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Optimal defence strategies for intercepting an incoming missile with inexpensive cooperating missiles

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Thesis

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Summary

Can a team of low-cost defending missiles provide a more cost-efficient approach to intercepting a high-performance attacking missile than a single high-cost defending missile? The trajectory of the high-performance attacking missile is unknown and predicting the possible paths brings uncertainties to intercept calculations. Although low-cost defending missiles may have limited abilities to detect and pursue the attacking missile, they will have the ability to cooperate with each other to see and intercept the attacking missile.

The objective is to find a cost-efficient solution that will maximise the probability of stopping the attacking missile. Two questions are addressed: what is the best strategy for a team of cooperating defending missiles, and what are the cost-efficient missile characteristics for a team of cooperating defending missiles? I have used simple models to explore these questions in the thesis. If teams of low-cost defending missiles appear to be cost-efficient with simple models then more realistic models should be used for future analysis.

In this thesis I develop a cooperative control strategy based on predicting the possible paths of the attacking missile from a known initial state to a known stationary target, then define the region that the attacking missile will be in when the defending missiles start seeking the attacking missile, and the region the attacking missile will be in when it can be intercepted. These two regions are used to determine points for the defending missiles to aim towards before operating their on-board seekers. The selection of *aim points* for the defending missiles to fly towards such that the defending missiles are able to maximise their coverage of the seek and intercept regions is a multi-dimensional unconstrained non-linear optimisation problem. The more paths that the defending missiles can see and hit, the greater the probability of intercepting the attacking missile. The optimisation problem can be solved using the Nelder-Mead method. Examples are given to demonstrate the effectiveness of the method. The examples track the progression of the engagement scenarios from initial detection of the attacking missile by ground-based radar, to launch of the defending missiles, and through to the point

when the defending missiles can see the attacking missile and switch to traditional intercept strategies.

The second part of the work develops a method for determining which characteristics of defending missiles give the best performance. A set of 1728 different defending missile teams was generated by varying the number of missiles, missile speed, seeker performance and missile manoeuvrability. Seeker range has the most influence on performance.

Although increasing the number of missiles can ensure good performance of a missile defence system, the upfront costs associated with multiple missiles can be large and so teams of cooperating missiles might not be a cost-effective solution. Future work could include extending the problem to have multiple attacking missiles, incorporating more detailed defence system models, validating performance and cost measures, comparing the performance of the Nelder-Mead method to other numerical methods, and comparing the overall control strategy with other strategies.

Declaration

I declare that:

This thesis presents work carried out by myself and does not incorporate without acknowledgment any material previously submitted for a degree or diploma in any university; to the best of my knowledge it does not contain any materials previously published or written by another person except where due reference is made in the text; and all substantive contributions by others to the work presented, including jointly authored publications, is clearly acknowledged.



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This opportunity to be a part of the Defence STEM Cadetship program has allowed me to gain industry and research experience. This has truly been an answer to my prayers.

For wealth certainly makes itself wings
Like an eagle that flies toward the heavens.
Book of Proverbs

Publications

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1 Introduction

The aim of this work is to determine whether replacing a single high-cost defending missile with a team of cooperating low-cost defending missiles can improve the performance and flexibility of missile defence systems and be more cost effective.

A missile has several key components [Fleeman and Schetz, 2012]: a seeker which enables the missile to detect and track the target, a computer which provides commands and coordinates individual functions of the missile, a warhead which supplies kinetic energy to the target, a flight control system which directs the missile in flight, a propulsion system which propels the missile towards the target, and stabilisers which provide the missile stability during flight.

The cost of a defending missile will generally increase with its capabilities and performance. Missiles that are low-cost generally have inferior components that result in reduced performance compared to high-cost missiles.

A defending missile's performance can be characterised by its:

- thrust profile, which will dictate the missile's speed over time
- maneuverability, which will dictate the missile's ability to change direction
- on-board seeker, which will dictate the missile's viewing angle and range
- communication rate, which will dictate the missile's ability to coordinate with other missiles and with the ground-based radar
- effectiveness, which determines how close the defending missile must get to the attacking missile
- reliability, which determines the missile's probability of stopping the attacking missile when within effective range.

Consider a missile defence system defending a known, stationary target against a high-performance attacking missile, as shown in Figure 1.1. Missile defence systems have many components, but for the purpose of this thesis we have simplified the missile defence system into:

- a ground-based radar which detects and measures position and velocity of the attacking missile,
- defending missiles which intercept and prevent the attacking missile from entering the *keep-out* zone of the protected stationary target,
- a fire control system which determines an intercept solution for the defending missiles, and
- a launch platform which launches the defending missiles.

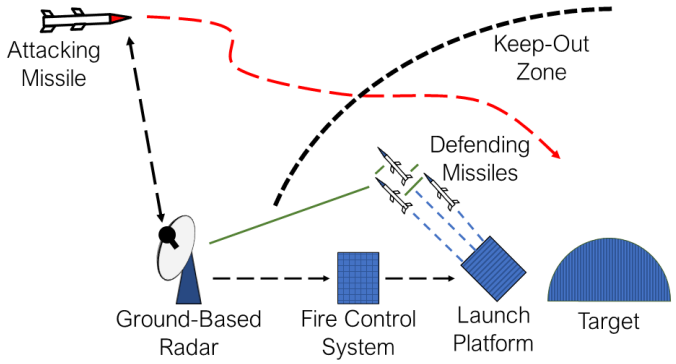


Figure 1.1: Components of the missile defence system.

The trajectory of the high-performance attacking missile is unknown and predicting the possible paths towards the stationary target brings uncertainties to intercept calculations. If there are multiple possible targets then the missile defence system must consider possible attacker paths towards each target. If a target is not stationary then the motion of the target would be known to the fire-control system.

A premise of this research is that although low-cost defending missiles may have limited abilities to detect and pursue the attacking missile, they will have the ability to cooperate with each other to seek and intercept the attacking missile. Although in practice a fully cooperative system with negligible communication delays is ambitious, other authors have made the same assumption for their research in cooperative control [Piet-Lahanier et al., 2013, Yuksek and Ure, 2016, Shaferman and Oshman, 2016, Rajagopalan et al., 2019]. If analysis shows that cooperating

low-cost defending missiles are not cost-effective even under ideal conditions then they will not be cost-effective under realistic conditions.

There are several key events that occur during a missile engagement:

1. The attacking missile is detected by the ground-based radar. The ground-based radar will track the attacking missile and provide position and velocity measurements to the fire control system. We assume that the ground-based radar will be able to send data to the defending missiles before and after launch.

2. The fire control system will determine how many defending missiles to launch, when to launch them, and where each defending missile should fly to before it turns on its seeker.

3. As the defending missiles fly out towards the attacking missile, information from the ground-based radar will be used by the fire control system to refine the possible paths of the attacking missile, which in turn will be used to refine the points that the defending missiles are flying towards.

4. Each defending missile will be equipped with a *seeker* that can detect the attacking missile. The defending missile seekers will provide continuous and more accurate measurements of the attacking missile position and velocity compared to the ground-based radar. The seeker performance of low-cost defending missiles may be significantly reduced in comparison to a high-cost defending missile. The defending missiles will turn on their on-board seekers at the time when the attacking missile is predicted to come within the seeking range of the defending missiles. By this time, the defending missile positions and directions should be organised such that they will be able to see and intercept all possible paths of the attacking missile (see Figure 1.2).

5. Low-cost defending missiles, in comparison to a high-cost defending missile, may have limited abilities to pursue an attacking missile, but they will have the ability to cooperate with each other. Once the attacking missile is detected by any defending missile, the team of defending missiles will coor-

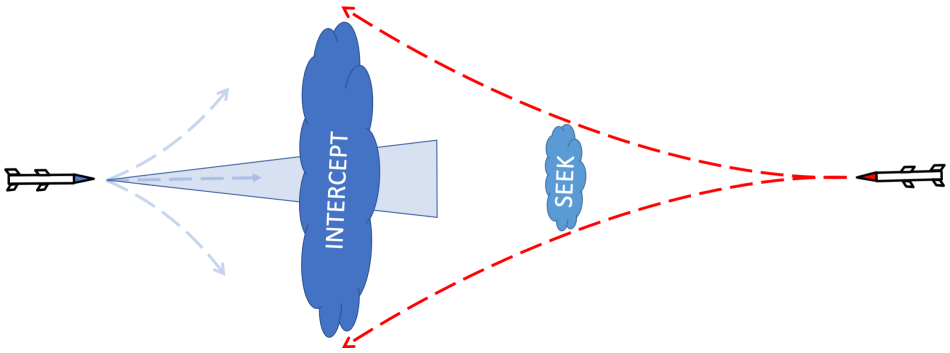


Figure 1.2: The intercept region and seek region.

dinate to track and intercept the attacking missile with as many defending missiles as possible.

The objective is to find a cost-efficient solution that will maximise the probability of stopping the attacking missile. Two questions are addressed: what is the best strategy for a team of cooperating defending missiles, and what are the best missile characteristics for a team of cooperating defending missiles?

Section 2 reviews relevant literature for the application of intercepting an attacking missile with a team of cooperating defending missiles. Section 3 describes the missile defence system with more detail and summarises the assumptions made to simplify the modelling. Even though the models are simple, they can lead to useful insights and conclusions which can be used in more detailed analysis, perhaps making the detailed analysis phase easier and shorter.

The cooperative control strategy (Section 4) is based on predicting the possible paths of the attacking missile from a known initial state to a known stationary target, then defining the region that the attacking missile will be in when the defending missiles start seeking the attacking missile, and the region the attacking missile will be in when it can be intercepted. These two regions are used to determine points for the defending missiles to aim towards before operating their on-board seekers. The selection of *aim points* for the defending missiles to fly towards such that the defending missiles are able to maximise their coverage of the seek and intercept regions is a multi-dimensional unconstrained non-linear optimisation problem. The optimisation determines whether it is better to have

thin coverage of the entire seek and intercept regions or concentrated coverage on parts of the seek and intercept regions to maximise the probability of stopping the attacking missile. In general, the more paths that each defending missile can see and hit, the greater the probability of stopping the attacking missile. The Nelder-Mead method [Nelder and Mead, 1965] is used to solve the optimisation problem, though any similar method could be used. Examples are given showing the simulation of the control strategy from first radar detection to when the defending missiles fly-out and turn-on their on-board seekers.

Section 5 presents a 'design of experiments' approach to generate a number of different defending missile teams and assess their probability of intercepting an attacking missile. A cost function is formulated with guidance from the Defence Science and Technology Group to evaluate cost effectiveness for each defending missile team. Pareto plots, an example of a variation of parameters approach, and main effect screening plots are included to help analyse the performance and cost of the different defending missile teams.

The methods described in this thesis can be used to guide further development for defending missile models. To ensure that this work remained unclassified and releasable, I developed simple 3D missile models. This thesis only considers defence against a single attacking missile. However, the work could be extended to analyse engagements with multiple attacking missiles.

2 Literature review

This review highlights some key methods presented in the literature for the application of intercepting an attacking missile with a team of cooperating defending missiles. This is an extension of the literature presented in a conference paper [Kapsis et al., 2021].

2.1 Calculating the intercept region

Prediction of where a maneuvering missile could be intercepted is an important element to this problem. Before a defending missile is able to see the attacking missile with its on-board seeker, the defending missile has to rely on the ground-based radar to provide measurements of the attacking missile's position and velocity. The update rate from the ground-based radar can be slow, and infrequent measurements makes it difficult to accurately predict the possible paths of the attacking missile [Shin et al., 2011]. A separate estimator can provide the missing information about the attacking missile. Most estimators assume some knowledge about the attacking missile's dynamic performance and provide some information about the conditional probability density function of the missile's state given some measurements [Shaviv and Oshman, 2017]. Shaviv and Oshman [2017] employed a computationally efficient sequential Monte Carlo method to estimate the attacking missiles's manoeuvre region. Doucet et al. [2001] discusses difficulties associated with various methods:

> ... real data can be very complex, typically involving elements of non-Guassianity, high dimensionality and nonlinearity, which conditions usually preclude analytic solution. ... many approximation schemes, such as the extended Kalman filter, Gaussian sum approximations and grid-based filters, have been proposed to surmount this problem. The first two methods fail to take into account all the salient statistical features of the processes under consideration, leading quite often to poor results. Grid-based filters, based on deterministic numerical integration methods, can lead to accurate results, but are difficult to implement and too computationally expensive to be of any practical use in high

> dimensions. Sequential Monte Carlo methods ... provide a convenient and attractive approach to computing the posterior distributions.

Piet-Lahanier et al. [2013] present a method that uses nonlinear model predictive control to derive missile guidance laws for multiple defending missiles intercepting a manoeuvring target. They describe the possible intercept points as a set whose boundaries depend on a noise function applied to the measurements of the attacking missile trajectory, acceleration limits of the attacking missile, and its intended target. The polyhedral boundary consists of a set of vertices and supporting hyper-planes and is used to characterise the intercept region. This approach avoids making assumptions that the attacking missile will perform a particular manoeuvre. Instead, the defending missile trajectories are guided to cover as much of the intercept region as possible with the assumption that the measurements and missile parameters remain within given bounds. A limitation of this method is that the boundary becomes complex as the number of vertices and supporting hyper-planes become large.

An alternative method may be to represent the boundary of the intercept region as a simpler convex set such as an ellipsoid [Durieu et al., 2001]. Shin et al. [2011] was able to apply a similar ellipsoidal approximation based on the method by Maksarov and Norton [2002] to characterise the boundary of the set of possible missile positions based on hypotheses on existing manoeuvres and bounds on feasible acceleration. They were able to combine this method with a guidance law that can guide the defending missiles to a good state for the intercept phase and also guide the defending missiles using measurements from only the ground-based radar.

2.2 Determining launch conditions

A task for the research is to determine launch parameters for the defending missiles such as launch coordinates and initial pitch and heading angles. Yuksek and Ure [2016] formulated an mixed integer nonlinear programming problem (MINLP) that optimised the selection of the number of defending missiles to launch, the coordinates to launch from and the initial launch directions. They emphasise that

the launch coordinates of the defending missiles can directly affect miss distance, while the initial launch direction can affect guidance performance. They use a probabilistic search algorithm to solve the MINLP and optimise for minimum miss distance and maximum efficiency in use of resources. Their method was applied to the problem of intercepting a ballistic attacking missile with multiple defending missiles. Their work seems highly relevant as they were able to embed full missile dynamics into their optimisation by using a collaborative extended Kalman filter algorithm. However, a ballistic missile has a deterministic path, therefore it is not clear whether their method is suitable for intercepting a high-performing, unpredictable missile.

2.3 Cooperative control strategies

Research on control of multi-vehicle systems performing cooperative tasks has been around since the late 1980s [Murray, 2007]. For the application of cooperative intercept using a team of cooperating missiles, cooperative control strategies include staggered launch, cooperative coverage and simultaneous intercept.

Shaferman and Oshman [2016] used a linear deterministic model to derive a staggered launch strategy for defending missiles. Staggering the defending missiles had a guidance and estimation benefit when compared to the non-staggered strategy. The guidance benefit was due to the attacking missile's inability to optimally evade all the defending missiles, and resulted in a 75% average miss distance reduction when compared to an one-to-one scenario. Trailing defending missiles were able to receive better quality measurements of the threat from leading defending missiles, and have more time to react and reduce their miss distance. The staggering strategy may be a promising cooperative control strategy for intercepting a high-performance missile with multiple inexpensive defending missiles.

Another cooperative control strategy is given by Su et al. [2018], who guide multiple inferior missiles to cooperatively cover the attacking missile intercept region. Their guidance goal was to maximise the interception probability such that each defending missile covers as much of the intercept region as possible. Defending missiles communicate during flight and adapt their covering strategy against evasive

strategies such as constant manoeuvring, random step manoeuvring and bang-bang manoeuvring. A simulation study by Shaferman and Oshman [2016] showed that the most effective evasion strategy is to conduct a well-timed direction reversal of a maximum effort maneuver, also known as a bang-bang maneuver. The adaptive covering strategy presented by [Su et al., 2018] gave smaller miss distance and higher successful interception probability than that of augmented proportional navigation guidance [Zarchan, 2012] and differential game guidance [Shinar et al., 2007] when used with a team of three defending missiles. Overall, the strategy given by [Su et al., 2018] is very similar to the strategy developed in this thesis. However, their cooperative strategy does not consider a seek region and their results are limited by their two-dimensional missile dynamics.

Rajagopalan et al. [2019] used a predictive regret-matching algorithm to guide the defending missiles such that they totally cover the attacking missile manoeuvrability range. Once the attacking missile manoeuvrability range is totally covered by the team of defending missiles then the proportional navigation guidance law is used to steer the defending missiles to intercept the attacking missile. A clear difference between this approach and that of Su et al. [2018] is that Su et al. [2018] do not transition the defending missiles to a different guidance law to intercept the attacking missile. Also Rajagopalan et al. [2019] measure their performance with a *zero-effort-miss* distance calculation that measures the distance that the defending missile would miss the attacking missile if the attacking missile continued along its present direction and the defending missile made no further direction changes [LI et al., 2019], whereas Su et al. [2018] predicts attacking missile positions at interception by a region of points and maximises the interception probability instead of minimising the miss distance. The concept of predicting intercept points and maximising the probability of intercept is similar to the method presented in this thesis. The results presented by Rajagopalan et al. [2019] are limited as they assumed that the defending missiles could see the attacking missile for the entire duration of their flight—they did not model separate fly-out and seek phases.

Zhou et al. [2018] implemented a guidance strategy for a team of defending missiles intending to intercept an attacking missile simultaneously. Their approach allowed missiles to communicate during flight and synchronise their intercept times. An-

other approach by In-Soo Jeon et al. [2006] assigned a common intercept time to each defending missile before launch. Predetermining an intercept time before launch is challenging as it is difficult to specify a predetermined intercept time for a high-performance missile with an uncertain trajectory [Su et al., 2018]. Previous methods to solve the cooperative simultaneous intercept problem were based on missiles' time-to-go and were only suitable for stationary threats [Jeon et al., 2010, Zhang et al., 2013, Zhou and Yang, 2016]. Zhou et al. [2018] based their approach on missile ranges-to-go—the distances between the defending missiles and the attacking missile—with the idea that if the defending missiles ranges-to-go are in coherence and decrease to zero simultaneously, then all the defending missiles will hit the target simultaneously. Their results did not include three dimensional missile dynamics, and they allowed their defending missiles to be faster and travelling in the same direction as the attacking missile.

