reaches its minimum quality level required for production ($Q_{\min}$), where $Q_{\min} \geq 0$. The relation among $Q_{\max}$, $Q_{\min}$ and $\tau_{\max}$ can be expressed by Equation (5).

$$\tau_{\max} = \frac{Q_{\max} - Q_{\min}}{k} \tag{5}$$

To ensure that all raw materials consumed for production meet the minimum quality level, the shipment interval has to be shorter than $\tau_{\max}$, leading to m in Equation (4) restricted by Equation (6).

$$m \geq \frac{\tau_{\text{run}}}{\tau_{\max}} \tag{6}$$

3.2 Shelf-life based price function

In food industries, the manufacturers must process the products complying with both quality standards and safety regulations. The information of the best consumption time limit printed on the packaging is an indicator of this assurance given by the manufacturers. Customers often use this information to further perceive the quality of the products on the shelves. A longer remaining life time is perceived to be a fresher product, thus they are willing to pay the product at its regular price. As their life time is approaching its expiration date, the customer's willingness to pay (WTP) starts decreasing linearly, such as found in pre-cut lettuce, pre-cut carrots, milk and yoghurt, or exponentially as can be seen in beef and chicken (Tsiros & Heilman, 2005). In this paper, a linear decrease of customer's WTP is considered.

Over one production cycle, the product is delivered to the retailers in n shipments. Thus, the age of batch i received by retailer j or E_{ij} may differ from that for other batches depending on the storage time before shipment. Since the product at retailer j is consumed at a constant rate of d_{ret_j} over one ordering cycle (τ_{run}), each product in a batch may have different pricing. To illustrate, for a given batch i in Figure 2 retailer sets the product price at $p_{\max}$ for products consumed before τ_{start}, or time when the customer's WTP starts decreasing (region I). After that point, the remaining stock is sold at discounted prices to entice demand (region II). Lastly, unsold products after reaching the shelf-life (τ_{sl}) are considered as outdated items with minimum value or $p_{\min}$ (region III).

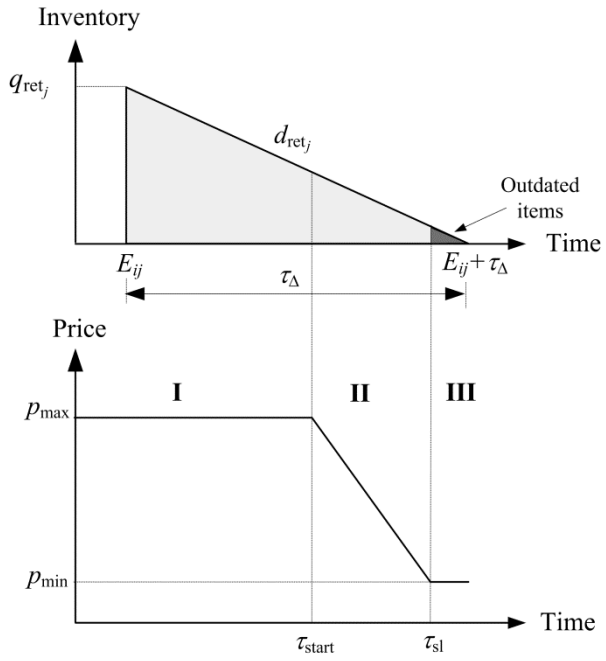


Figure 2. Shelf-life based price function of a batch of q_{ret_j}

Mathematically, this shelf-life based pricing can be represented by Equation (7).

$$p(t)=\begin{cases} p_{\max} & t<\tau_{\text{start}} \quad \text{Region I} \\ p_{\min}+\dfrac{p_{\max}-p_{\min}}{\tau_{\text{sl}}-\tau_{\text{start}}}\left(\tau_{\text{sl}}-t\right) & \tau_{\text{start}}\le t<\tau_{\text{sl}} \quad \text{Region II} \\ p_{\min} & t\ge\tau_{\text{sl}} \quad \text{Region III} \end{cases} \tag{7}$$

It is expected the retailers only receive a batch having E_{ij} less than τ_{start} to generate more revenue. Thus, any batch *i* received by the retailer would fall into the following three cases depending on the time of the last product consumed or $E_{ij} + \tau_{\Delta}$ (Figure 2).