Having cooperative defending missiles is advantageous because it is harder for an attacking missile to evade a team of defending missiles than a single missile, since the attacking missile may not be able to detect all the defending missiles simultaneously and determine the best evasion strategy to counter the entire team of defending missiles [John B. Hoffman and Warren J. Boord, 2016].

Intercepting an attacking missile with a team of low-cost cooperating missiles is still a fairly new research application in cooperative control. My contribution is to use many possible attacking missile paths to represent the attacking missile's manoeuvre region, and define a seek and intercept region based on a seek and intercept time respectively. I also consider the fact that having more defending missiles intercepting a particular attacking missile path will increase the probability of stopping the attacking missile, which allows the optimisation to determine how to best distribute the defending missiles across the seek and intercept regions. Furthermore, understanding how design characteristics of the defending missiles can affect performance and suitability of the chosen control strategy, to the best of my knowledge, has not been investigated.

3 Missile defence modelling and simplifying assumptions

Developing a control strategy for cooperating missile defence requires modelling of the attacking missile and the missile defence system. I have used relatively simple models to explore the question of whether a team of low-cost defending missiles is more cost-effective than a single high-cost defending missile. If a team of low-cost defending missiles appears to be cost-effective with simple models then the models can be made more realistic for future analysis. This section discusses some general models and simplifying assumptions. More details of my models are presented in Section 4.

3.1 Coordinate system

Figure 3.1 depicts an Earth fixed coordinate system with the origin $(0, 0, 0)$ at the location of the stationary target. As the missile defence system will operate over small regions, the curvature of the earth can be ignored [Jenkins, 1984].

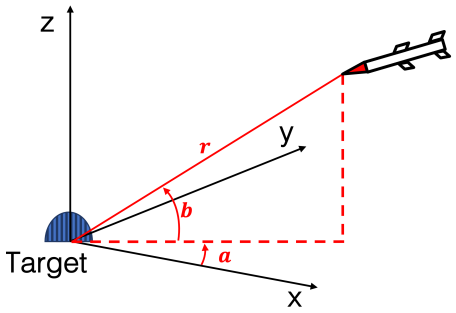


Figure 3.1: The Cartesian coordinate system

The coordinates of the attacking missile and defending missiles will be modelled in Cartesian (x, y, z) coordinates where the x axis points east, the y axis points north, and the z axis points up. Alternatively, missile coordinate components can also be expressed as a range r (i.e. distance to the attacking missile) and two angles, the azimuth angle a about the z-axis, and the elevation angle b from the xy-plane [Yuksek and Ure, 2016].

3.2 Ground-based radar

As an attacking missile approaches the target, a ground-based radar system will provide measurements of its position (range r, azimuth angle a, elevation angle b) and velocity (range rate $\dot{r} = \frac{dr}{dt}$, azimuth rate $\dot{a} = \frac{da}{dt}$ and elevation rate $\dot{b} = \frac{db}{dt}$) where t is time. The key parameters of ground-based radar are [John B. Hoffman and Warren J. Boord, 2016]:

- detection range,
- range and range rate accuracy, measured as standard deviations σ_r and $\sigma_{\dot{r}}$ with the true value as the mean,
- angle and angular rate accuracy for
  - azimuth angle: σ_a and $\sigma_{\dot{a}}$,
  - elevation angle: σ_b and $\sigma_{\dot{b}}$, and
- time between successive measurements.

Each radar measurement is the sum of the actual value of the parameter being measured and the measurement error. These errors are usually modelled as random variables with Gaussian distributions [Macfadzean, 1992]. Figure 3.2 shows an example of the position measurements (orange) corresponding to the actual position (blue) for a ballistic missile path. In this case the errors are [Wolff, 1998]:

$$\begin{aligned}
\sigma_r &= 70 \text{ metres} \\
\sigma_{\dot{r}} &= 30 \text{ metres/second} \\
\sigma_a &= 0.1/(2\pi) \text{ radians} \\
\sigma_{\dot{a}} &= 0.05/(2\pi) \text{ radians/second} \\
\sigma_b &= 0.1/(2\pi) \text{ radians} \\
\sigma_{\dot{b}} &= 0.05/(2\pi) \text{ radians/second.}
\end{aligned}$$

Accurately estimating an attacking missile position and velocity involves applying a filter to smooth position measurements and estimate velocity. There are two types of filters that are commonly used [Macfadzean, 1992]:

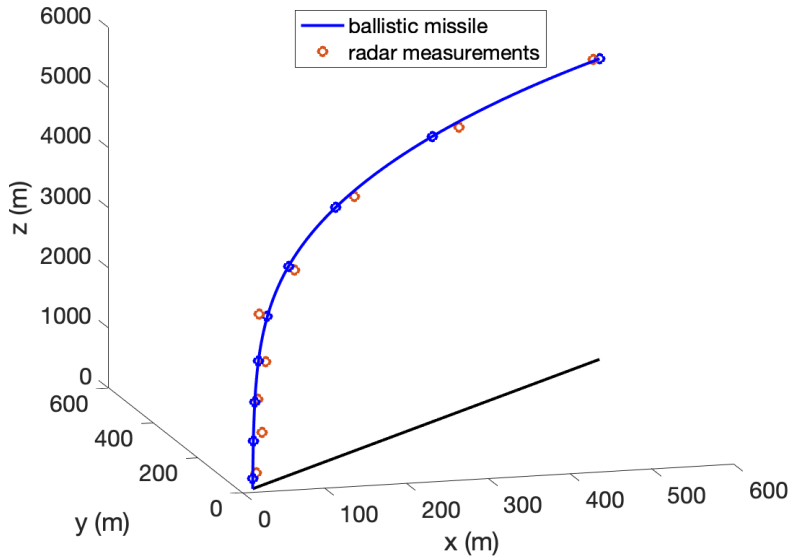


Figure 3.2: Ballistic trajectory (blue) and ground-based radar measurements (orange)

- an α–β filter estimates position and speed from position measurements only, with correction terms for distance and speed depending on the observed distance error and constants α and β
- a Kalman filter uses a more detailed system model, with variable gains that are adjusted from sample to sample.

Spatial errors in radar measurements will lead to errors in the predicted missile path. For non-ballistic missiles, which can be steered, these errors will generally be small compared to deviations in the path due to the manoeuvrability of the attacking missile (see Section 4.1). Errors estimating speed will impact the time of intercept. If the attacking missile is traveling faster than estimated then the defending missile will need to see and intercept the attacking missile sooner, and if the attacking missile is traveling slower than measured then the defending missile will have to see and intercept the attacking missile later. I assume that these types of errors will be compensated for by updating measurements from the ground-based radar and from the defending missile seekers during the engagement.

3.3 Missile dynamics

The motion of a missile is determined by four forces (Figure 3.3):

- *thrust force* $\boldsymbol{F}_{\mathrm{t}}$, which acts in the direction of the missile body to propel the missile forwards
- *steering force* $\boldsymbol{F}_{\mathrm{s}}$, is equivalent to the lift force which acts perpendicular to direction of travel to steer the missile
- *drag force* $\boldsymbol{F}_{\mathrm{d}}$, which opposes the direction of travel and slows the missile
- *gravitational force* $\boldsymbol{F}_{\mathrm{g}}$ which acts downwards.

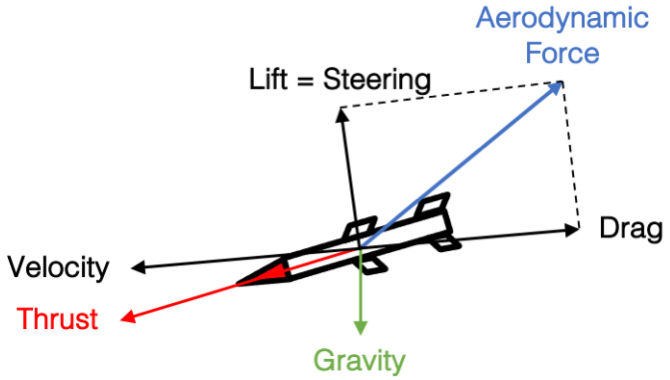


Figure 3.3: Forces acting on a missile

A missile propulsion system will develop a thrust force $\boldsymbol{F}_{\mathrm{t}}$ by burning fuel to produce a high pressure gas which is ejected behind the missile at high velocity [Raymer, 1992]. Missiles are able to generate lateral acceleration $\boldsymbol{a}_{\mathrm{lat}}$ by using movable control surfaces. The lateral acceleration that can be generated by a missile is limited by aerodynamic and structural constraints [Raymer, 1992]. The force from the lateral acceleration acts perpendicular to the velocity vector to steer the missile. Therefore, the steering force $\|\boldsymbol{F}_{\mathrm{s}}\| \leq ma_{\max}$ where m is the mass of the missile and lateral acceleration is limited by $a_{\max}$, gives a minimum turning radius. The drag force on a missile with velocity vector $\boldsymbol{v}$ will be a combination of form drag and skin drag [Raymer, 1992]. The gravitational force on a missile is $\boldsymbol{F}_{\mathrm{g}} = m\boldsymbol{g}$ where $\boldsymbol{g} = (0, 0, -9.81)\,\mathrm{ms}^{-2}$ is the acceleration due to gravity. Suppose a missile is at position $\boldsymbol{p} = (x, y, z)$ with velocity $\boldsymbol{v} = (v_x, v_y, v_z)$. The motion of

the missile is given by the differential equations

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{v}$$
$$m\frac{d\boldsymbol{v}}{dt} = \boldsymbol{F}_{\mathrm{t}} + \boldsymbol{F}_{\mathrm{d}}(\boldsymbol{v}) + \boldsymbol{F}_s + m\boldsymbol{g}.$$

A *ballistic* trajectory has no thrust ($\boldsymbol{F}_{\mathrm{t}} = 0$) and no steering force ($\boldsymbol{F}_{\mathrm{s}} = 0$). A *non-ballistic* trajectory will have thrust force $\boldsymbol{F}_{\mathrm{t}}$ and steering force $\boldsymbol{F}_{\mathrm{s}}$, which may vary with time.

3.4 Attacking missile trajectories

Dunham and Wilson [2020] warn that it is no longer adequate to assume that adversarial missile trajectories will be a predictable parabolic arc. Non-parabolic trajectories make it hard to determine the intended path to the target. Thus, it is desirable for missile defence systems to wait and respond when the threat is within close range of the intended target. The challenge is to develop a control strategy that can be used by the defending missile system to successfully intercept a threat capable of a non-ballistic trajectory.

In Section 4 we use Bézier curves to approximate attacking missile paths, since they are easy to generate and allow the attacking missile to manoeuvre while respecting maximum manoeuvrability constraints.

3.5 Modelling the defending missiles

A particular *type* of low-cost defending missile will have performance that is characterised by its thrust profile, maneuverability, seeker characteristics, communication rate, effectiveness and reliability. The combination of these characteristics will influence the size of the defending missiles. For example, more thrust requires more weight, better seeker performance requires more power, better hit lethality requires more weight, and better maneuverability requires a lighter, smaller missile. The benefits of designing a lighter weight missile include lower production cost, lower logistics cost, smaller size, higher firepower for a wight-limited

launch platform and better mission flexibility as multiple launch platforms can be used, such as bombers, fighter aircraft, helicopters, ships, submarines and ground vehicles [Fleeman, 2001].

Thrust Missiles will have a predetermined thrust profile that will influence the speed throughout the flight as well as the duration of flight. The thrust phase of a missile's flight lasts only a few seconds. After burning all the on-board fuel, a missile should be able to sustain their speed until the end of their flight.

In Section 4, it is assumed that both the attacking and defending missiles will travel at constant speeds, since defending missiles will generally intercept the attacking missile well after the thrust phase has finished. However, future work could assess the impact of varying speed on the proposed control strategy.

Manoeuvrability The defending missiles will have a minimum turning radius determined by the maximum lateral acceleration constraint at a given missile speed. In Section 4 we use optimised geometric Hermite curves [Yong and Cheng, 2004] to approximate defending missile paths; they are more complicated than Bézier curves but are convenient and allow us to find efficient missile paths with controlled curvature.

On-board seeker The on-board seeker of defending missiles can either be an infrared seeker, which provides measurements of the direction of the attacking missile, or a radar seeker which provides measurements for direction and range of the attacking missile. The dominant estimation error is the estimation of the acceleration of the attacking missile, which must be inferred from direction measurements. Hence, intercept navigation using measurements of direction and range is not much better in practice than proportional navigation using direction measurements only [Shaferman and Oshman, 2016]. Proportional navigation is explained in [Shneydor, 1998]. Radar seekers require a pulse transmitter and receiver. The missile diameter and shape of the nose will determine whether the radar system can be carried by the missile or whether an external system is required to illuminate the target. Infrared seekers do not require an external system to operate [Shaferman

and Oshman, 2016]. However, the performance of infrared seekers is sensitive to rain, snow, haze and fog [Macfadzean, 1992].

Radar seekers can be classified as either a search seeker, having low accuracy but longer range, or a track seeker, having high accuracy and short range [Macfadzean, 1992]. The pulse repetition frequency (PRF) of a radar determines how often a pulse is emitted. A radar may also provide a doppler measurement that can be used to measure velocity. Table 3.1 shows the parameters for a radar seeker and an infrared seeker.

Table 3.1: Parameters for the on-board seeker

Parameter	Radar	Infrared
angular resolution	depends on system	depends on system
range	depends on system	depends on system
accuracy	depends on range	depends on system
measurement frequency	PRF	continuous
range measurement	yes	no
position measurement	yes	yes
doppler measurement	yes	no

Ground-based radar will typically be used to detect an attacking missile and give its initial position and speed. The defending missiles will use measurements from the ground-based radar to guide towards the attacking missile. Once the defending missiles are able to see the attacking missile, they will transition to track the attacking missile with their high accuracy on-board seeker. The key seeker parameters used in Sections 4 and 5 are the defending missile on-board seeker range and seeker angle. Seeker errors have not been considered. However, seeker errors could be incorporated into an overall probability of hit.

Communication We will assume that defending missiles can all communicate with each other throughout the entire time of flight, with negligible communication delays. The ground-based radar will send measurements of the attacking missile periodically. The defending missiles need to communicate their states with each other to cooperatively seek and intercept the attacking missile.

Effectiveness Defending missiles can stop an attacking missile by either hitting it directly or by exploding nearby. It is preferable for the defending missiles to directly hit the attacking missile with as much energy as possible [John B. Hoffman and Warren J. Boord, 2016], and the more missiles that can hit the attacking missile then the higher the likelihood of mission success. The probability of a defending missile not intercepting the attacking missile can be modelled as an increasing function of the *miss distance*—the minimum distance between the defending missile and the attacking missile. In Section 4 we assume that a defending missile will not stop the attacking missile unless it gets within a specified distance of the attacking missile. If it does get close enough, effectiveness is modelled by a specified 'probability of fail'.

Reliability Missiles usually do not have redundancy and each subsystem must function reliably for success [Fleeman, 2001]. According to Fleeman [2001], the measure of reliability depends on the number of parts and the type of parts. Moving parts tend to be less reliable than non-moving parts, and software reliability has replaced mechanical reliability as a major contributor to failure. Fleeman [2001] suggests that if designers want to increase reliability then they should consider minimising the number of subsystems, minimise number of parts, minimise moving parts, and minimise complexity of the subsystems. A missile can be highly reliable and still miss the target. There are other factors that directly contribute to a missile missing its target, such as radar errors, intercept calculations, guidance and control calculations, and manoeuvrability constraints. I have assumed that the defending models are perfectly reliable, but reliability could be incorporated into the effectiveness model.

3.6 Conclusion

There are several factors that should be considered when modelling a missile defence scenario, including coordinate system, ground-based radar errors, intercept region calculations, missile dynamics, missile trajectories, and several missile characteristics. I will make some simplifying assumptions in my formulation of a missile defence system in Section 4:

- The curvature of the earth can be ignored as the missile defence system will operate over small regions.
- Ground-based radar spatial errors can be ignored as their impact on the attacking missile path prediction errors will be insignificant compared to the impact of attacking missile manoeuvrability.
- Ground-based radar errors estimating the attacking missile speed can be ignored as the timing for intercept will be updated and corrected during the engagement.
- Attacking and defending missiles will travel at constant speeds. For both the attacking missile and the defending missiles, the engagement will occur well after their initial thrust phases and the drag force which gradually reduces the missiles speed is likely to be small in comparison to the left (steering) force.
- Defending missile seeker errors will be incorporated into the probability of a particular missile failing to hit the attacking missile.
- Communication delays and restrictions between the team of defending missiles are negligible.
- Detailed miss distance calculations are replaced by a simple calculation—if a defending missile gets within a specified distance of the attacking missile then we will consider that as a hit with a given probability of failure.

The next section describes the actual models used and formulates an optimisation problem.

4 Control strategy for defending missiles

This section develops an optimal control strategy for a team of defending missiles. An earlier 2D version of this work was presented at the 2021 MODSIM conference [Kapsis et al., 2021].

We consider a simplified 3D problem where the curvature of the Earth is ignored. We want to defend against an attacking missile that is assumed to fly towards a stationary target at position $(0, 0, 0)$. The coordinate system has x pointing east, y pointing north and z pointing up. Suppose the attacking missile is detected at time t_0 by a ground-based radar, and is at position $\mathbf{P}_0$ with velocity $\mathbf{v}_\text{a}$. The attacking missile is able to perform aerodynamic manoeuvres, thus its future path towards the target is uncertain. We have up to M defending missiles that can cooperate to seek and intercept the attacking missile. The defending missiles must prevent the attacking missile from entering the predetermined *keep-out* zone of the protected target. I assume that the target is stationary, but it would be straightforward to extend the methods I developed to protect a moving asset, or a group of assets.

The proposed method to defend against the attacking missile has the following steps:

1. given a radar measurement of the position and velocity of the attacking missile, determine possible paths of the attacking missile towards the stationary target. In the situation where there could be multiple possible targets, then predicted attacking missile paths should be generated towards each target.

2. approximate an *intercept time* when the team of defending missiles will aim to intercept the attacking missile before it reaches the *keep-out* zone

3. calculate a *seek time* when the team of defending missiles will activate their seekers and aim to detect the attacking missile; seek time depends on the intercept time and the detection range of the defending missile seekers

4. define a *seek region* comprising points on the predicted paths where the attacking missile could be at the calculated seek time

5. define an *intercept region* comprising points on the predicted paths where the attacking missile could be at the chosen intercept time
6. define an *aim point* for each defending missile to fly towards so the defending missiles will be able to effectively seek and intercept the attacking missile
7. generate a path for each defending missile from its position to its aim point
8. determine the appropriate *launch time* for each defending missile to reach their aim point when the attacking missiles reaches the seek region

Prior to launching the defending missiles, and during their fly-out phase, the ground-based radar will provide periodic updates on the attacking missile's position and velocity. At every radar update, the predicted paths of the attacking missile, intercept time, seek time, seek region, intercept region, aim points and defending missile paths will change. The challenge is to find aim points that will minimise the probability of missing the attacking missile and ensure the defending missile paths remain feasible.

4.1 Predicting attacking missile paths

An attacking missile path is a smooth curve that originates at the attacking missile position $\mathbf{P}_0$ and terminates at the stationary target location $\mathbf{P}_3$. Missiles are limited in their ability to manoeuvre by aerodynamic and structural constraints [Raymer, 1992], which define the maximum curvature of the path. We use cubic Bezier curves [Hazewinkel, 1997] to represent possible attacking missile paths, as they are smooth, easy to generate and allow the missile to manoeuvre while respecting maximum manoeuvrability constraints. Increasing the number of predicted attacking missile paths will better represent the manoeuvre region of the attacking missile. Although not every possible path or type of manoeuvre can be represented, this method appears to give a realistic envelope of paths and can be used to estimate the density of possible paths in any region. Figure 4.1 gives an example of a Bezier curve.

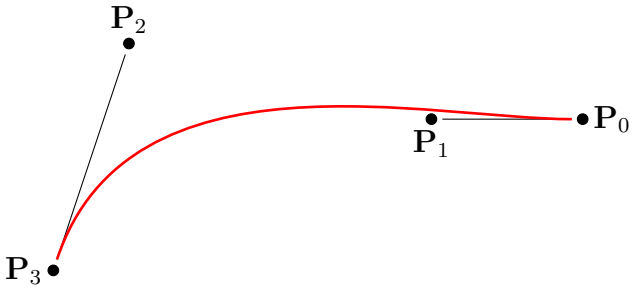


Figure 4.1: Control points for a Bezier curve.

These parametric curves have the form

$$B(u) = \sum_{k=0}^{3} \binom{3}{k} u^k (1-u)^{3-k} \mathbf{P}_k, \quad u \in [0,1]$$

where the *control points* are:

$\mathbf{P}_0$: the initial position of the attacking missile, detected by the radar

$\mathbf{P}_1$: a point ahead of the attacking missile that is on the line through the current missile position in the current missile direction

$\mathbf{P}_2$: a point on a line through the target that defines a possible direction from which the attacking missile will approach the target

$\mathbf{P}_3$: the position of the target.

Different paths are generated by modifying $\mathbf{P}_1$ and $\mathbf{P}_2$ while $\mathbf{P}_0$ and $\mathbf{P}_3$ remain fixed. Our examples typically have 500 different paths to give a good approximation of the probability that the attacking missile will pass through any spatial region. We assume that the attacking missile will travel at a constant speed throughout its flight. Figure 4.2 shows 500 sample missile paths (orange lines) for an attacking missile travelling at $\mathbf{v}_\mathrm{a} = (-0.6\,\mathrm{km\,s^{-1}}, 0\,\mathrm{km\,s^{-1}}, 0\,\mathrm{km\,s^{-1}})$ when detected at $\mathbf{P}_0 = (30\,\mathrm{km}, 0\,\mathrm{km}, 7\,\mathrm{km})$ (red point). Paths with curve radius less than 7.339 km have been discarded since they have a lateral acceleration greater than $5g$ where g is gravitational acceleration ($9.81\,\mathrm{ms^{-2}}$). The actual path of the attacking missile (red line) is unknown to the defending missile team.

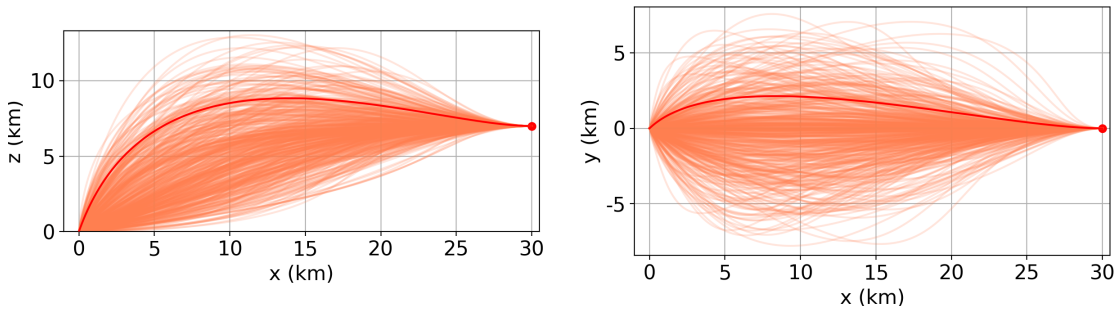


Figure 4.2: 500 possible attacking missile paths (orange lines) in the xz plane (left) and xy plane (right) originating at position $\mathbf{P}_0 = (30\,\text{km}, 0\,\text{km}, 7\,\text{km})$ (red point) with velocity $\mathbf{v}_\text{a} = (-0.6\,\text{km}\,\text{s}^{-1}, 0\,\text{km}\,\text{s}^{-1}, 0\,\text{km}\,\text{s}^{-1})$. The actual path of the attacking missile is indicated by the red line.

4.2 Choosing an intercept time

Let t_0 be the time when the attacking missile is first detected by the ground-based radar. Each defending missile j will be launched at some time $t_{\text{L}j} > t_0$ and will fly towards an unique aim point where it will turn on its seeker to seek and then attempt to intercept the attacking missile. An aim point specifies the time, position and direction $(t_{\text{da}j}, \mathbf{P}_{\text{da}j}, \mathbf{d}_{\text{da}j})$ of the defending missile j. The 'da' in the subscript indicates 'defending missile at the aim point'.

The target will have a *keep-out* zone which we can consider to be the defending missile's minimum intercept range r_i from the target. If we wait for the attacking missile to get closer to the target then the remaining time that the attacking missile has to manoeuvre towards its target reduces and the manoeuvrability region of the attacking missile becomes smaller, which will make it easier to intercept. Section 4.14 gives an example of the intercept region for keep-out zones at 15 km and 20 km.

For the team of defending missiles to intercept the attacking missile close to the keep-out zone, the time that the attacking missile would hit the target is defined by approximating the path of the attacking missile by a straight line between its position $\mathbf{P}_0$ and the position of the target $\mathbf{P}_3$, and calculating the time at which

the attacking missile will reach the keep-out distance r_i as

$$t_\mathrm{i} = t_0 + \frac{|\mathbf{P}_3 - \mathbf{P}_0| - r_\mathrm{i}}{|\mathbf{v}_\mathrm{a}|}. \tag{4.1}$$

For an attacking missile travelling at $\mathbf{v}_\mathrm{a} = (-0.6\,\mathrm{km\,s^{-1}}, 0\,\mathrm{km\,s^{-1}}, 0\,\mathrm{km\,s^{-1}})$ at position $\mathbf{P}_0 = (30\,\mathrm{km}, 0\,\mathrm{km}, 7\,\mathrm{km})$, the target intercept time is $t_\mathrm{i} = 26.34$ seconds for a keep-out zone at $r_\mathrm{i} = 15\,\mathrm{km}$.

Equation 4.1 underestimates the target intercept time, which means that the defending missiles will be launched earlier than needed and so intercept the attacking missile outside the keep-out zone. The difference between the earliest intercept time and the actual intercept time will be less than a few seconds.

4.3 Calculating a seek time

Each defending missile will have a seeker with a detection range of r_s. The aim point for each defending missile is chosen such that each defending missile will arrive at its aim point at the same time t_s as the attacking missile enters the seek region. The attacking missile cannot be intercepted unless it is seen by at least one defending missile. The seek time t_s occurs before the intercept time t_i because the attacking missile will reach the seek region before it reaches the intercept region, so $t_\mathrm{s} < t_\mathrm{i}$.

Assume that the attacking missile and a defending missile are flying towards each other along a straight line at a constant speed and will intercept at time t_i. If the attacking missile is traveling at constant speed $|\mathbf{v}_\mathrm{a}|$ and the defending missile at constant speed $|\mathbf{v}_\mathrm{d}|$ then the closing speed is $|\mathbf{v}_\mathrm{a}| + |\mathbf{v}_\mathrm{d}|$. The defending missile can detect the attacking missile when the two missiles are at distance r_s apart. Thus the earliest time at which seeking should begin is:

$$t_\mathrm{s} = t_\mathrm{i} - \frac{r_\mathrm{s}}{|\mathbf{v}_\mathrm{a}| + |\mathbf{v}_\mathrm{d}|}. \tag{4.2}$$

Equation 4.2 underestimates the seek time, which means that the defending missiles will arrive at their aim points and turn on their seekers earlier than needed

to intercept the attacking missile outside the keep-out zone. This is a strategic decision to ensure that the attacking missile does not arrive at the seek region earlier than the defending missiles, while keeping the calculations simple.

4.4 Seek region

The seek region comprises points in time and space where the attacking missile could be when it is first detected by the team of defending missiles. The seek points are points $\mathbf{P}_i(t_\mathrm{s})$ that occur along each predicted attacking missile path i at time t_s, as shown by the blue points in Figure 4.3.

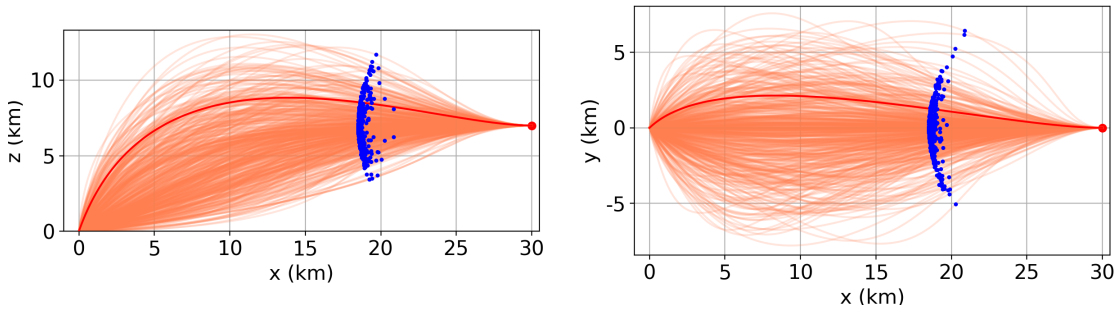


Figure 4.3: The seek region contains the set of possible attacking missile positions (blue seek points) in the the xz plane (left) and xy plane (right) that occur at time t_s along each predicted attacking missile path (orange lines).

4.5 Intercept region

The intercept region comprises points in time and space where the attacking missile could be when it is intercepted by one or more of the defending missiles. The defending missiles must reach the intercept region at the same time as the attacking missile. The intercept points can be approximated by points $\mathbf{P}_i(t_\mathrm{i})$ that occur along each predicted attacking missile path at time t_i, as shown by the red points in Figure 4.4.

4.6 Defending missile detection region

A defending missile's seeker will usually have a narrow field of view. The detection region of a defending missile is modelled as a conical sector of a sphere. When a

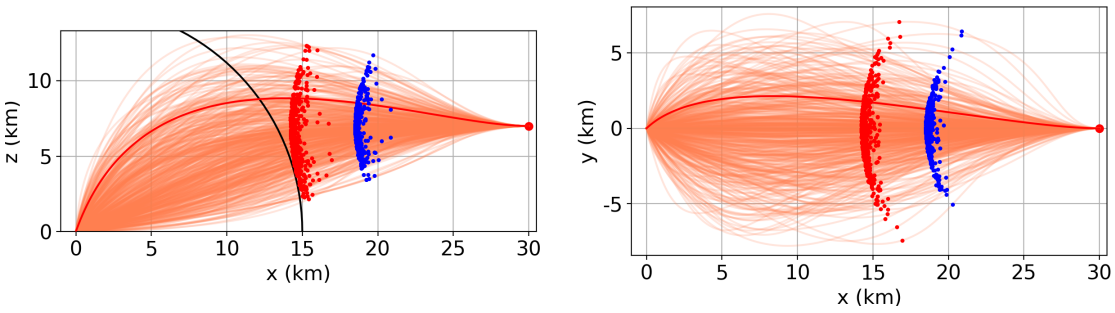


Figure 4.4: The intercept region contains the set of possible attacking missile positions (red intercept points) in the xz plane (left) and xy plane (right) that occur at time t_{i} along each predicted attacking missile path (orange lines). The keep-out zone is 15 km from the target and is shown by the black arc in the xz plane (left image).

defending missile j is at its aim point it will have position $\mathbf{P}_{\mathrm{da}j}$ and be heading in direction $\mathbf{d}_{\mathrm{da}j}$. The detection range is r_{s} and the angular field of view is θ_s from the heading in any direction. A defending missile will be able to see a predicted attacking missile path if the predicted seek point along that path can be detected by a defending missiles on-board seeker:

- the distance between the aim point and seek point is less than the seeker's detection range, that is $|\mathbf{P}_i(t_{\mathrm{s}}) - \mathbf{P}_{\mathrm{da}j}| \leq r_{\mathrm{s}}$, and
- the angle between the aim point direction $\mathbf{d}_{\mathrm{da}j}$ and the direction towards the seek point $\mathbf{P}_i(t_{\mathrm{s}})$ is less than the defending missiles seeker field of view, that is

$$\arccos\left(\frac{(\mathbf{P}_i(t_{\mathrm{s}}) - \mathbf{P}_{\mathrm{da}j}) \cdot \mathbf{d}_{\mathrm{da}j}}{|\mathbf{P}_i(t_{\mathrm{s}}) - \mathbf{P}_{\mathrm{da}j})||\mathbf{d}_{\mathrm{da}j}|}\right) \leq \theta_{\mathrm{s}}.$$

Once a path is seen by one defending missile, it is considered to be seen by all defending missiles. The more paths that the defending missiles can see, the more likely it is that the attacking missile will be detected. Once one defending missile detects the attacking missile then all the defending missiles will manoeuvre to intercept the attacking missile.

Figure 4.5 shows the portion of the seek region (blue points) that is covered by a defending missile at $\mathbf{P}_{\mathrm{da}j} = (11.547\,\mathrm{km}, -0.284\,\mathrm{km}, 4.332\,\mathrm{km})$ with a seeker view

angle of $\pm 5^\circ$ and a detection range of 10 km. The image on the right shows the seek points in the yz plane that the defending missile can see (green points) and the seek points that the defending missile cannot see (pale blue points). The aim point in this case (blue point in the left diagram) was found using the optimisation process described in Sections 4.11 and 4.12.

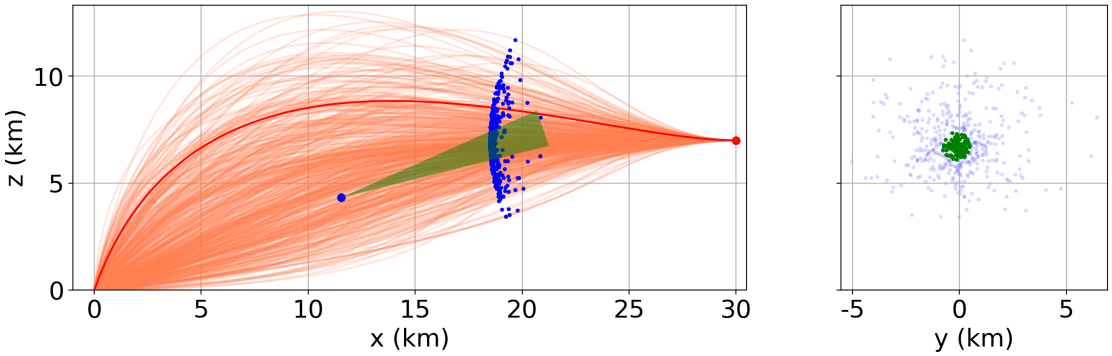


Figure 4.5: The defending missile's detection region (green region) from the aim position $\mathbf{P}_{\mathrm{a}j} = (11.547\,\mathrm{km}, -0.284\,\mathrm{km}, 4.332\,\mathrm{km})$ (single blue point) and the seek region (blue points). The image on the right shows the seek points, in the yz plane, that the defending missile can see (green points) and the seek points that the defending missile cannot see (pale blue points).

4.7 Defending missile manoeuvre region

Suppose defending missile j has an aim point position and direction $(\mathbf{P}_{\mathrm{da}j}, \mathbf{d}_{\mathrm{da}j})$. The defending missiles have a minimum turning radius r_{dt} corresponding to its maximum lateral acceleration. Assuming a defending missile j reaches its aim point at time t_{s}, it must then manoeuvre to intercept the attacking missile at time t_{i}. The manoeuvre region of each defending missile is approximated as a conical sector of a sphere with the center point at the aim point position $\mathbf{P}_{\mathrm{da}j}$. The half-angle of the manoeuvre cone is limited by the minimum turning radius of the defending missile j and the time that the defending missile j has to reach the intercept region:

$$\theta_{\mathrm{m}} = \frac{(t_{\mathrm{i}} - t_{\mathrm{s}})|\mathbf{v}_{\mathrm{d}}|}{r_{\mathrm{dt}}}.$$

A point in the intercept region can be intercepted by the defending missile j if the angle between the defending missile j's aim direction $\mathbf{d}_{\mathrm{da}j}$ and intercept point

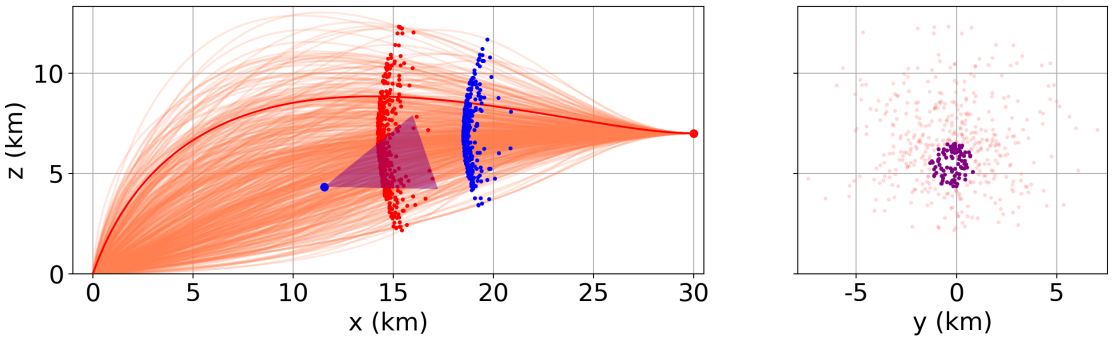


Figure 4.6: The defending missile's manoeuvre region (purple region) from the aim position $\mathbf{P}_{\mathrm{a}j} = (11.547\,\mathrm{km}, -0.284\,\mathrm{km}, 4.332\,\mathrm{km})$ (single blue point). The image on the right shows the intercept points, in the yz plane, that the defending missile can hit (purple points) and the intercept points that the defending missile cannot hit (pale red points).