Case 1: $E_{ij} + \tau_{\Delta} < \tau_{\text{start}}$, then the revenue per year obtained from this batch is defined by Equation (8).

$$R_{ij}(n,T) = \frac{d_{\text{ret}_j}}{T} p_{\max} \tau_{\Delta} \tag{8}$$

Case 2: $\tau_{\text{start}} \le E_{ij} + \tau_{\Delta} < \tau_{\text{sl}}$, then the revenue per year of this batch is shown in Equation (9).

$$R_{ij}(n,T) = \frac{d_{\text{ret}_j}}{T} \left[p_{\max} \left(\tau_{\text{start}} - E_{ij} \right) + \int_{\tau_{\text{start}}}^{E_{ij}+\tau_{\Delta}} p(t)dt \right] \tag{9}$$

Case 3: $E_{ij} + \tau_{\Delta} \le \tau_{\text{sl}}$ then Equation (10) represents the revenue per year function of this batch.

$$R_{ij}(n,T) = \frac{d_{\text{ret}_j}}{T} \left[p_{\max} (\tau_{\text{start}} - E_{ij}) + \int_{\tau_{\text{start}}}^{\tau_{\text{sl}}} p(t)dt + p_{\min} (E_{ij} + \tau_{\Delta} - \tau_{\text{sl}}) \right] \tag{10}$$

3.3 Model of an integrated production-inventory policy

For ease of explanation consider a supply chain with a manufacturer and two retailers. The production-inventory policy is coordinated through the following actions. The manufacturer processes the demand from all retailers during *T* and then ships them twice ($n = 2$) at the same interval to each retailer. The first shipments are instantly after the production meets the batch size (q_{ret_j}), while the subsequent deliveries are performed in intervals of τ_{Δ}. During the production run time (τ_{run}), the raw material is ordered twice ($m = 2$) in a size of $q_{\text{raw}} = D.T/m$. Thus, the inventory profiles at all actors can be seen in Figure 3. Banerjee et al. (2007) addressed this multiple integer coordination approach for non-deteriorating items.

As can be seen from Figure 3, a pessimistic approach is used to calculate the age of each batch, i.e. it is the length between the time when the first product in a batch is produced

and the time when the batch is shipped. Given $q_{\text{ret}_j} = d_{\text{ret}_j}.T/n$, then the age of batch 1 for all retailers (E_{1j}) can be written by Equation (11), while for the remaining batches (i = 2, 3, ..., n) their ages can be formulated by Equation (12).

$$E_{1j} = \frac{d_{\text{ret}_j} T}{nP} \tag{11}$$

$$E_{ij} = \frac{d_{\text{ret}_j} T}{nP} + (i-1)\left[\frac{T}{n} - \sum_{j=1}^{N} \frac{n d_{\text{ret}_j} T}{nP}\right] \tag{12}$$