$\mathbf{P}_i(t_{\mathrm{i}})$ is less than the angle of the defending missile's manoeuvre cone such that

$$\arccos\left(\frac{(\mathbf{P}_i(t_{\mathrm{i}}) - \mathbf{P}_{\mathrm{da}j}) \cdot \mathbf{d}_{\mathrm{da}j}}{|\mathbf{P}_i(t_{\mathrm{i}}) - \mathbf{P}_{\mathrm{da}j}||\mathbf{d}_{\mathrm{da}j}|}\right) \leq \theta_{\mathrm{m}}.$$

Figure 4.6 shows the portion of the intercept region (red points) that is covered by a defending missile at $\mathbf{P}_{\mathrm{a}j} = (11.547\,\mathrm{km}, -0.284\,\mathrm{km}, 4.332\,\mathrm{km})$ with a minimum turning radius of 16.310 km and a maximum lateral acceleration of $4g$ and constant speed of $0.8\,\mathrm{km\,s^{-1}}$. This is the same scenario and aim point as in Figure 4.5. The image on the right shows the intercept points in the yz plane that the defending missile can hit (purple points) and the intercept points that the defending missile cannot hit (pale red points).

4.8 Aim points

Each defending missile j is assigned an aim point that it will fly towards after it is launched. An aim point $(t_{\mathrm{da}j}, \mathbf{P}_{\mathrm{da}j}, \mathbf{d}_{\mathrm{da}j})$ comprises an arrival time $t_{\mathrm{da}j}$, a position $\mathbf{P}_{\mathrm{da}j}$ and a unit direction vector $\mathbf{d}_{\mathrm{da}j}$ of the defending missile j. Once each defending missile reaches its respective aim point, it will activate its on-board seeker. The aim points should be positioned such that the defending missiles will be able to cover as much of the seek and intercept regions as possible. The more

paths that the defending missiles can see, the more likely it is that the defending missiles will be able to detect the attacking missile, and the more paths that the defending missiles can hit, the more likely it is that the defending missile will be able to intercept the attacking missile. Once a defending missile has detected the attacking missile, then all the defending missiles will manoeuvre to intercept the attacking missile.

After launch, the defending missiles will receive periodic updates of the attacking missile's position and direction from the ground-based radar. At every radar update, the predicted paths of the attacking missile, intercept time, seek time, seek region, intercept region, aim points and defending missile paths will change. The challenge is to ensure that the defending missile paths toward their respective aim point remains feasible after launch. An optimisation problem has been formulated in Section 4.11.

4.9 Defending missile paths

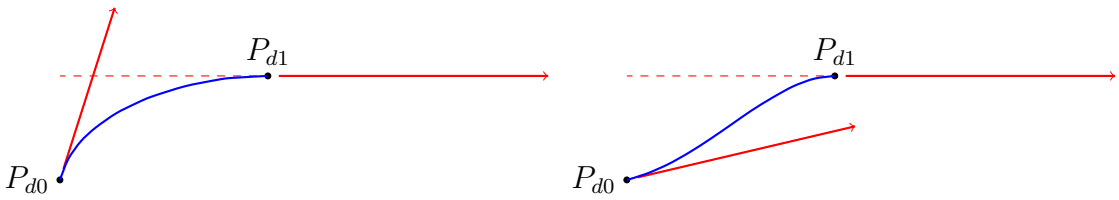
The path for each defending missile is formed from a current position and direction to its aim point using an optimised geometric Hermite curve with minimum strain energy [Yong and Cheng, 2004]. This type of curve is found by optimizing the magnitudes of the endpoint tangent vectors, a_0 and a_1, in the Hermite interpolation process so that the *strain energy* of the curve is minimised. Minimising the strain energy is equivalent to minimising the integral of the average curvature squared along the path. Using optimised geometric Hermite curves is convenient and ensures that the defending missile paths do not have too much curvature. Given a defending missile's position $\mathbf{P}_{\mathrm{d}0}$ and direction $\mathbf{d}_{\mathrm{d}0}$ and respective aim point position $\mathbf{P}_{\mathrm{d}1}$ and direction $\mathbf{d}_{\mathrm{d}1}$, the optimised geometric Hermite curve [Yong and Cheng, 2004] is expressed as

$$\mathbf{Q}(t) = (2t+1)(t-1)^2\mathbf{P}_{\mathrm{d}0} + (-2t+3)t^2\mathbf{P}_{\mathrm{d}1} + (1-t)^2 t a_0 \mathbf{d}_{\mathrm{d}0} + (t-1)t^2 a_1 \mathbf{d}_{\mathrm{d}1}$$

where

$$a_0 = \frac{6((\mathbf{P}_{d1} - \mathbf{P}_{d0}) \cdot \mathbf{d}_{d0})(\mathbf{d}_{d1} \cdot \mathbf{d}_{d1}) - 3((\mathbf{P}_{d1} - \mathbf{P}_{d0}) \cdot \mathbf{d}_{d1})(\mathbf{d}_{d0} \cdot \mathbf{d}_{d1})}{4(\mathbf{d}_{d0} \cdot \mathbf{d}_{d0})(\mathbf{d}_{d1} \cdot \mathbf{d}_{d1}) - (\mathbf{d}_{d0} \cdot \mathbf{d}_{d1})^2}$$
$$a_1 = \frac{3((\mathbf{P}_{d1} - \mathbf{P}_{d0}) \cdot \mathbf{d}_{d0})(\mathbf{d}_{d0} \cdot \mathbf{d}_{d1}) - 6((\mathbf{P}_{d1} - \mathbf{P}_{d0}) \cdot \mathbf{d}_{d1})(\mathbf{d}_{d0} \cdot \mathbf{d}_{d0})}{(\mathbf{d}_{d0} \cdot \mathbf{d}_{d1})^2 - 4(\mathbf{d}_{d0} \cdot \mathbf{d}_{d0})(\mathbf{d}_{d1} \cdot \mathbf{d}_{d1})}.$$

Figure 4.7 shows two example defending missile paths, from launch to a horizontal aim point. Figure 4.7a has a high launch angle, whereas Figure 4.7b has a lower launch angle. The blue curves are the paths defined by optimised geometric Hermite curve. The red vectors are the optimal endpoint tangent vectors. With the high launch angle, the defending missile curves downwards throughout its path. With the low launch angle, the missile curves up and then down.



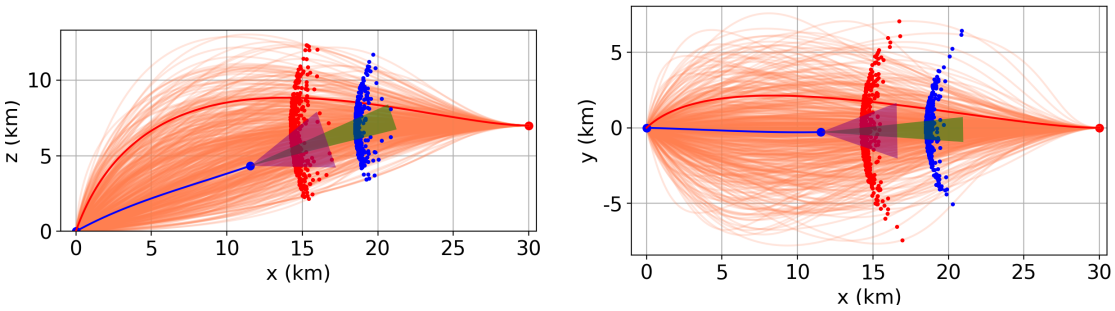
(a) High launch angle.

(b) Low launch angle.

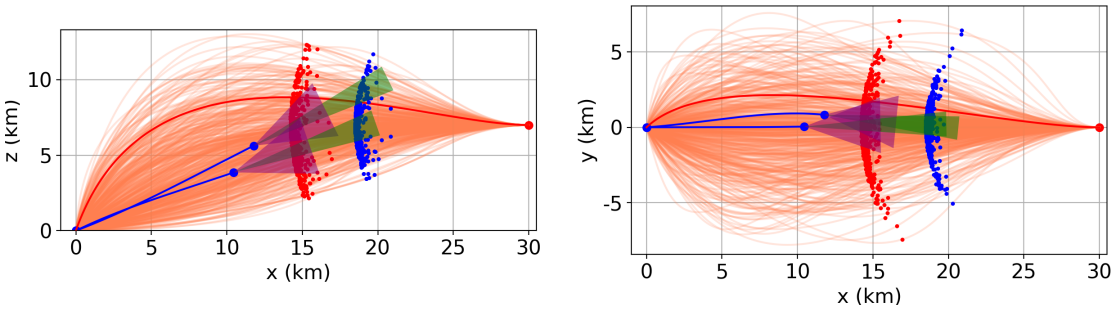
Figure 4.7: Optimised geometric Hermite curves (blue line) and optimised endpoint tangent vectors (red lines).

A good launch angle should ensure that the fly-out path to the aim point has low curvature, and that future positions and directions on the fly-out path will be suitable for generating future paths when the position of the attacking missile is updated from ground-based radar measurements. We found, by trial and error, that setting the launch directions to intercept the aim direction at a point 1/10 of the way between $x = x_{d0}$ and $x = x_{d1}$, as shown in Figure 4.7a, gave good results. We prefer the defending missiles to be launch higher because if the attacking missile decides to take an extreme path with higher altitude then the defending missiles will have time to better position themselves to cover those paths. A lower launch angle will limit the defending missiles ability to cover extreme paths.

Figure 4.8 shows the optimised geometric Hermite curves (blue lines) for a single defending missile (Figure 4.8a) and two defending missiles (Figure 4.8b) before launch. The aim points have been generated by solving the optimisation problem outlined in Section 4.11.



(a) One defending missile path at initial radar detection (time t_0).



(b) Two defending missile paths at initial radar detection (time t_0).

Figure 4.8: Examples of defending missile paths for one and two missiles at initial radar detection (time t_0).

4.10 Launch time

The aim point for each defending missile includes the seek time $t_{\mathrm{da}j} = t_{\mathrm{s}}$ at which the defending missile should arrive at the aim point and turn on its seeker. If the length of the fly-out path for defending missile j is l_j then the launch time required is

$$t_{\mathrm{dl}j} = t_{\mathrm{s}} - \frac{l_j}{v_{\mathrm{d}j}}$$

where $v_{\mathrm{d}j}$ is the speed of the defending missile.

The length of a fly-out path can be found by numerical integration.

4.11 Optimising aim points

Suppose there are M defending missiles. Each defending missile j will fly towards its respective aim point $(t_{\mathrm{da}j}, \mathbf{P}_{\mathrm{da}j}, \mathbf{d}_{\mathrm{da}j})$. The objective is to choose aim points that maximise the likelihood of stopping the attacking missile. It is more convenient to write this objective as minimising the probability of missing the attacking missile with every defending missile. Suppose there are N predicted attacking missile paths. The team of defending missiles can intercept the predicted attacking missile path i only if path i is seen by at least one defending missile. The probability of the team of defending missiles missing path i is:

$$m_i = \begin{cases} 1 & \text{if path } i \text{ cannot be seen} \\ f^{\mu_i} & \text{if path } i \text{ can be seen} \end{cases}$$

where f is the probability of an individual defending missile failing to stop the attacking missile even though it can manoeuvre to an intercept point, and μ_i is the number of defending missiles that can feasibly intercept path i. The probability of the team of defending missiles missing the attacking missile is

$$P_{\mathrm{miss}} = \sum_{i=1}^{\mathrm{N}} p_i m_i$$

where p_i is the probability that the attacking missile will take the predicted path i, and where $\sum_i^N p_i = 1$. A similar probability of miss has been described in Azzollini et al. [2021].

The manoeuvre region of the attacking missile is not uniformly distributed. Furthermore, an attempted intercept on a particular path may fail, so the probability of missing the attacking missile on a particular path decreases with the number of attempted intercepts. The optimisation needs to determine whether it is beneficial to thinly cover the entire seek and intercept regions or to concentrate coverage in dense areas of the seek and intercept regions to minimise the probability of missing the attacking missile. In general, the more paths that each defending missile can see and hit, the greater the probability of stopping the attacking missile.

After launching the defending missiles, periodic updates of the attacking missile's position and direction are received from the ground-based radar. At every update, new aim points with new defending missile paths are calculated. Defending missiles have restricted manoeuvrability (turning rate), therefore the new aim points need to be positioned such that each defending missile can correct its path towards its new aim point.

Suppose defending missile j has direction vector $\mathbf{d}_{\mathrm{d}jk}$ at the k^{th} radar update where $k \geq 0$ and the updated aim point vector is $\mathbf{d}_{\mathrm{da}jk}$. We can approximate the angles that the defending missile will have to turn through by the angles:

- $|\theta_{\mathrm{d}}|$ between the current direction and the line from the current location to the aim point location, and
- $|\theta_{\mathrm{da}}|$ between the line from the current location to the aim point location and the required direction at the aim point.

The direction vectors (red lines) and the angles that the defending missile will turn through are illustrated in Figure 4.9. In this case, the defending missile will have to turn right away from its current path then left to achieve the aim direction.

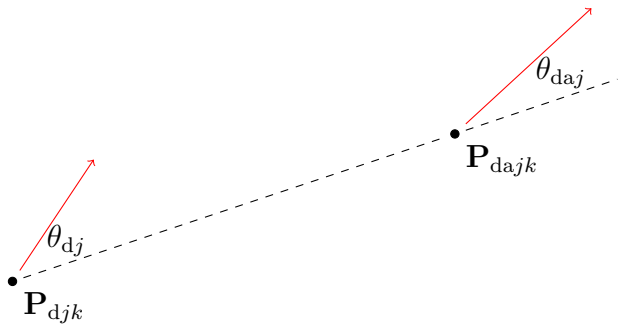


Figure 4.9: Approximating the angles the defending missile j will turn through.

The total change in heading for the two direction changes will be $\Theta_j = |\theta_{\mathrm{d}j}| + |\theta_{\mathrm{da}j}|$ where

$$\theta_{\mathrm{d}j} = \arccos \frac{(\mathbf{P}_{\mathrm{da}jk} - \mathbf{P}_{\mathrm{d}jk}) \cdot \mathbf{d}_{\mathrm{d}jk}}{|\mathbf{P}_{\mathrm{da}jk} - \mathbf{P}_{\mathrm{d}jk}||\mathbf{d}_{\mathrm{d}jk}|}$$

and

$$\theta_{\mathrm{da}j} = \arccos \frac{(\mathbf{P}_{\mathrm{da}jk} - \mathbf{P}_{\mathrm{d}jk}) \cdot \mathbf{d}_{\mathrm{da}jk}}{|\mathbf{P}_{\mathrm{da}jk} - \mathbf{P}_{\mathrm{d}jk}||\mathbf{d}_{\mathrm{da}jk}|}.$$

We could introduce a hard constraint $\Theta \leq \Theta_{\max}$, but this would mean introducing Lagrange multipliers and Karush-Kuhn-Tucker conditions to solve a non-linear constrained optimisation problem [Beavis and Dobbs, 1990]. Instead, since smaller angle changes are more desirable, we add a simple penalty function to our objective function to give the new objective function

$$\zeta = P_{\mathrm{miss}} + qP_{\Theta}$$

where the penalty for turns is given by the mean of the angles squared

$$P_{\Theta} = \frac{\sum_{j=1}^{M} \Theta_j^2}{M}$$

and where q is the penalty weight [Bazaraa et al., 2006]. As the defending missiles approach the intercept region, it becomes less feasible for them to achieve high angle changes to their aim points because of the limited time and space available for them to manoeuvre. We increase the penalty weight at every radar update to avoid large angle changes as the defending missiles get closer to the aim points. We use

$$q_{k+1} = q_0(\log(k+1) + 1).$$

Other penalty functions P_{Θ} were considered, including the mean angle Θ_j, root mean square, maximum and maximum square of the angles. We found that the mean of the angles squared provided a lower probability of miss on average compared to the other penalty functions. Some of the results are shown in Figure 4.10. The aim of the experiment was to find a penalty function that required a low penalty weight to minimise the objective function without compromising the probability of miss. The results show the different penalty weights that gave a maximum angle no greater than 20 degrees for three defending missiles. The mean of squared penalty function gave low probability of miss without the angles becoming too large.

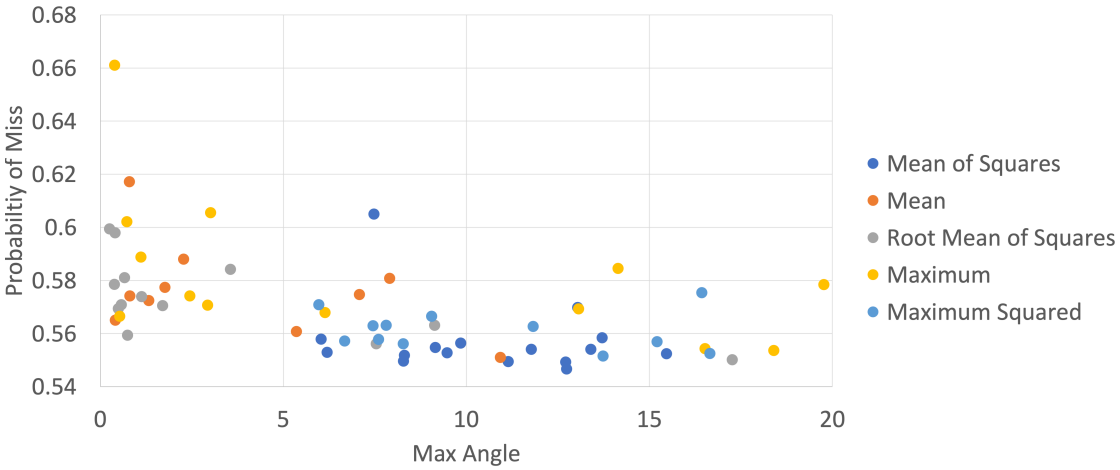


Figure 4.10: Comparing different penalty functions, where the x axis shows the maximum angle generated and the y axis shows the probability of miss.

4.12 Solving the optimisation problem

We solved the unconstrained non-linear optimisation problem using the Nelder-Mead method [Nelder and Mead, 1965], which is a direct search method commonly used to solve unconstrained multi-dimensional problems and non-linear problems [Gao and Han, 2010]. Azzollini et al. [2021] implemented the Nelder-Mead method for their problem because it does not require derivatives, is easy to implement and can handle multidimensional non-linear optimisation. Other algorithms such as compass search or the pattern search method of Hooke and Jeeves [1961] are reliable but sometimes slow in practice compared to the Nelder-Mead method[KOLDA et al., 2003].

We use the Nelder-Mead method to minimize the probability of miss P_{miss} by searching for good aim points for each defending missile j. Each aim point has 5 variables:

$$(x_{\text{da}j}, y_{\text{da}j}, z_{\text{da}j}, \alpha_{\text{da}j}, \beta_{\text{da}j}) \tag{4.3}$$

where $\alpha_{\text{da}j}$ is the direction angle in the xy plane and $\beta_{\text{da}j}$ is the direction angle in the xz plane. The Nelder-Mead method maintains a non-degenerate simplex—a geometric figure in $5M$ dimensions of non-zero volume that is the convex hull of $5M + 1$ vertices [Lagarias et al., 1998]. Because the number of variables increases

by 5 for each defending missile, the dimension of the optimisation problem could become too large to solve in a reasonable time for a large number of defending missiles. However, the method was fast enough for analysis of examples with up to 10 missiles.

The initial simplex is a set of random aim points where the direction vector of each aim point passes through both the intercept region and the seek region. Figure 4.11 is an example of an initial aim point solution for a team of 10 defending missiles.

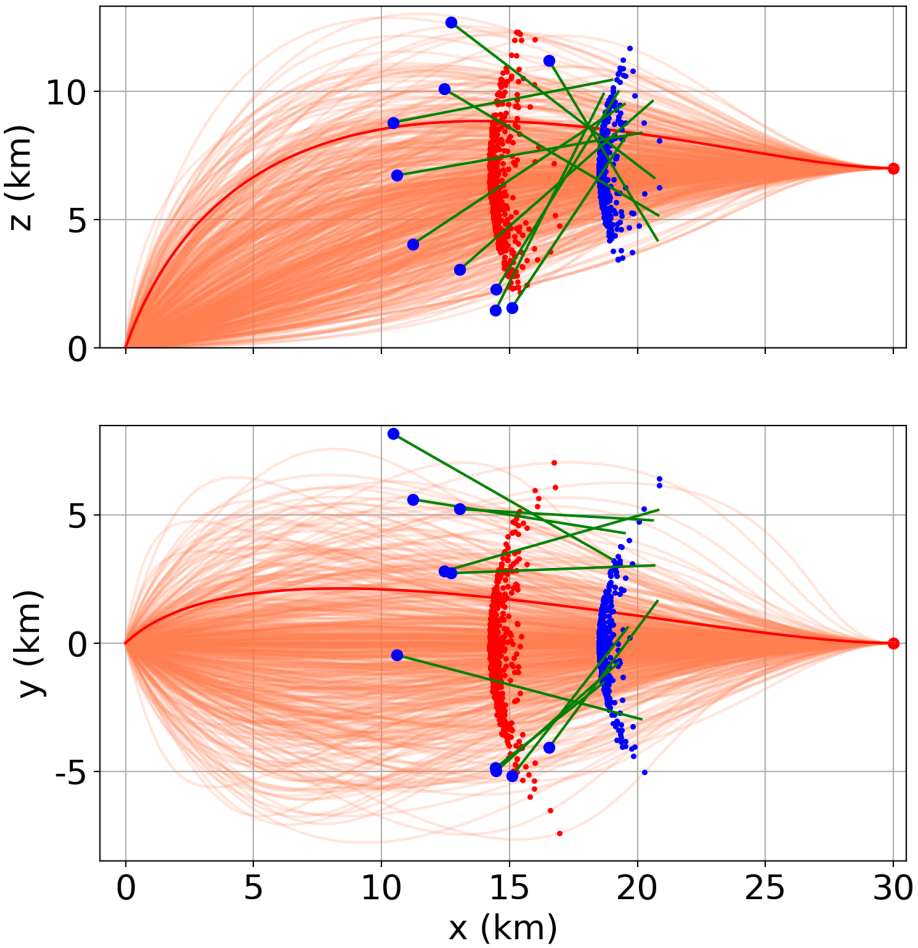
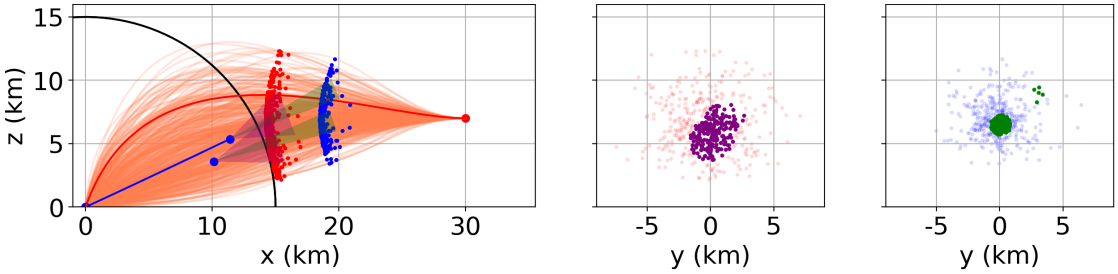


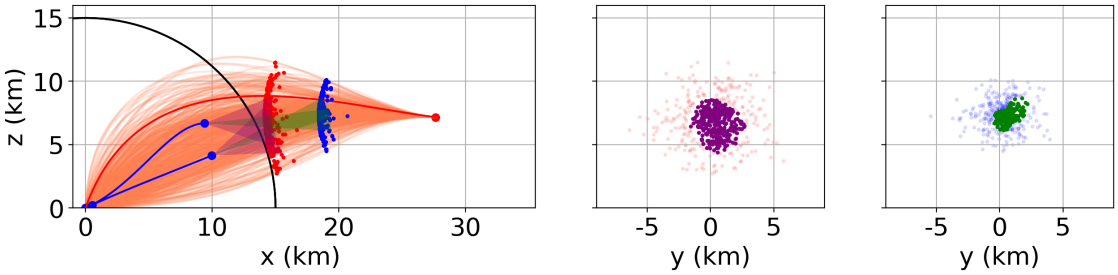
Figure 4.11: Random initial aim point solution for 10 defending missiles. The top image shows the aim vectors (green lines) in the xz plane and the bottom image shows the aim vectors in the xy plane.

4.13 Radar updates

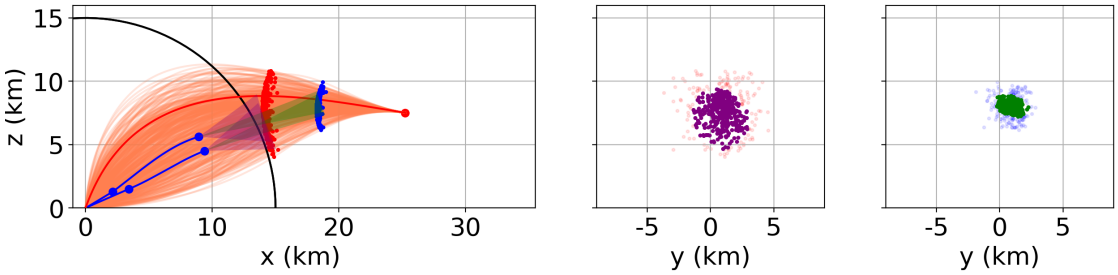
Prior to launching the defending missiles, and during their fly-out phase, the ground-based radar will provide periodic updates on the attacking missile's position and velocity. At every radar update, the predicted paths of the attacking missile, intercept time, seek time, seek region, intercept region, aim points and defending missile paths will be re-calculated with the new position and velocity of the attacking missile. This means that after launch, defending missiles may receive updated aim points to fly towards. Consequently, they may reach their new aim point sooner or later than when the attacking missile reaches the updated seek region. Figure 4.12 is an example of radar updates given for an attacking missile initially detected at time $t_0 = 0$ at position $\mathbf{P}_0 = (30\,\text{km}, 0\,\text{km}, 7\,\text{km})$ (red point) with velocity $\mathbf{v}_\text{a} = (-0.6\,\text{km}\,\text{s}^{-1}, 0\,\text{km}\,\text{s}^{-1}, 0\,\text{km}\,\text{s}^{-1})$. The attacking and defending missiles are travelling at constant speeds $|\mathbf{v}_\text{a}| = 0.6\,\text{km}\,\text{s}^{-1}$ and $|\mathbf{v}_\text{d}| = 0.8\,\text{km}\,\text{s}^{-1}$ respectively. Aim points are calculated for two defending missiles. The defending missiles each have a probability of failing to intercept $f = 0.5$ and a minimum turning radius of $d_\text{tr} = 16.31\,\text{km}$ (maximum lateral acceleration of $4g$). The initial penalty weight is $q_0 = 0.16$. The probability of miss at launch time was $P_\text{miss}(t_0) = 0.8265$, and was reduced to $P_\text{miss}(t_3) = 0.36$ after three radar updates. If the team of defending missiles were able to cover all the seek and intercept points then the probability of miss could be minimised to $P_\text{miss} = 0.25$.



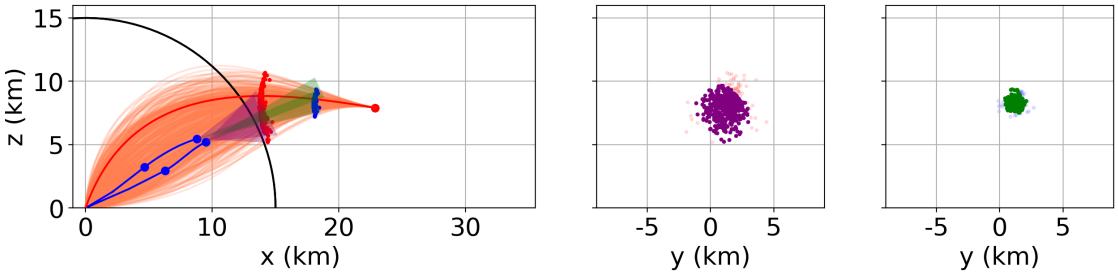
(a) Aim points at initial radar detection ($P_{\text{miss}} = 0.8265$). The first defending missile will be launched 3.29 seconds from initial detection.



(b) Aim points at second radar detection ($P_{\text{miss}} = 0.675$). The second defending missile will be launched 4.857 seconds from initial detection.



(c) Aim points at third radar detection ($P_{\text{miss}} = 0.5475$).



(d) Aim points at fourth radar detection ($P_{\text{miss}} = 0.36$).

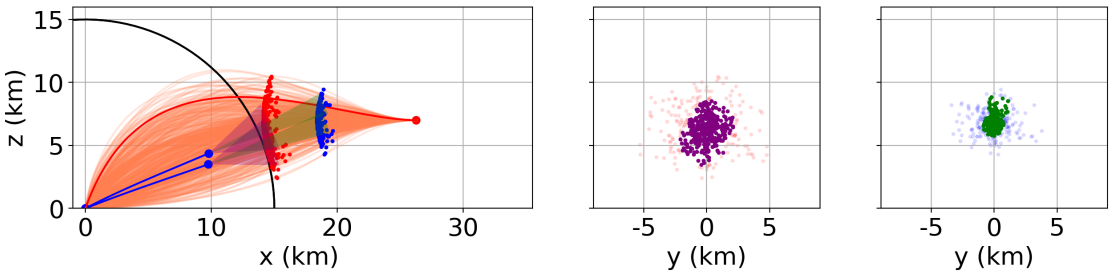
Figure 4.12: An example with a team of two defending missile where radar updates are provided every 4 seconds. The middle image provides the intercept points (red points) in the yz plane and the right-most image provides the seek points (blue points) in the yz plane.

4.14 Keep-out range

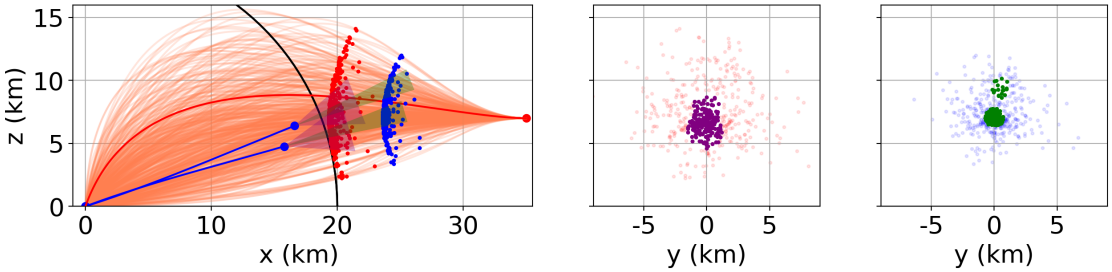
This section shows the influence of the keep-out range on the probability of miss for an example with two defending missiles. Suppose the attacking missile is travelling at a constant speed of $0.6\,\mathrm{km\,s^{-1}}$ and the defending missiles at constant speed $0.8\,\mathrm{km\,s^{-1}}$. The attacking missile is limited to $5g$ manoeuvres and the defending missiles $4g$ manoeuvres. The defending missiles have an individual probability of fail of 0.1.

Figure 4.13 shows the intercept and seek regions when the keep-out zones (black arc) are 15 km (Figure 4.13a) and 20 km (Figure 4.13b). The larger keep-out range gives a bigger seek region and a bigger intercept region.

When we optimise the positions of the aim points (blue points are the aim point positions, purple cones are the defending missile manoeuvre region, green cones are the defending missile detection region), the probability of miss for two defending missiles at time of launch are 0.41 for a keep-out zone at 15 km and 0.74 for a keep-out zone at 20 km. The probability of miss is better when the intercept region is closer to the target.



(a) Example intercept and seek regions when keep-out range is 15 km. The probability of miss for two defending missiles at time of launch is 0.41.

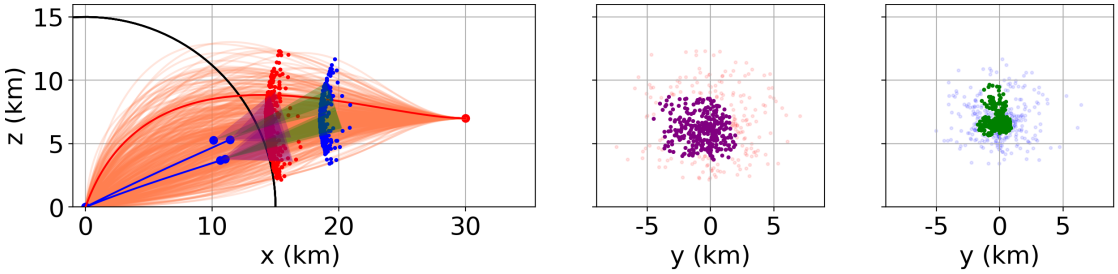


(b) Example intercept and seek regions when keep-out range is 20 km. The probability of miss for two defending missiles at time of launch is 0.74.

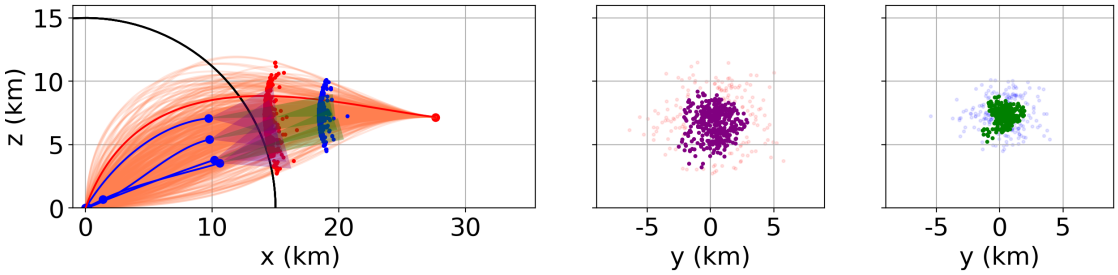
Figure 4.13: Example of the change in intercept and seek regions for different values of the keep-out range.

4.15 Four-missile example

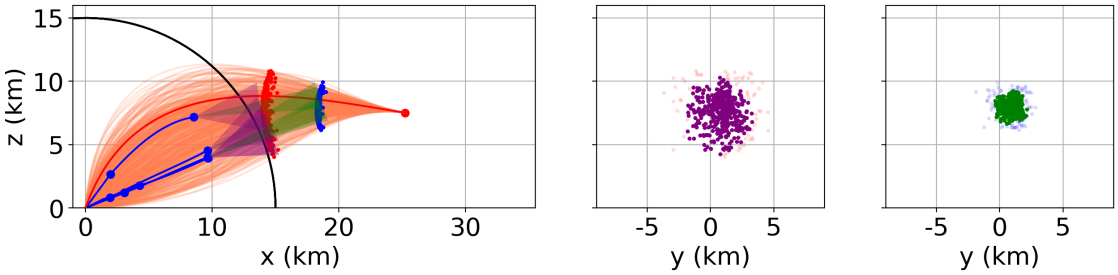
The example shown in Figure 4.14 is for an attacking missile initially detected at time $t_0 = 0$ at position $\mathbf{P}_0 = (30\,\text{km}, 0\,\text{km}, 7\,\text{km})$ (red point) with velocity $\mathbf{v}_\text{a} = (-0.6\,\text{km}\,\text{s}^{-1}, 0\,\text{km}\,\text{s}^{-1}, 0\,\text{km}\,\text{s}^{-1})$. The attacking and defending missiles are travelling at constant speeds $|\mathbf{v}_\text{a}| = 0.6\,\text{km}\,\text{s}^{-1}$ and $|\mathbf{v}_\text{d}| = 0.8\,\text{km}\,\text{s}^{-1}$ respectively. Aim points are calculated for four defending missiles. The defending missiles each have an individual probability of failing to intercept $f = 0.1$ and a minimum turning radius of $d_\text{tr} = 16.310\,\text{km}$ (maximum lateral acceleration of $4g$). The initial penalty weight is $q(t_0) = 0.16$. The probability of miss at launch time was $P_\text{miss}(t_0) = 0.5671$, and was reduced to $P_\text{miss}(t_3) = 0.0695$ after three radar updates. If the team of defending missiles were able to cover all the seek and intercept points then the probability of miss would be $P_\text{miss} = 0.0001$.