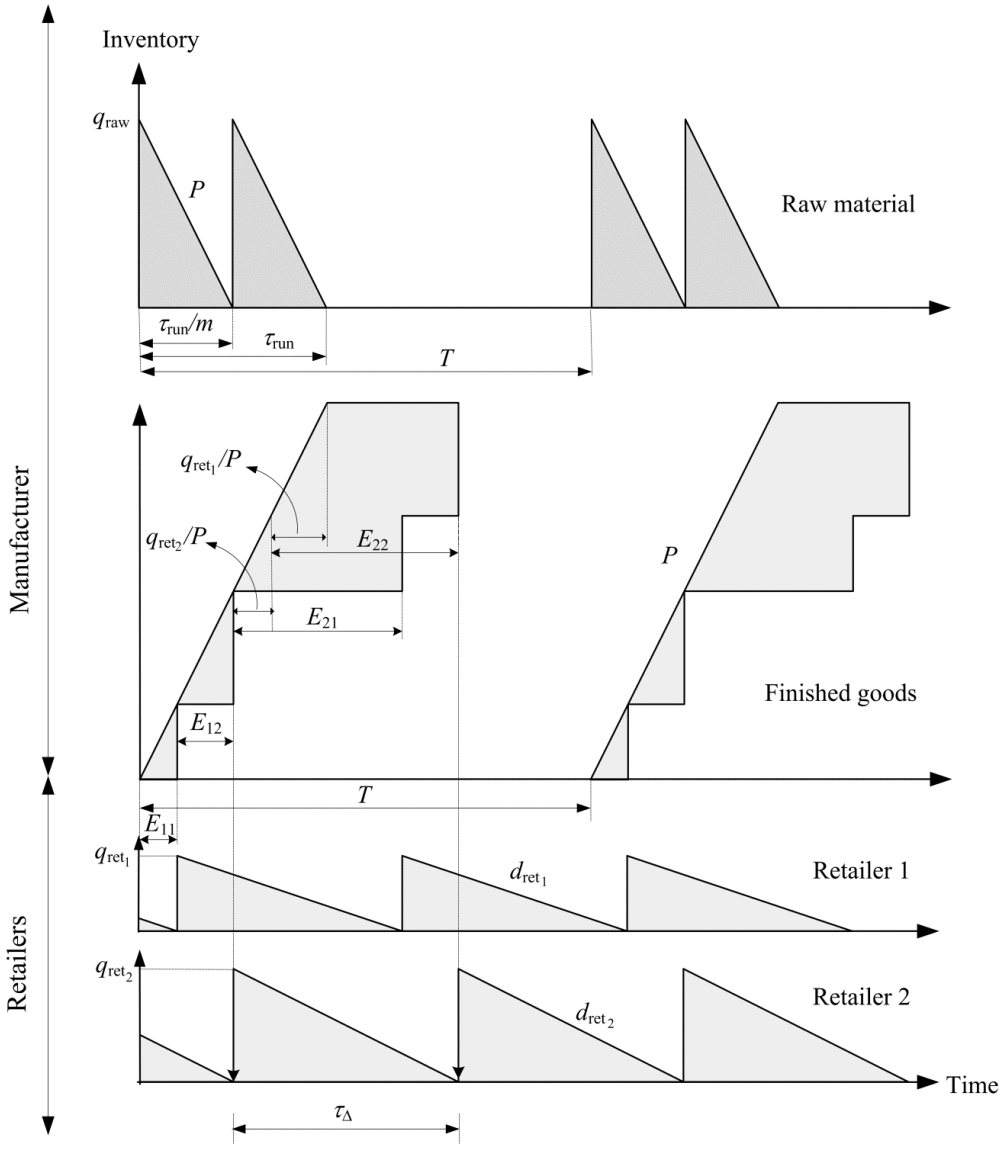


Figure 3. Inventory profile of an integrated manufacturer-retailers system

In the configuration aforementioned, the manufacturer incurs costs for procuring the raw material. Total of the considered costs per year, $TC_{\text{raw}}(m,T)$, is the sum of the purchasing cost, the delivering cost, the inventory holding cost and the quality loss cost which are respectively shown in Equation (13).

$$TC_{\text{raw}}(m,T) = c_{\text{raw}} D + A_{\text{raw}} \frac{m}{T} + H_{\text{raw}} \frac{D^2 T}{2mP} + c_{\text{loss}} \frac{mP}{T} \int_0^{\frac{DT}{mP}} \Delta Q(t) dt \qquad (13)$$

The manufacturer also bears a number of costs related to handling the finished goods before shipping them to the retailers. The average inventory of finished goods at the manufacturer ($\bar{I}_{\text{mfc}}$) can be calculated by subtracting the average retailer's inventory from the total system's inventory resulting in Equation (14). Detailed steps of this approach can be found in the previous studies such as in Hill (1997) and Prasetyo & Lee (2013).

$$\bar{I}_{\text{mfc}} = \sum_{j=1}^{N} d_{\text{ret}_j} \sum_{l=1}^{j} \frac{d_{\text{ret}_l} T}{nP} - \frac{T}{2} \sum_{j=1}^{N} d_{\text{ret}_j} \left(1 - \frac{D}{P} - \frac{1}{n}\right) \qquad (14)$$

The cost of carrying the finished goods inventory is obtained by multiplying this average inventory with the cost for holding the inventory per unit item per year.

Therefore, the total cost for processing and handling the finished goods at the manufacturer, $TC_{\text{mfc}}(n,T)$, comprises the manufacturing cost, the setup cost and the inventory carrying cost that are respectively presented by Equation (15).

$$TC_{\text{mfc}}(n,T) = c_{\text{mfc}} D + \frac{A_{\text{mfc}}}{T} + H_{\text{mfc}} \left[\sum_{j=1}^{N} d_{\text{ret}_j} \sum_{l=1}^{j} \frac{d_{\text{ret}_l} T}{nP} - \frac{T}{2} \sum_{j=1}^{N} d_{\text{ret}_j} \left(1 - \frac{D}{P} - \frac{1}{n}\right) \right] \qquad (15)$$

Given the manufacturer's selling price per unit product c_{ret} then total revenue per year of the manufacturer, $TP_{\text{mfc}}(m,n,T)$, can be obtained by deducting the total cost from the total revenue, as expressed by Equation (16).

$$TP_{\text{mfc}}(m,n,T) = c_{\text{ret}} D - TC_{\text{raw}}(m,n,T) - TC_{\text{mfc}}(m,n,T) \qquad (16)$$

On the other hand, all retailers bear the relevant costs consisting the purchasing cost, the ordering cost and the carrying cost that are respectively formulated by Equation (17).

$$TC_{\text{ret}}(n,T) = c_{\text{ret}} D + \frac{n}{T}\sum_{j=1}^{N} A_{\text{ret}_j} + \sum_{j=1}^{N} \frac{H_{\text{ret}_j} d_{\text{ret}_j} T}{2n} \tag{17}$$

The total profit per unit time of all the retailers, $TP_{\text{ret}}(n,T)$, is acquired by subtracting Equation (17) from the total revenue per unit time as shown in Equation (18).

$$TP_{\text{ret}}(n,T) = \sum_{j=1}^{N}\sum_{i=1}^{n} R_{ij}(n,T) - TC_{\text{ret}}(n,T) \tag{18}$$

Finally, the total profit of the whole supply chain, $TP(m,n,T)$, is obtained by adding the total profit of the manufacturer and the retailers. Therefore, the integrated production-inventory policy can be expressed by the following equations.

Maximise:

$$TP(m,n,T) = TP_{\text{mfc}}(m,n,T) + TP_{\text{ret}}(n,T) \tag{19}$$

Subject to:

$$P \geq D; \tag{20}$$

$$m \geq \frac{\tau_{\text{run}}}{\tau_{\max}}; \tag{21}$$

$$E_{ij} < \tau_{\text{start}}; \text{ for } i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, N \tag{22}$$

$$T > 0; \tag{23}$$

$$m, n > 0 \text{ and integer}; \tag{24}$$

Equation (19) denotes the total profit of the supply chain that should be maximised while ensuring that the production rate is able to meet all demand, constraint in Equation (20). Equation (21) guarantees no raw material for production beyond its minimum quality level, while Equation (22) ensures all batches arrive at the retailers' warehouse before the customer's WTP starts declining. Further, Equations (23) and (24) denote the decision variables constraints.

The optimisation of production-inventory policy for the whole supply chain is achieved by simultaneously specifying the production cycle time (T), the raw material delivery frequency over this production cycle time (m) and the number of finished goods shipments to retailers over the production cycle time (n). Unfortunately, from the appearance of the model i.e. Equations (19) – (24), this would be analytically intractable to formulate the explicit closed forms of the decision variables. Some researchers, for instance Banerjee et al. (2007) and Siajadi et al. (2006) applied heuristic procedures to deal with similar problems. However, the existences of constraints Equations (21) and (22) lead to similar procedures hardly being implemented. Therefore, in this research a meta-heuristic approach based on genetic algorithms is utilised to simultaneously optimise the decision variables. The genetic algorithms are capable of finding near global optimal or global optimal solution to high scale problems. Aytug et al. (2003) surveyed on the applications of the genetic algorithms in dealing with problems in the area of production and management and showed that, for the majority cases, the genetic algorithms tend to outperform other meta-heuristic approaches.

4. Numerical examples

The optimisation for the numerical test is performed using genetic algorithm (GA) tool in MATLAB R2009a on a notebook with processor Intel®Core i5-480M 2.66GHz with 320GB hard drive and 4GB RAM. The parameter setting obtained from a preliminary experiment is as follows: the population size is 50 individuals, the reproduction is performed by crossover using scatter chromosome function with a rate of 0.7, two-individual elitism, and migration at an interval of 20 generations with a rate of 0.2, the stopping rule is set by the maximum of 100 generations, while the remaining parameters are set to their default values. The lower and upper limits of the decision variables m, n and T are respectively [1 1 0] and [100 100 1]. Due to a natural stochastic behaviour of the algorithm, the GA is run for 20

times for each case and then the decision variables yielding the best value are selected as the solution for the case.

The numerical test is performed with two main objectives: (i) to evaluate the performance of the proposed model which is achieved by comparing the total profit obtained from the proposed model to that of from the benchmark model, and (ii) to examine the behaviour of the model due to parameter changes which is conducted by altering some parameters, e.g. the ratio of total demand to production rate (α), the deterioration rate (k), the quality loss cost (c_{loss}), the customer's WTP decreasing time (τ_{start}) and the product shelf-life (τ_{sl}), obtaining the new optimal solutions and then analysing the results.

In the first test, parameters in Banerjee et al. (2007) are adopted with a slight modification to suit the proposed configuration (Table 1).

Table 1. Parameters in the numerical test

Notation	**Value**	
d_{ret_j}	[5,000 3,000 4,000]	unit/year
P	60,000	unit/year
c_{loss}	25	$/unit quality/year
c_{raw}	15	$/unit
c_{mfc}	8	$/unit
c_{ret}	35	$/unit
p_{max}	50	$/unit
p_{min}	0	$/unit
k	0.5	unit quality/year
A_{raw}	40	$/delivered lot
A_{mfc}	750	$/one-setup
A_{ret_j}	[50 50 50]	$/shipment
H_{raw}	10	$/unit/year
H_{mfc}	15	$/unit/year
H_{ret_j}	[20 20 20]	$/unit/year
Q_{max}	1	unit quality
Q_{min}	0.8	unit quality

η	0	
τ_{sl}	0.1096	year
τ_{start}	0.0822	year

Using these parameters, the production-inventory policy problem is then solved using two approaches, i.e. accommodating the quality loss (P1) and disregarding the quality loss (P2). For the former case, the proposed model, Equations (19) – (24), is optimised using the GA to find the maximum total profit. For the latter case, parameters related to quality loss in Equation (19) – (24) are set as follows: $c_{loss} = 0$, while τ_{start} and $\tau_{sl} = \infty$ to indicate that the quality degradation is not considered in the policy (benchmark model). The problem is then solved using the GA to find the decision variables. The total profit of the benchmark model is obtained by entering these decision variables into Equation (19) to reflect the actual system's profit when the decision is made without considering the quality loss of raw material and finished goods. The decision variables, the total cost and revenue for the manufacturer and the retailers, the total profit and the difference of the profit in percentage are summarised in Table 2.