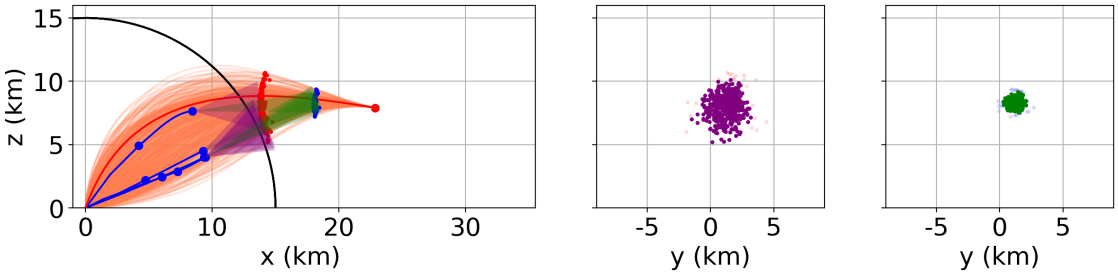
(a) Aim points at initial radar detection ($P_{\text{miss}} = 0.5671$). The first defending missile will be launched in 1.9 seconds and the second in 3.76 seconds from initial detection.



(b) Aim points at second radar detection ($P_{\text{miss}} = 0.3844$). The third defending missile will now be launched in 4 seconds and the fourth in 4.34 seconds from initial detection.



(c) Aim points at third radar detection ($P_{\text{miss}} = 0.1976$).

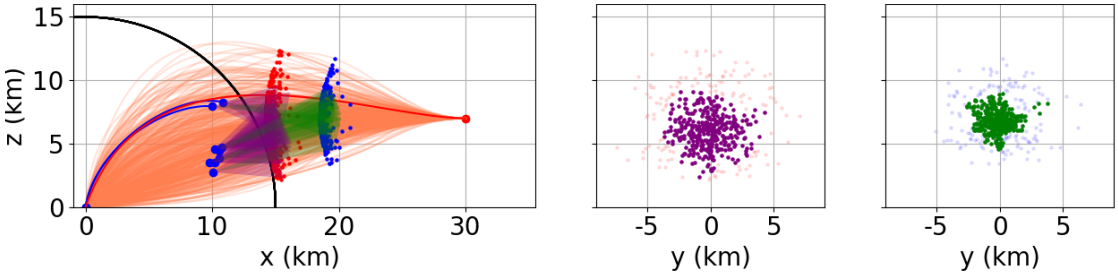


(d) Aim points at fourth radar detection ($P_{\text{miss}} = 0.0695$).

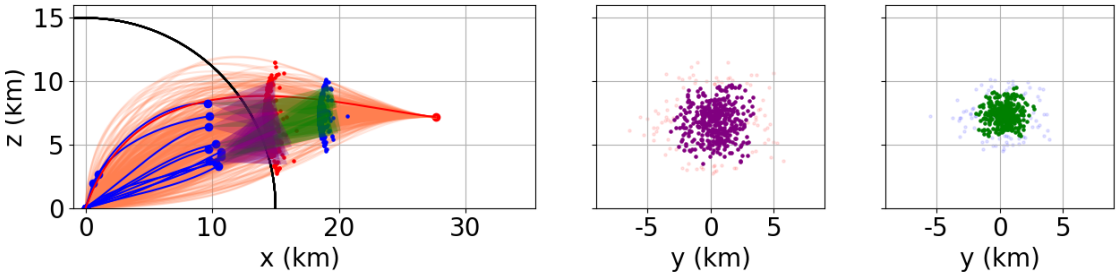
Figure 4.14: An example with a team of four defending missiles where radar updates are provided every 4 seconds. The middle image provides the intercept points (red points) in the yz plane and the right-most image provides the seek points (blue points) in the yz plane.

4.16 Ten-missile example

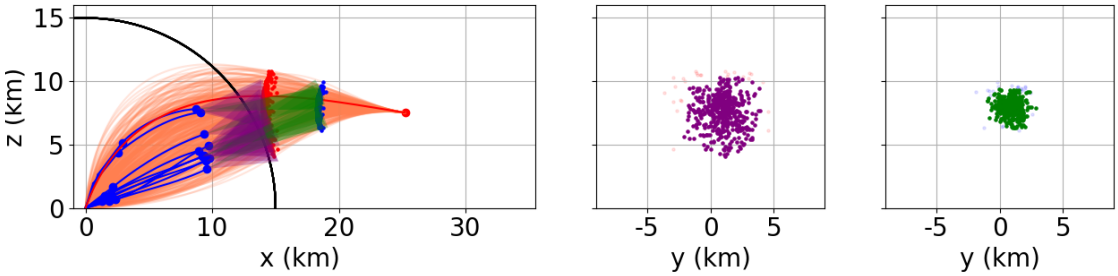
The example shown in Figure 4.15 is for an attacking missile initially detected at time $t_0 = 0$ at position $\mathbf{P}_0 = (30\,\text{km}, 0\,\text{km}, 7\,\text{km})$ (red point) with velocity $\mathbf{v}_\text{a} = (-0.6\,\text{km}\,\text{s}^{-1}, 0\,\text{km}\,\text{s}^{-1}, 0\,\text{km}\,\text{s}^{-1})$. The attacking and defending missiles are travelling at constant speeds $|\mathbf{v}_\text{a}| = 0.6\,\text{km}\,\text{s}^{-1}$ and $|\mathbf{v}_\text{d}| = 0.8\,\text{km}\,\text{s}^{-1}$ respectively. Aim points are calculated for ten defending missiles. The defending missiles each have an individual probability of failing to intercept $f = 0.1$ and a minimum turning radius of $d_\text{tr} = 16.310\,\text{km}$ (maximum lateral acceleration of $4g$). The initial penalty weight is $q(t_0) = 0.16$. The probability of miss at launch time was $P_\text{miss}(t_0) = 0.3203$, and was reduced to $P_\text{miss}(t_3) = 0.0102$ after three radar updates. If the team of defending missiles were able to cover all the seek and intercept points then the probability of miss would be $P_\text{miss} = 10^{-10}$.



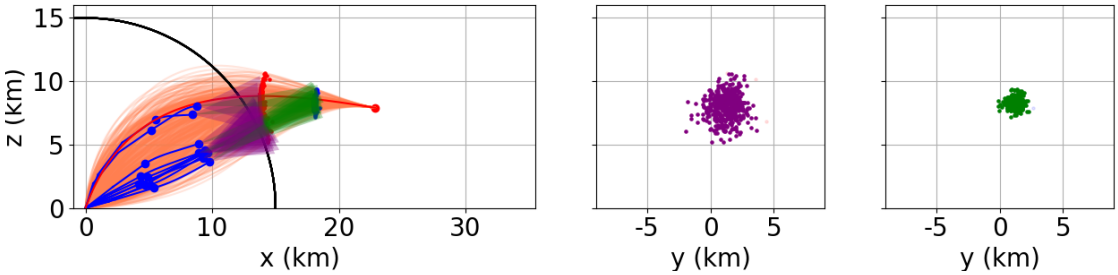
(a) Aim points at initial radar detection ($P_{\text{miss}} = 0.3203$). The first defending missile will be launched in 0.39 seconds and the second in 1.41 seconds from initial detection.



(b) Aim points at second radar detection ($P_{\text{miss}} = 0.2056$). The remaining eight defending missile will now be launched between 4.47 and 6.19 seconds from initial detection.



(c) Aim points at third radar detection ($P_{\text{miss}} = 0.0734$).



(d) Aim points at fourth radar detection ($P_{\text{miss}} = 0.0102$).

Figure 4.15: An example with a team of ten defending missiles where radar updates are provided every 4 seconds. The middle image provides the intercept points (red points) in the yz plane and the right-most image provides the seek points (blue points) in the yz plane.

4.17 Calculation times

The calculations of optimal aim points were done using Python code running on a laptop computer. The `minimize` function from the SciPy library was used to solve the optimisation problem, using the Nelder-Mead method. The SciPy library uses fast, compiled code. The time taken to calculate optimal aim points after each ground-based radar update was:

- 6–10 seconds for the example in Figure 4.12, with two defending missiles
- 27–36 seconds for the example in Figure 4.14, with four defending missiles
- 3–4.5 minutes for the example in Figure 4.15, with ten defending missiles.

These calculation times are longer than would be required for a real-time missile defence system, but are acceptable for proving the concept and for analysing the best type of missile defence team (Section 5.2).

Figure 4.16 shows that the calculation time increases as the number of defending missiles increase (explained in Section 4.12). This is because the Nelder-Mead method needs to find an aim point for each defending missile, find the predicted paths that each defending missile can see and hit, and then calculate the probability of miss that is achieved by the team of defending missiles.

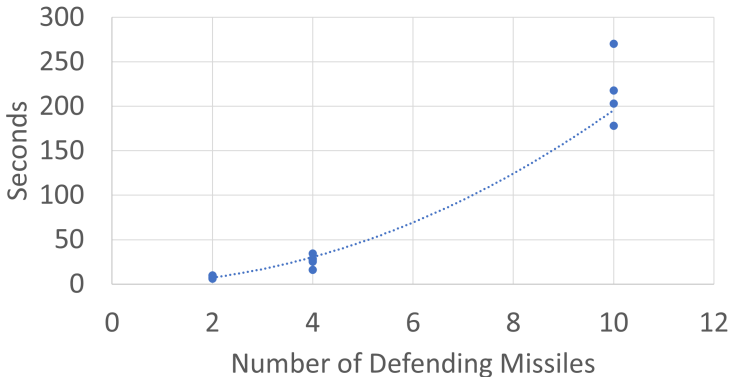


Figure 4.16: The relationship between calculation time and number of defending missiles.

If multiple cooperative defending missiles are a cost-effective approach then speeding up these calculations will be important. The time taken to find the predicted

paths that a defending missile can see and hit increases linearly with the number of predicted attacking paths. One way to improve calculation time would be to implement a parallel calculation to find the predicted paths that each defending missile can see and hit.

4.18 Conclusion

This section describes a control strategy for intercepting a high-performance attacking missile with a team of cooperating defending missiles. The method generates a set of possible paths that an attacking missile could take, then finds defending missile paths that will see and intercept as many of these paths as possible to minimise the probability of missing the attacking missile.

The problem of choosing trajectories for the defending missiles can be formulated as an unconstrained non-linear programming problem, where the objective is to choose aim points for the defending missiles to maximise their coverage of the seek and intercept regions. The more predicted attacking missile paths that the defending missiles are able to see and hit, the greater the probability of intercepting the attacking missile. A challenge was to ensure that the defending missile paths remained feasible after launch. Introducing a term to the objective function that penalises large steering angles achieved this, whilst keeping the optimisation problem unconstrained so that it could be solved using numerical methods such as the Nelder-Mead method.

Examples were used to demonstrate the effectiveness of the method. These examples track the progression of engagement scenarios from initial detection of the attacking missile by the radar, through to launch of the defending missiles, and through to the point when the defending missiles are able to see the seek region and switch to traditional intercept strategies.

Section 4.14 showed that a closer keep-out zone allows the manoeuvrability region of the attacking missile to become smaller, thus requiring fewer defending missiles to effectively cover the seek and intercept regions.

The following could be considered for future work:

- Investigating the influence of radar errors. It was assumed that errors in the measurement of initial position and speed would give insignificant variation in the possible paths of the attacking missile compared to the variations due to the manoeuvrability of the attacking missile. However, radar errors will enlarge the seek and intercept regions and provide a more difficult area for the defending missiles to cover.

- Incorporating terminal guidance for the defending missiles once they have seen the attacking missile. Typically defending missiles will resort to a proportional navigation guidance scheme once they are able to see the attacking missile [Shneydor, 1998]. For the purpose of my analysis, I assumed that a defending missile would be able to hit the attacking missile if its path was within the manoeuvre region of the defending missile. Even though this is a considerable simplification, it will allow us to produce useful insights into the cost-effectiveness of multiple cooperating missiles (Section 5).

- Comparing the performance of the control strategy with other strategies such as the staggered launch strategy [Shaferman and Oshman, 2016] or the predictive regret-matching algorithm [Rajagopalan et al., 2019]. Descriptions of these strategies are provided in Section 2.

- Comparing the performance of the Nelder-Mead method to other numerical methods.

- Evaluating the performance of the cooperative strategy with missile models that include variation in speed along the trajectories of both the attacking and defending missiles.

5 Determining the best type of defending missile

The aim of the research is to develop methods for determining whether replacing a single high-cost defending missile with a team of cooperating low-cost defending missiles can improve performance of missile defence systems and be more cost-effective. Section 4 gave a method for determining the best way to operate a team of defending missiles. This section develops a method for determining the best type of defending missile to use, given various defending missile configurations and costs.

The cost of a defending missile team will depend on many factors, including factors related to the performance of the defending missile. Cost information is not readily available, so to illustrate the method a simple cost function is defined that depends on four missile performance parameters: speed, maximum lateral acceleration, seeker range and seeker angle (Section 5.1). Sections 5.2–5.5 presents a 'design of experiments' approach to analyse a range of defending missile types and assess the probability of a team of defending missiles intercepting an attacking missile.

5.1 Cost function

The cost of a defending missile team is dependant on the characteristics of the defending missiles. For example, a longer seeker range will cost more than a shorter seeker range. For each key defending missile parameter we use a general function to map the value of that parameter onto a cost component. The total cost of a missile is the aggregate of these cost components,

$$C = M\left(c_v(v_\mathrm{m}) + c_a(a_\mathrm{max}) + c_r(r_\mathrm{s}) + c_\theta(\theta_\mathrm{s})\right)$$

where M is the number of defending missiles, $c_v : \mathbb{R} \to \mathbb{R}$ is the cost function of the defending missile's nominal speed v_m, $c_a : \mathbb{R} \to \mathbb{R}$ is the cost function of the defending missile's maximum lateral acceleration a_max, $c_r : \mathbb{R} \to \mathbb{R}$ is the cost function of the defending missile's seeker range r_s and $c_\theta : \mathbb{R} \to \mathbb{R}$ is the cost function of the defending missile's seeker angle θ_s.

Experts from the Defence Science and Technology group indicated that:

- the cost for maximum speed and lateral acceleration is equal to the cost of maximum seeker range and seeker angle,
- there is a large variation of cost for speed and seeker range,
- there is a small variation of cost for lateral acceleration, and
- increasing the seeker angle does not have much impact on cost because small seeker angles require more expensive technology and larger diameter missiles.

With guidance from these experts and [Fleeman, 2001], I have estimated linear cost functions for the four characteristics of the defending missiles:

$$\begin{aligned} c_v(v_\mathrm{m}) &= 0.00052\ v_\mathrm{m} &&+ 0.311 \\ c_a(a_\mathrm{max}) &= 0.00340\ a_\mathrm{max} &&+ 0.136 \\ c_r(r_\mathrm{s}) &= 0.00002\ r_\mathrm{s} &&+ 0.223 \\ c_\theta(\theta_\mathrm{s}) &= 0.00660\ \theta_\mathrm{s} &&+ 0.198. \end{aligned}$$

Figure 5.1 shows these cost functions for defending missile speed, maximum lateral acceleration, seeker range and seeker angle.

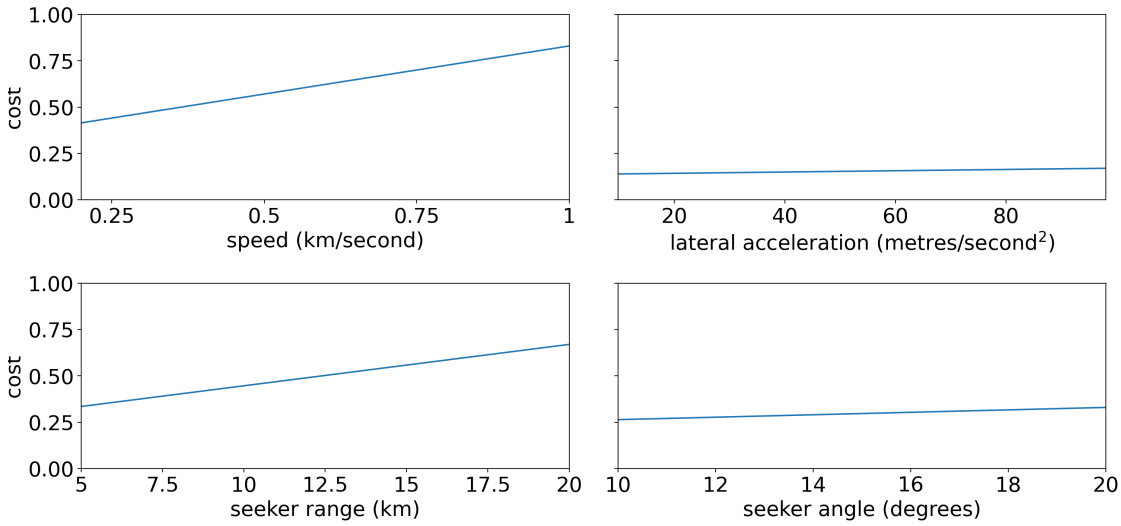


Figure 5.1: Cost functions for defending missile speeds (top-left), maximum lateral acceleration (top-right), seeker range (bottom-left), and seeker angle (bottom-right).

5.2 Experiment design

I used a 'design of experiments' method [Montgomery, 2017] to plan and conduct experiments and analyse the resulting data to understand how cost and probability of hit are influenced by various performance factors of the defending missile team. The factors I considered are:

- the four defending missile design factors discussed in Section 5.1 — speed, maximum lateral acceleration, seeker range and seeker angle,
- the keep-out range for the defending missiles,
- the number of defending missiles in the team, and
- the probability of an individual defending missile failing to stop the attacking missile.

Montgomery [2017] says that

> ... two-level factorial and fractional factorial designs should be the cornerstone of industrial experimentation for product and process development, troubleshooting, and improvement ...

and that

> ... three-level factorial and fractional designs are much less useful because the designs are relatively large even for a modest number of factors, and most of the small fractions have complex alias relationships that would require very restrictive assumptions regarding interactions to be useful.

There are, however, situations where it is insightful to include factors with more than two levels. For instance, for lateral acceleration we have included four levels ($1g, 2g, 5g$ and $10g$) to allows us to see that an increase in lateral acceleration from $5g$ to $10g$ does not improve probability of hit as much as an increase in lateral acceleration from $2g$ to $5g$ (see Figure 5.7). The relationship between lateral acceleration and probability of hit is not linear.

The optimisation method used in Section 4 to determine the probability of hit for a team of defending missiles is fast enough that I decided to implement a full factorial multi-level design for this analysis, whereby all possible combinations of the levels of seven factors presented in Table 5.1 are investigated. The computation time to evaluate all 1728 experiments was 8.66 hours on a 2.9 GHz 6-Core Intel Core i9 processor.

Table 5.1: Factors and levels for missile type analysis.

Factor	Levels
keep-out range (km)	15, 30
number of missiles	$1, 2, 5, 10$
probability of fail	$0.0, 0.4, 0.9$
speed ($\mathrm{km\,s^{-1}}$)	$0.2, 0.6, 1$
lateral acceleration ($\mathrm{m\,s^{-2}}$)	$1g, 2g, 5g, 10g$
seeker range (km)	5, 10, 20
seeker angle (degrees)	$10, 20$

The engagement scenario has one attacking missile and a team of defending missiles. Parameters relating to the attacking missile are the same for all experiments. The attacking missile is traveling at a constant speed of $0.6\,\mathrm{km\,s^{-1}}$ starting at an altitude of $7\,\mathrm{km}$ and can manoeuvre with a maximum lateral acceleration of $5g$. The following results evaluate the optimised probability of hit calculated at the time the defending missiles are launched. Typically, the probability of hit will improve after the defending missiles are launched.

5.3 Pareto front

We are interested in defending missile configurations that minimise cost and maximise the probability of hitting (and stopping) the attacking missile. A defending missile configuration is said to *Pareto-dominate* another configuration if the cost is at least as good, the performance is at least as good, and one or both of cost or performance is strictly better. The set of all non-dominated, or *Pareto-efficient*, solutions forms a *Pareto front*, named after the Italian economist Vilfredo Pareto. For every solution that is not on the Pareto front, there is a better solution on the front.

For example, consider the four defending missile teams shown in Table 5.2. The first three teams have the same basic missile type, and the cost increases with the number of missiles. The fourth team is a single, faster missile.

Table 5.2: Cost and performance of four different missile teams.

defending team label	number of defenders	speed ($\mathrm{km\,s^{-1}}$)	lateral acceleration ($\mathrm{m\,s^{-2}}$)	seeker range (km)	seeker angle (deg)	cost
A	1	0.2	$5g$	20	20	1.72
B	2	0.2	$5g$	20	20	3.44
C	10	0.2	$5g$	20	20	17.2
D	1	1.0	$5g$	20	20	2.13

Figure 5.2 shows the probability of hit for the four defending missile teams. The two rows of subplots correspond to different keep-out ranges. The three columns correspond to different probabilities of fail for individual missiles—a typical value is 0.4, and the values either side are extreme values designed to show the impact of increasing or decreasing the probability of failure.

Each subplot shows the relationship between cost and probability of hit for each team of defending missiles. The points connected by the purple line in each subplot form the Pareto front. The pale points in the interior of the region are the Pareto-dominated solutions. Adding more defending missiles increases the probability of hit.

Each subplot shows that changing from 1 to 2 (orange) to 10 (red) basic missiles increases both the cost and performance, as expected. Furthermore, 1 fast missile (light blue) is better than 1 basic missile (dark blue) in all cases.

If the probability of an individual missile failing is 0 then 1 fast missile dominates 2 basic missiles and 10 basic missiles. In four of the six sub-graphs, 1 fast missile dominates 2 basic missiles.

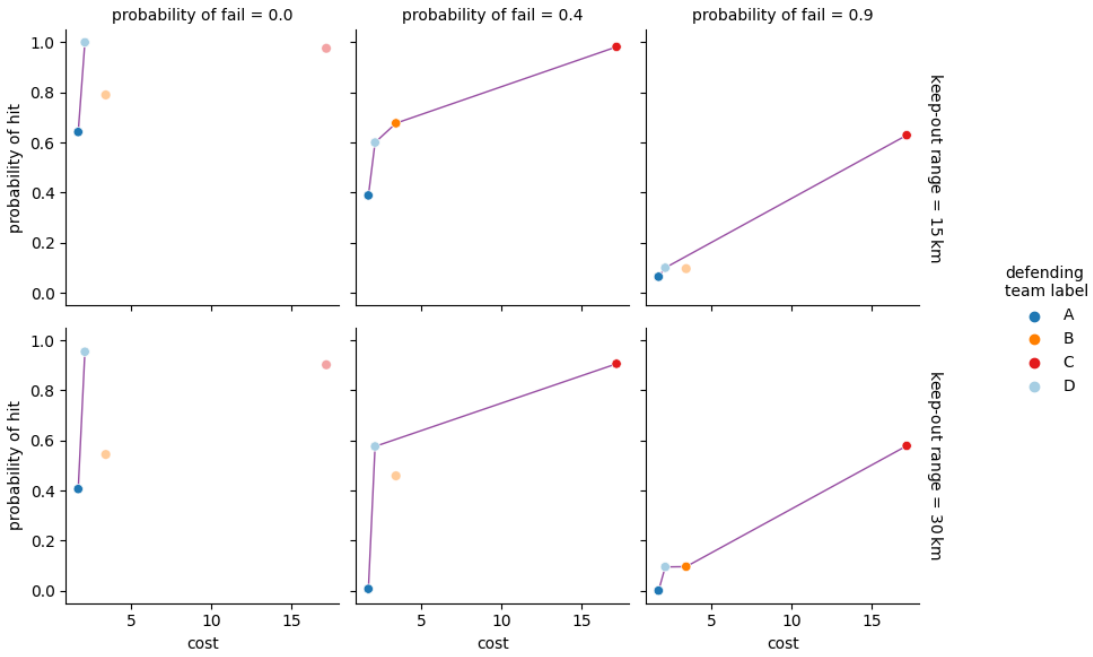


Figure 5.2: Cost and probability of hit for 4 defending missile teams. The points connected by the purple line form the Pareto front. Each subplot shows results for probability of fail equal to 0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom).

Figure 5.3 shows the cost and probability of hit for every defending missile team investigated. Typically defending missiles will have a probability of fail of 0.4. When the probability of fail is 0.0, a single defender is able to provide the best performance at the lowest cost. However, I have not been able to quantify the extra cost for a smaller probability of fail.

Some trends that can be observed from the graphs are:

- Keep-out range does not have much of an influence on the results for the values considered.
- When each defending missile has a low probability of fail then teams of 1 or 2 missiles achieve a high probability of hit. As the probability of fail of individual missiles becomes higher, more missiles are required to achieve an acceptably high probability of hit.

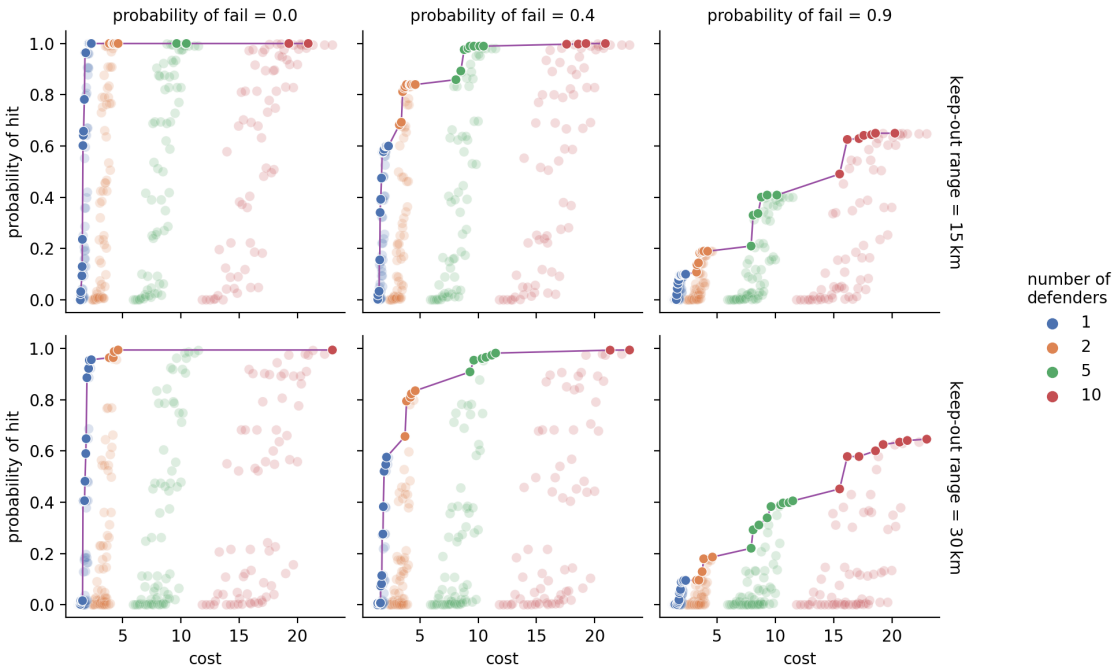


Figure 5.3: Cost and probability of hit for the defending missile teams. The points connected by the purple line form the Pareto front. Each subplot shows results for probability of fail equal to 0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom). Each color shows results for 1 (blue), 2 (orange), 5 (green) and 10 (red) missiles in the defending missile team.

- The cost of the defending missile teams fall into distinct bands determined by the number of missiles in the team. On the Pareto front, increasing the number of missiles in the team always increases performance but also always increases the cost.
- At the top edge of the Pareto fronts, increasing the number of missiles can give a large increase in cost for a small improvement in performance.
- At the left edge of the Pareto fronts, particularly when the probability of failure is low, a small increase in cost can give a large increase in performance.

The defending missile teams presented in Tables 5.3–5.6 are the Pareto-efficient points shown in Figure 5.3 that have a probability of hit greater than 0.6. A common feature among the Pareto-efficient teams is a seeker range of 20 km and, when keep-out range is 30 km, a seeker angle of 20 degrees.

Table 5.3 shows the performance and cost of defending missile teams with one defending missile. The blank cells indicate that when probability of fail is 0.4 or 0.9 there are no Pareto-efficient points with a probability of hit greater than 0.6. The final column is the ratio performance/cost—bigger is better. With one missile, a speed of 0.6 km s^{-1} is suitable for both keep-out ranges. When keep-out range is increased to 30 km then it is desirable to increase the maximum lateral acceleration and seeker angle to $5g$ and 20 degrees respectively.

Table 5.3: Pareto-efficient missile teams with one missile

keep-out range (km)	probability of fail	number of defenders	speed (km s^{-1})	lateral acceleration (m s^{-2})	seeker range (km)	seeker angle (deg)	$\frac{\text{performance}}{\text{cost}}$
15	0.0	1	0.6	2g	20	10	$\frac{0.96}{1.76} \approx 0.55$
15	0.4	1					
15	0.9	1					
30	0.0	1	0.6	5g	20	20	$\frac{0.89}{1.93} \approx 0.46$
30	0.4	1					
30	0.9	1					

Table 5.4 shows performance and cost of defending missile teams with two defending missiles. A probability of hit greater than 0.6 cannot be achieved when the probability of fail of individual missiles is 0.9. A speed of $0.6\,\mathrm{km\,s^{-1}}$ is common. When keep-out range is 15 km and probability of fail is decreased to 0.4 the required maximum lateral acceleration is $2g$ and the required seeker angle is 10 degrees. The required maximum lateral acceleration and seeker angle are lower then when the probability of fail is 0.0, because it is more cost-effective to have a single missile with higher performance than two missiles with lower performance when probability of fail is 0.0.

Table 5.4: Pareto-efficient missile teams with two missiles

keep-out range (km)	probability of fail	number of defenders	speed ($\mathrm{km\,s^{-1}}$)	lateral acceleration ($\mathrm{m\,s^{-2}}$)	seeker range (km)	seeker angle (deg)	$\frac{\text{performance}}{\text{cost}}$
15	0.0	2	0.6	5g	20	20	$\frac{1}{3.85} \approx 0.26$
15	0.4	2	0.6	2g	20	10	$\frac{0.81}{3.52} \approx 0.23$
15	0.9	2					
30	0.0	2	0.6	5g	20	20	$\frac{0.96}{3.85} \approx 0.25$
30	0.4	2	0.6	5g	20	20	$\frac{0.80}{3.85} \approx 0.21$
30	0.9	2					

Table 5.5 shows performance and cost of defending missile teams with five defending missiles. A speed of $0.6\,\mathrm{km\,s^{-1}}$ is common. No Pareto-efficient solution was found by the optimiser for a five defending missile team for keep-out range 30 km and probability of fail 0.0. As the number of defending missiles increases, the optimisation search space increases because the number of variables have increased, therefore finding the best solution becomes harder and takes more time. In this case, the best solution found for five defending missiles had a probability of hit slightly less than the probability of hit for the cheaper team of two missiles, so the five-missile solution was not on the Pareto front.

Table 5.5: Pareto-efficient missile teams with five missiles

keep-out range (km)	probability of fail	number of defenders	speed ($\mathrm{km\,s^{-1}}$)	lateral acceleration ($\mathrm{m\,s^{-2}}$)	seeker range (km)	seeker angle (deg)	$\frac{\text{performance}}{\text{cost}}$
15	0.0	5	0.6	5g	20	20	$\frac{1}{9.63} \approx 0.10$
15	0.4	5	0.6	2g	20	10	$\frac{0.98}{8.80} \approx 0.11$
15	0.9	5					
30	0.0	5					
30	0.4	5	0.6	5g	20	20	$\frac{0.95}{9.63} \approx 0.10$
30	0.9	5					

Table 5.6 shows performance and cost of defending missile teams with ten defending missiles. The Pareto-efficient solutions show that as keep-out range is increased, the required speed and maximum lateral acceleration are also increased for each probability of fail.

Table 5.6: Pareto-efficient missile teams with ten missiles

keep-out range (km)	probability of fail	number of defenders	speed ($\mathrm{km\,s^{-1}}$)	lateral acceleration ($\mathrm{m\,s^{-2}}$)	seeker range (km)	seeker angle (deg)	$\frac{\text{performance}}{\text{cost}}$
15	0.0	10	0.6	5g	20	20	$\frac{1}{19.25} \approx 0.05$
15	0.4	10	0.6	2g	20	10	$\frac{0.99}{17.59} \approx 0.06$
15	0.9	10	0.2	2g	20	20	$\frac{0.63}{16.18} \approx 0.04$
30	0.0	10	1	10g	20	20	$\frac{0.99}{23.00} \approx 0.04$
30	0.4	10	1	5g	20	20	$\frac{0.99}{21.33} \approx 0.05$
30	0.9	10	0.6	5g	20	20	$\frac{0.63}{19.25} \approx 0.03$

For the missile teams shown in Figure 5.3, the highest performance missiles give high probability of hit with only 1 or 2 missiles, particularly when the probability of individual missile failure is low. Figure 5.4 shows the relationship between cost and probability of hit when speed, maximum lateral acceleration and seeker range are capped at $0.6\,\mathrm{km\,s^{-1}}$, $5g$ and 10 km respectively. These lower performing missiles require more missiles in the team to achieve a given probability of hit.

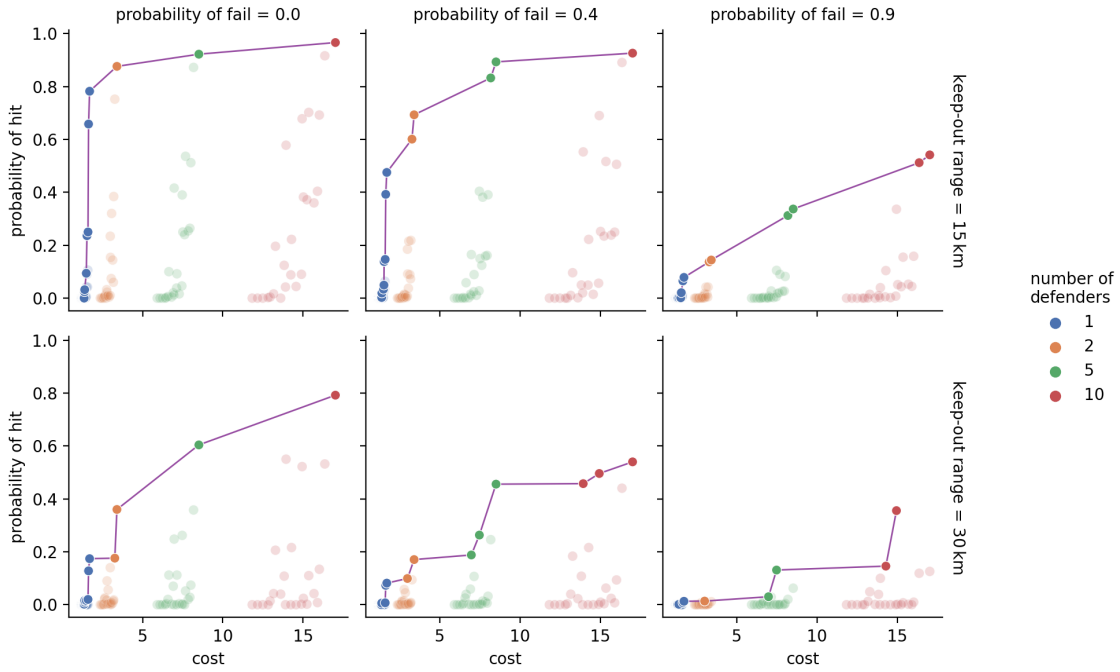


Figure 5.4: The relationship between cost and probability of hit for defending missile teams with reduced seeker, speed and maximum lateral acceleration performance. The points connected by the purple line form the Pareto front. Each plot shows results for probability of fail equal to 0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom). Each color shows results for 1 (blue), 2 (orange), 5 (green) and 10 (red) missiles in the defending missile team.

5.4 Sensitivity analysis on a Pareto-efficient solution

This section gives a sensitivity analysis on a Pareto-efficient solution to investigate which missile factors are important. The base solution is a Pareto-efficient solution for the case when keep-out range is 15 km and probability of fail is 0.4. This solution is shown in Figure 5.5, and has a probability of hit of 0.3924 and cost 1.6359. The defending missile team has one missile with a speed of $0.6\,\mathrm{km\,s^{-1}}$, maximum lateral acceleration of $5g$, seeker range of 10 km and seeker angle of 10 degrees.