Table 2. Total cost, revenue, and profit of the three policies

Model	m	n	T (year)	TC_{mfc}	TC_{ret}	R_{mfc}*	$\sum\sum R$**	TP	%
P1	2	2	0.0877	290,079	428,683	420,000	599,994	301,232	7.9%
P2	2	2	0.1037	289,461	429,115	420,000	577,670	279,094	

* R_{mfr} represents the manufacturing's revenue ($c_{ret} \times D$)
** $\sum\sum R$ denotes the total revenue of all retailers

From Table 2, the proposed policy (P1) returns in a total profit of $301,232, while the benchmark policy (P2) yields a total profit of $279,094. This demonstrates that accommodating the quality loss in the production-inventory policy could potentially improve the performance of the whole system by 7.9%. The manufacturer's production cycle time for

P1 is shorter than that of P2 to anticipate the quality degradation of the raw material and finished goods. For the manufacturer, this shorter production cycle would reduce the cost of quality loss; however, the manufacturer total cost could increase due to a higher production setup cost per unit time. The retailers are benefitted from this shorter production cycle time as the number of outdating products would be lower, hence the revenue increases. This fact demonstrates that although as an integrated system the proposed policy leads to a better total profit, the profit share among the actors in the supply chain could be unfairly distributed. In a decentralised decision making environment, where members of the supply chain act based on their own objectives, a profit sharing mechanism, i.e. a fair incentive scheme is recommended to rectify the disadvantaged parties.

The next numerical test investigates the effect of changes in some parameters on the total profit of the proposed model. Given α is the ratio of total demand rate to production rate, the value of α is shifted from 0.1 to 0.9 by changing the production rate. The problem is then optimised using the GA with aforementioned setting. The result is then presented in Table 3 and Figure 4.

Table 3. Total profit at different α

D	*P*	α	*m*	*n*	*T*	*TP*
12,000	120,000	0.1	2	2	0.0865	301,348
	60,000	0.2	2	2	0.0877	301,232
	40,000	0.3	3	2	0.0906	301,139
	30,000	0.4	4	3	0.1049	301,199
	24,000	0.5	5	3	0.1123	301,564
	20,000	0.6	6	4	0.1348	302,066
	17,143	0.7	8	5	0.1654	302,775
	15,000	0.8	11	6	0.2066	303,764
	13,333	0.9	16	9	0.2924	305,258

As shown in Table 3, the value of α affects the obtained decision variables. As α increases, the production cycle time tends to be longer with more numbers of inbound and outbound material shipments. Further, α also influences the total gained profit. Figure 4 demonstrates the pattern of the total profit at different α. It is interesting to note that increasing the value of α or setting the production rate close to the demand rate tends to improve the performance of the supply chain. However, we could not claim that this pattern is applicable for any parameter setting in the experiment, instead this finding indicates that when the production rate is adjustable, optimising it would potentially further improve the performance of the supply chain. This is in line with the investigation in Blackburn & Scudder (2009) where the picking rates of melons and sweet corns are recommended to be controlled to reduce the value loss across the supply chain and then the profit is maximised.

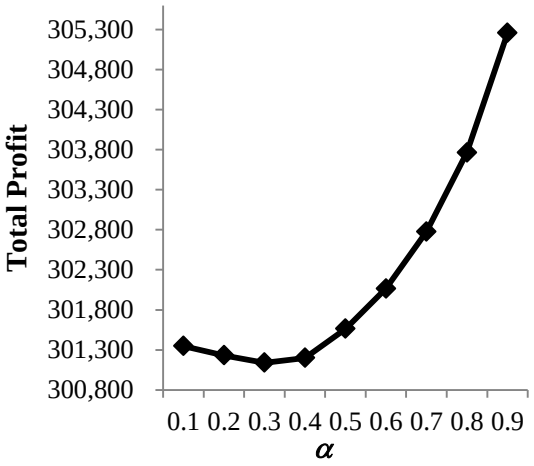


Figure 4. Relation of α to the total profit

Further numerical test is aimed to evaluate the effect of changes in the rate of raw material quality loss (k) and penalty cost (c_{loss}) on the performance of the developed model. Different values of k (0.3, 0.5, 0.7 & 0.9) and c_{loss} (25, 50, 75, 100 & 125) are observed. The summary of outcome is presented in Table 4 and Figure 5.

Table 4. The optimised decisions for various quality loss rates and penalty costs

	k = 0.3				k = 0.5				k = 0.7				k = 0.9			
c_{loss}	m	n	T	TP	m	n	T	TP	m	n	T	TP	m	n	T	TP
25	2	2	0.0878	301,495	2	2	0.0877	301,232	3	2	0.0878	300,996	3	2	0.0878	300,820
50	2	2	0.0877	301,100	3	2	0.0878	300,732	3	2	0.0878	300,381	4	2	0.0878	300,057
75	3	2	0.0878	300,820	3	2	0.0877	300,293	4	2	0.0878	299,859	4	2	0.0878	299,464
100	3	2	0.0878	300,556	4	2	0.0878	299,925	4	2	0.0877	299,399	5	2	0.0878	298,943
125	3	2	0.0877	300,293	4	2	0.0878	299,596	5	2	0.0878	298,996	5	2	0.0877	298,469

In general, for raw material having highest quality loss rate (k = 0.9) and penalty cost (c_{loss} = 125), the total profit is at its lowest level of $298,469 with the highest number of raw material shipments (m = 5). As c_{loss} increases the total profit diminishes at a linear trend for any value of k as clearly observed from Figure 5.

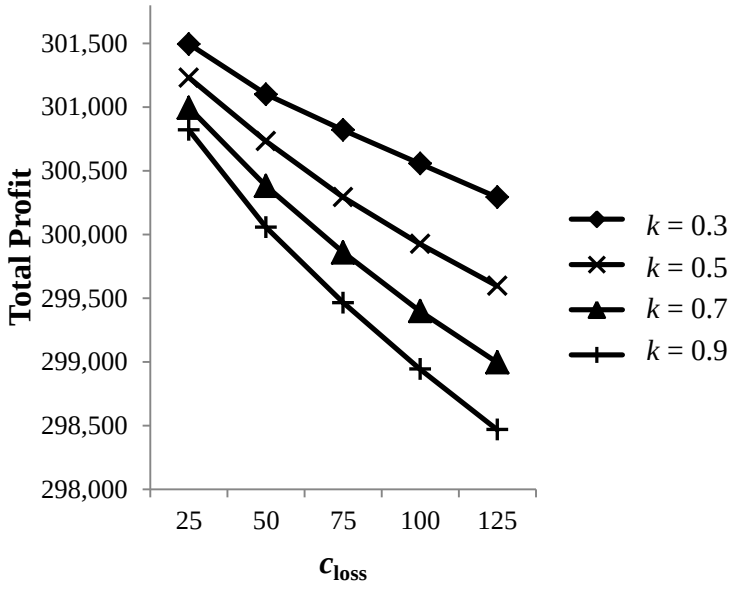


Figure 5. Effect of various deterioration rates and penalty costs on total profit

Besides, the change influences the procurement strategy, i.e. the number of raw material deliveries (m) tends to be higher. This is to ensure that the raw material consumed for production having a better quality to avoid a bigger penalty cost. Similarly, for any value of

c_{loss} the total profit of the supply chain declines as k increases (Figure 5). For the observed cases, the values of n and T tend to relatively remain steady.

In the last numerical test, the effect of changes to customer perception of product quality on the system performance is investigated by observing different values of τ_{start} and τ_{sl}. The value of τ_{start} is varied from $0.25\tau_{sl}$, $0.5\tau_{sl}$, to $0.75\tau_{sl}$. Note that a wider gap between τ_{sl} and τ_{start} indicates an environment where customers tend to have a strong response to the product's remaining shelf-life. In addition, in this numerical test, various values of the shelf-life (τ_{sl}), from 0.0548 year ($\approx$ 20 days) to 0.1644 year ($\approx$ 60 days) are examined. This test reflects different characteristics of finished goods. Certain food products have a longer shelf-life such as dried noodles and cookies, while other products may be expired in a couple of days after production. The result of this experiment is summarised in Table 5 and Figure 6.

Table 5. The system performance due to various τ_{sl} and τ_{start}

No	τ_{sl}	τ_{start}											
		$0.25\tau_{sl}$				$0.5\tau_{sl}$				$0.75\tau_{sl}$			
		m	*n*	*T*	*TP*	*m*	*n*	*T*	*TP*	*m*	*n*	*T*	*TP*
1	0.055	1	2	0.0195	258,096	1	1	0.0270	284,234	1	2	0.0446	293,765
2	0.082	1	2	0.0269	276,354	1	2	0.0455	294,002	2	2	0.0663	299,440
3	0.110	1	2	0.0341	285,653	2	2	0.0600	298,268	2	2	0.0877	301,232
4	0.137	1	2	0.0538	291,104	2	2	0.0743	300,368	3	2	0.1040	301,505
5	0.164	2	2	0.0639	294,550	2	2	0.0884	301,245	3	2	0.1040	301,505

As seen from Table 5, at any value of the investigated product's shelf-life (τ_{sl}), in circumstances where the customer's willingness to pay (WTP) decreases early, the manufacturer tends to adopt a shorter production cycle. Besides, the total profit declines nonlinearly as indicated by the widening gaps between the lines at different points of τ_{sl} (Figure 6).