I performed a sensitivity analysis by varying each parameter, one at a time, using the results from the 1728 experiments, and evaluating the change in performance or cost per change in factor level. The results are shown in Table 5.7. Increasing the seeker angle to 20 degrees has the greatest performance improvement per increase in cost and gives another Pareto-efficient solution. Increasing the seeker range to 20 km also gives another Pareto-efficient solution, and the most gain in probability of hit. Increasing the speed to $1\,\mathrm{km\,s^{-1}}$ does not give a Pareto-efficient solution. Increasing the number of defending missiles to two is the most costly variation, and does not give a Pareto-efficient solution. Increasing the maximum lateral acceleration to $10g$ increased cost without giving a performance benefit.

Table 5.7: Sensitivity analysis about a Pareto-efficient solution with one missile, constant speed of $0.6\,\mathrm{km\,s^{-1}}$, maximum lateral acceleration of $5g$, seeker range of 10 km and seeker angle of 10 degrees.

Factor F	$\Delta P_{\mathrm{hit}}/\Delta F$		$\Delta C/\Delta F$		$\Delta P_{\mathrm{hit}}/\Delta C$
number of missiles	0.2086	per missile	1.6359	per missile	0.1275
speed	0.2160	per $\mathrm{km\,s^{-1}}$	0.5188	per $\mathrm{km\,s^{-1}}$	0.4164
lateral acceleration	−0.0002	per $9.81\,\mathrm{m\,s^{-2}}$	0.0336	per $9.81\,\mathrm{m\,s^{-2}}$	−0.0072
seeker range	0.0186	per km	0.0223	per km	0.8328
seeker angle	0.0083	per degree	0.0066	per degree	1.2545

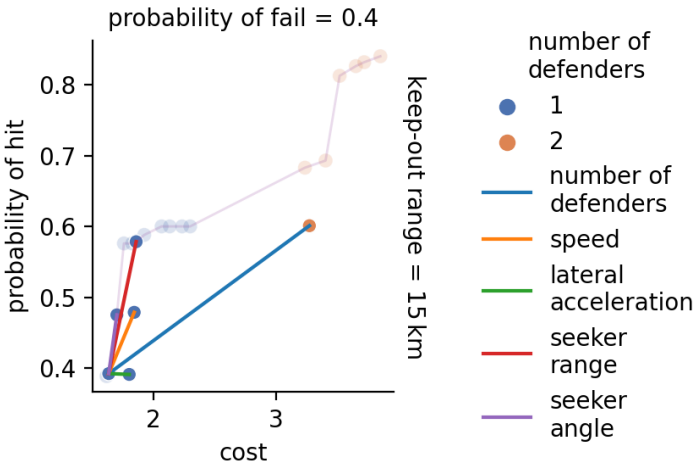


Figure 5.5: A section of the Pareto front for keep-out range 15 km and probability of fail 0.4. The points along the Pareto front are connected by a dim purple line. Given the Pareto-efficient solution with probability of hit 0.3924 and cost 1.6359 units, we have identified the solutions where; the number of defending missiles is increased (blue line), speed is increased (orange), maximum lateral acceleration is increased (green line), seeker range is increased (red line) and seeker angle is increased (purple line).

5.5 Screening experimental factors

Screening is a technique for identifying factors that have significant effects on the objective function [Montgomery, 2017]. Figures 5.6–5.9 show the variation of probability of hit for speed, maximum lateral acceleration, seeker range and seeker angle. The lines in each plot show the mean probability of hit at each level of each factor, where each color indicates the number of defending missiles. The mean probability of hit for a factor and level considers performance over all levels of all other factors. By observing these plots we can determine the influence of each factor on probability of hit. A steep line indicates that the factor has significant influence in probability of hit.

Figure 5.6 shows the interaction of mean probability of hit with defending missile speeds of 0.2, 0.6, and 1 $\mathrm{km\,s^{-1}}$. The attacking missile is traveling at a constant speed of 0.6 $\mathrm{km\,s^{-1}}$. When keep-out range is 15 km, increasing the defending missile speed from 0.2 $\mathrm{km\,s^{-1}}$ to 0.6 $\mathrm{km\,s^{-1}}$ has a significant influence on probability of hit. Further increasing the defending missile speed to 1 $\mathrm{km\,s^{-1}}$ gives a smaller increase in performance.

Figure 5.7 shows the interaction of mean probability of hit with defending missile maximum lateral acceleration levels of $1g$, $2g$, $5g$, and $10g$. The attacking missile is capable of maximum lateral acceleration of $5g$ manoeuvres. Performance increases with maximum lateral acceleration, but the improvement reduces as maximum lateral acceleration increases.

Figure 5.8 shows the interaction of mean probability of hit with defending missile seeker ranges of 5, 10, and 20 km. These plots show a significant increase in performance with seeker range across the entire range of values. However, seeker ranges greater than 20 km are difficult to achieve in practice.

Figure 5.9 shows the interaction of mean probability of hit with defending missile seeker angles of 10 and 20 degrees. These plots show that seeker angle has relatively little influence on probability of hit.

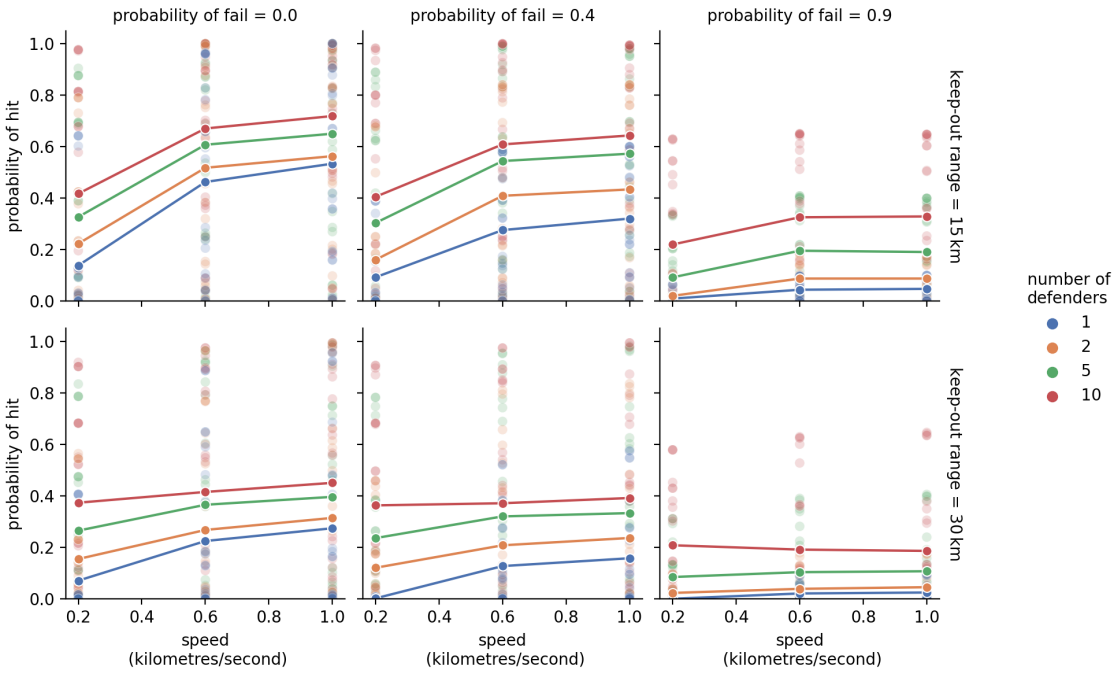


Figure 5.6: The interactions of mean probability of hit with defending missile speed. Each plot shows results for probability of fail equal to 0.0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom). Each color shows results for 1 (blue), 2 (orange), 5 (green) and 10 (red) missiles in the defending missile team.

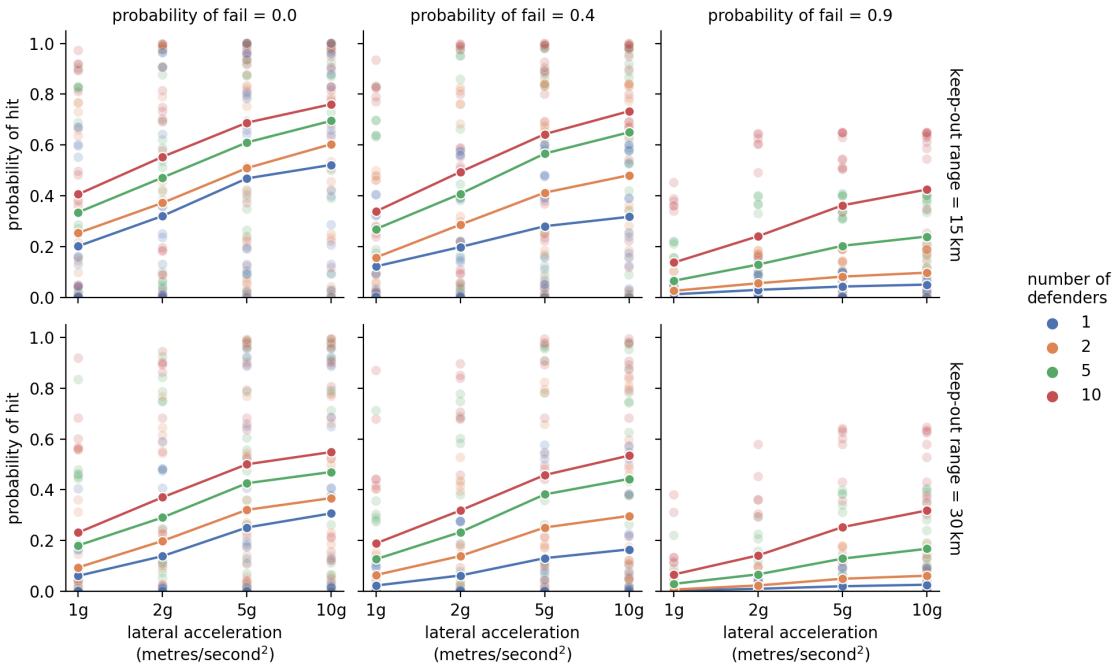


Figure 5.7: The interactions of mean probability of hit with defending missile maximum lateral acceleration. Each plot shows results for probability of fail equal to 0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom). Each color shows results for 1 (blue), 2 (orange), 5 (green) and 10 (red) missiles in the defending missile team.

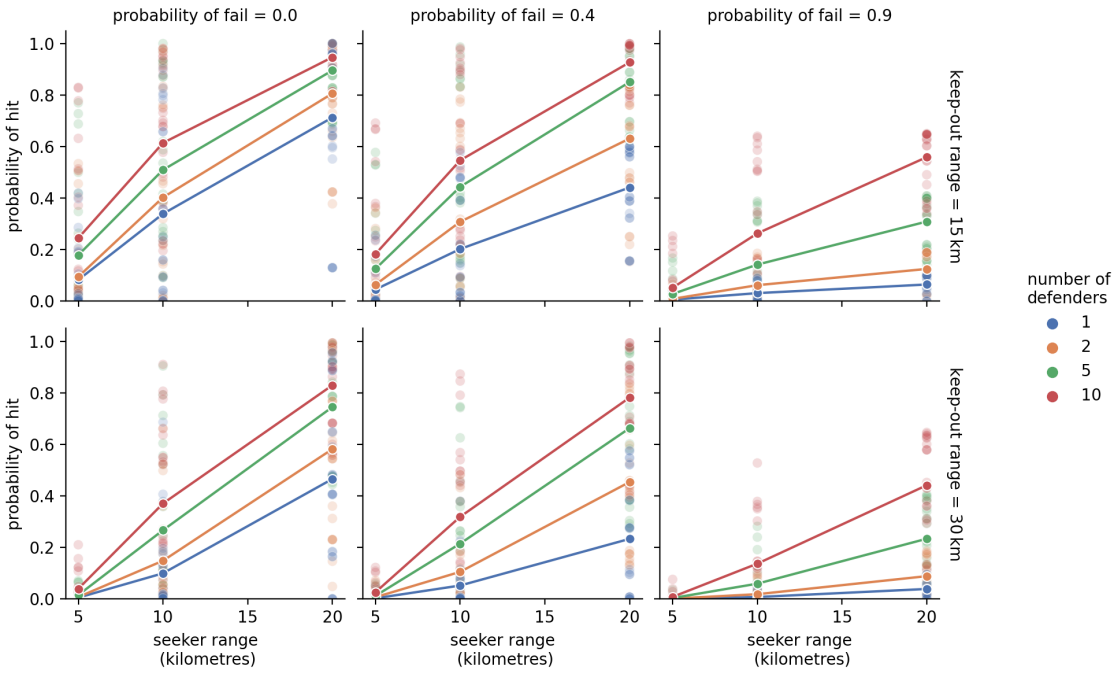


Figure 5.8: The interactions of mean probability of hit with defending missile seeker range. Each plot shows results for probability of fail equal to 0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom). Each color shows results for 1 (blue), 2 (orange), 5 (green) and 10 (red) missiles in the defending missile team.

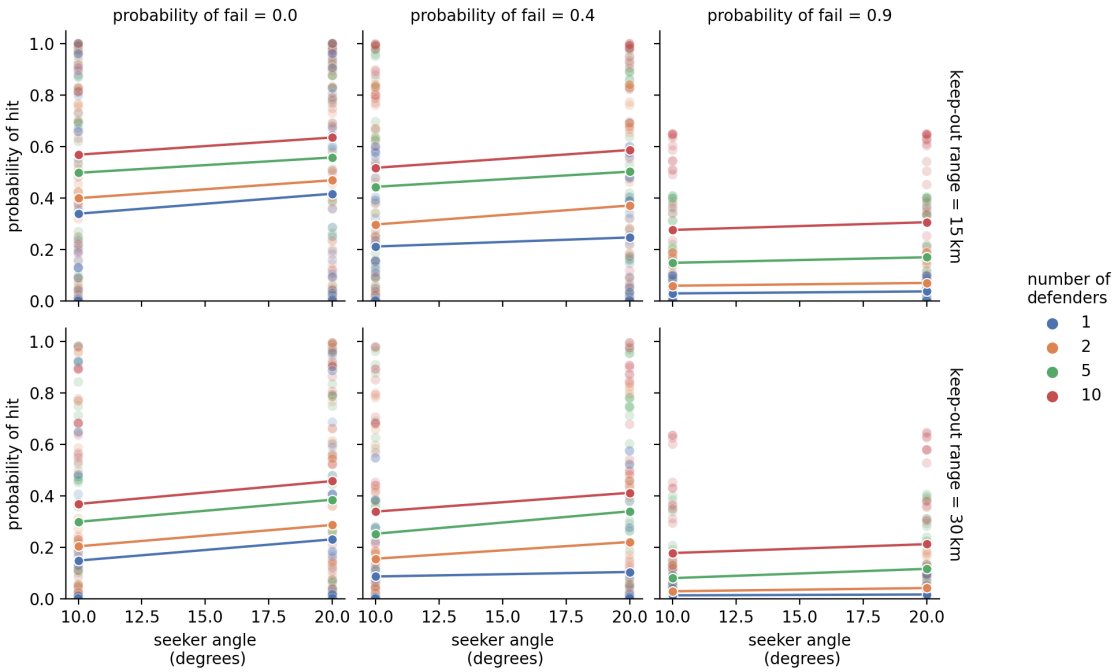


Figure 5.9: The interactions of mean probability of hit with defending missile seeker angle. Each plot shows results for probability of fail equal to 0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom). Each color shows results for 1 (blue), 2 (orange), 5 (green) and 10 (red) missiles in the defending missile team.

The variation of cost for each factor is not considered in this analysis. However, for a missile team to have good performance it would be reasonable to have a high seeker range, and speed and lateral acceleration at least equivalent to the attacking missile.

5.6 Conclusion

A 'design of experiments' approach can be used to assess the performance and cost of different configurations of defending missiles, to determine whether replacing a single high-cost defending missile with a team of cooperating low-cost defending missiles can improve performance of missile defence systems and be more cost-effective.

I conducted 1728 simulation experiments to evaluate the influence of keep-out range, number of defending missiles, probability of fail, speed, maximum lateral acceleration, seeker range and seeker angle on the probability of hit at time of launch.

In our engagement scenario example, seeker range had the most influence on the probability of hitting the attacking missile, and seeker angle the least. Increasing the number of defending missiles had the most cost. Having more than one defending missile provides a performance benefit when the probability of an individual defending missile's failure is high. However, there are situations where having more than one defending missile is not as cost-effective as having a single better performing defending missile. Doing the same analysis with more realistic performance parameters and costs may give a different result.

6 Conclusion

This thesis develops methods that can be used to determine whether replacing a single high-cost defence missile with a team of cooperating low-cost defence missiles can improve performance and flexibility of missile defence systems and be more cost effective. The first method determines the best way to operate a team of cooperating defense missiles so that the probability of hitting an attacking missile is maximised. The second method determines which defending missile features give the best performance.

The problem of how to control a team of cooperating defence missiles to seek and intercept a manoeuvring attacking missile can be formulated as a multi-dimensional unconstrained non-linear optimisation problem, where the objective is to choose fly-out paths for the defending missiles that maximise their coverage of the possible paths of the attacking missile. The more paths that the defending missiles are able to see and hit, the greater the probability of intercepting the attacking missile. A challenge was to ensure that the defending missile paths remained feasible after launch. Introducing a term to the objective function that penalises large steering angles achieved this, whilst keeping the optimisation problem unconstrained so that it could be solved using numerical methods such as the Nelder-Mead method [Nelder and Mead, 1965].

Example scenarios showed that fewer defending missiles were required to effectively cover the seek and intercept regions if the planned intercept region was closer to the stationary target.

The method for deciding which defending missile characteristics give the best team performance is based on a 'design of experiments' approach to determine whether replacing a single high-cost defending missile with a team of cooperating low-cost defending missiles can improve performance of missile defence systems and be more cost effective. A set of 1728 different defending missile teams was generated by varying the number of missiles, missile speed, seeker performance and missile manoeuvrability. Seeker range had the most influence on the probability of hitting the attacking missile, and seeker angle the least. Although increasing the number of missiles can ensure good performance of a missile defence system, the upfront

costs associated with multiple missiles can be large and so teams of cooperating missiles might not be a cost effective solution.

The following could be considered for future work:

- Defending against multiple attacking missiles with multiple cooperating defending missiles.
- Incorporating more detailed defence system models into the methods.
- Investigating the best keep-out range to maximise performance of the control strategy.
- Use of more accurate performance and cost measures to assess the control strategy and defending missile teams.
- Comparing the performance of the control strategy with other strategies such as the staggered launch strategy [Shaferman and Oshman, 2016] or the predictive regret-matching algorithm [Rajagopalan et al., 2019].
- Comparing the performance of the Nelder-Mead method to other numerical methods.

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