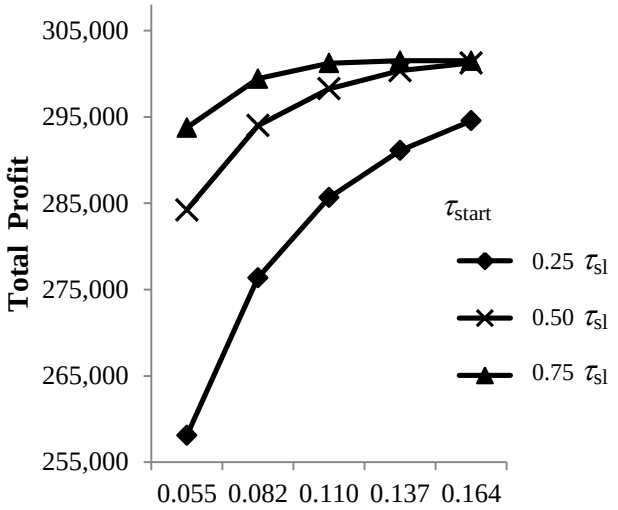


Figure 6. τ_{sl} vs. total profit for various τ_{start}

This widening discrepancy is more obvious for a shorter τ_{sl}, i.e. for $\tau_{sl} = 0.0548$, reducing τ_{start} incrementally at $0.25\,\tau_{sl}$ from $0.75\,\tau_{sl}$ to $0.25\,\tau_{sl}$ leads to a total profit reduction of \$9,531 and \$26,138 respectively. This demonstrates that in any location where the business is operated, the customer's perception of the product quality is important to be carefully examined by management. Apart from that, it can be seen from Table 5 that for every value of τ_{start} and the total profit declines nonlinearly as τ_{sl} is linearly decreased. Figure 6 shows this pattern for different τ_{start} ($0.25\,\tau_{sl}$, $0.5\,\tau_{sl}$, $0.75\,\tau_{sl}$) with a substantial total profit reduction observed for $\tau_{start} = 0.25\,\tau_{sl}$. This finding means that food products with a shorter shelf-life need to be handled more accurately to ensure that the supply chain system runs in its best performance. In addition, any available method for increasing the shelf-life of the product, for instance by improving the packaging or handling in a cold chain system, should be continuously examined. From this experiment, the profit improvement resulted from extending τ_{sl} can be clearly observed, thus the investment can be economically analysed prior to making a final decision.

5. Conclusion

In this paper, a mathematical model representing an integrated production-inventory policy for food products in a single-vendor multi-buyer supply chain system has been established. Two different quality characteristics of the materials in the form of both raw material and finished goods are addressed. In contrast to the majority of studies on perishable items where the quality deterioration is reflected by the instantaneous extinction of the product quantity, the proposed research deals with the problem by adopting a kinetic model and shelf-life based pricing approach for the quality deterioration of the raw material and finished goods respectively. This representation is better suited for many cases in reality than the previous studies which are only suitable for a limited type of perishable products, such as radioactive substances. The profit of the whole supply chain is maximised by simultaneously specifying the manufacturer's production cycle time, the number of raw material arrivals and the number of finished goods shipments over one production cycle.

From the numerical tests, it is shown that the proposed policy gives better results in terms of the total gained profit, particularly compared to the integrated policy disregarding the product quality loss. In managing the production-inventory policy for food products, it is recommended that the production rate should be considered as a variable to be controlled to further enhance the supply chain performance. In addition, both the speed of the raw material quality deterioration and the customer's perception of the product quality based on its remaining shelf-life are important aspects to consider as they substantially contribute to the total profit of the supply chain. In particular for products with short remaining shelf-life, it could be economical to extend the expired dates applying technologies in packaging, storing or handling.

It is important to note that the proposed integrated mechanism can be further improved by accommodating a different number of shipments for each retailer to reduce the inventory and to promote fresher products in the batch. Apart from that, incorporating the

production rate as one of the decision variables will significantly strengthen the contribution, particularly when the supply chain operates in stochastic environments. These can be substantial avenues for future research in this area. Further, in term of the optimisation method, defining a conclusive heuristic procedure to find the optimal or nearly optimal solution is worth of investigation in forthcoming research, particularly for addressing small scale problems. The optimised results obtained from applying the GA as shown in this paper is beneficial to be used as benchmark for evaluating the performance of the procedure as also found in Chan & Kingsman (2007).

Acknowledgement

The first author would like to thank the Australian Government Overseas Aid Program (AusAid) for supporting this research**.**

References

Aytug, H., Khouja, M., Vergara, F., 2003. Use of genetic algorithms to solve production and operations management problems: a review. International Journal of Production Research, 41(17), 3955 – 4009. Doi: 10.1080/00207540310001626319

Banerjee, A., Kim, S., Burton, J., 2007. Supply chain coordination through effective multi-stage inventory linkages in a JIT environment. International Journal of Production Economics, 108 (1), 271 – 280. Doi:10.1016/j.ijpe.2006.12.015

Blackburn, J., Scudder, G., 2009. Supply chain strategies for perishable products: The case of fresh produce. Production and Operations Management, 18 (2), 129 – 137. Doi: 10.1111/j.1937-5956.2009.01016.x

Chan, C.-K., Kingsman, B.G., 2007. Coordination in a single-vendor multi-buyer supply chain by synchronizing delivery and production cycles. Transportation Research Part E: Logistics and Transportation Review, 43 (2), 90 – 111. Doi:10.1016/j.tre.2005.07.008

Chang, C.-T., Teng, J.-T., Goyal, S.K., 2010. Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. International Journal of Production Economics, 123 (1), 62 – 68. Doi:10.1016/j.ijpe.2009.06.042

Chung, K.-J., Cárdenas-Barrón, L.E., Ting, P.-S., 2014. An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit. International Journal of Production Economics, 155, 310 – 317. Doi:10.1016/j.ijpe.2013.12.033

Food and Agriculture Organization, International Fund for Agricultural Development, World Food Program, 2014. The State of Food Insecurity in the World 2014. Strengthening the enabling environment for food security and nutrition. Rome: FAO. http://www.fao.org/3/a-i4030e.pdf

Geetha, K.V., Uthayakumar, R., 2010. Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments. Journal of Computational and Applied Mathematics, 233 (10), 2492 – 2505. Doi:10.1016/j.cam.2009.10.031

Ghare, P.M., Schrader, G.F., 1963. A model for exponentially decaying inventory. Journal of Industrial Engineering, 14 (5), 238 – 243.

Gustavsson, J., Cederberg, C., Sonesson, U., Van Otterdijk, R., Meybeck, A., 2011. Global food losses and food waste. Food and Agriculture Organization of the United Nations, Rom.

Heldman, D.R., 2011. 2 - Kinetic models for food systems. Food preservation process design, Academic Press, San Diego, 19 – 84.

Hill, R.M., 1997. The single-vendor single-buyer integrated production-inventory model with a generalised policy. European Journal of Operational Research, 97 (3), 493 – 499. Doi: 10.1016/S0377-2217(96)00267-6

Labuza, T.P., 1982. Shelf-life dating of foods. Food & Nutrition Press: Westport, Conn., USA.

Maihami, R., Nakhai Kamalabadi, I., 2012. Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. International Journal of Production Economics, 136 (1), 116 – 122. Doi:10.1016/j.ijpe.2011.09.020

Miœ, A., 2003. Influence of the storage of wheat flour on the physical properties of gluten. International Agrophysics, 17 (2), 71 – 75.

Ouyang, L.-Y., Wu, K.-S., Yang, C.-T., 2006. A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Computers & Industrial Engineering, 51 (4), 637 – 651. Doi: 10.1016/j.cie.2006.07.012

Park, K.S., 1983. An integrated production-inventory model for decaying raw materials. International Journal of Systems Science, 14 (7), 801 – 806. Doi: 10.1080/00207728308926498

Prasetyo, H., Lee, S.-H., 2013. Comment on "Simultaneous determination of multiproduct batch and full truckload shipment schedules". International Journal of Production Economics, 144 (2), 405 – 408. Doi: 10.1016/j.ijpe.2013.03.019

Raafat, F., 1985. A production-inventory model for decaying raw materials and a decaying single finished product system. International Journal of Systems Science, 16 (8), 1039 – 1044. Doi: 10.1080/00207728308926498

Rau, H., Wu, M.-Y., Wee, H.-M., 2004. Deteriorating item inventory model with shortage due to supplier in an integrated supply chain. International Journal of Systems Science, 35 (5), 293 – 303. Doi: 10.1080/00207720410001714833

Rong, A., Akkerman, R., Grunow, M., 2011. An optimization approach for managing fresh food quality throughout the supply chain. International Journal of Production Economics, 131 (1), 421 – 429. Doi:10.1016/j.ijpe.2009.11.026

Shah, N.H., Soni, H.N., Patel, K.A., 2013. Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Omega, 41 (2), 421 – 430. Doi:10.1016/j.omega.2012.03.002

Siajadi H., Ibrahim R.N., Lochert P.B., 2006. Joint economic lot size in distribution system with multiple shipment policy. International Journal of Production Economics, 102 (2), 302-16. Doi:10.1016/j.ijpe.2005.04.003

Soni, H.N., 2013. Optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment. International Journal of Production Economics, 146 (1), 259 – 268. Doi:10.1016/j.ijpe.2013.07.006

Taleizadeh, A.A., Noori-daryan, M., Cárdenas-Barrón, L.E., 2015. Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items. International Journal of Production Economics, 159, 285 – 295. Doi:10.1016/j.ijpe.2014.09.009

Tsiros, M., Heilman, C.M., 2005. The effect of expiration dates and perceived risk on purchasing behavior in grocery store perishable categories. Journal of Marketing, 69 (2), 114 – 129.

Wu, J., Skouri, K., Teng, J.-T., Ouyang, L.-Y., 2014. A note on "optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment". International Journal of Production Economics, 155, 324 – 329. Doi:10.1016/j.ijpe.2013.12.017

Wu, K.-S., Ouyang, L.-Y., Yang, C.-T., 2006. An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. International Journal of Production Economics, 101 (2), 369 – 384. doi:10.1016/j.ijpe.2005.01.010

Yang, P.-C., Wee, H.-M., 2003. An integrated multi-lot-size production inventory model for deteriorating item. Computers & Operations Research, 30 (5), 671 – 682. Doi:10.1016/S0305-0548(02)00032-1

Yu, Y., Wang, Z., Liang, L., 2012. A vendor managed inventory supply chain with deteriorating raw materials and products. International Journal of Production Economics, 136 (2), 266 – 274. Doi:10.1016/j.ijpe.2011.11.029