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The role of motion-based metaphors in enhancing mathematical thought: A perspective from embodiment theories of cognition

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Abstract

Motion-based metaphors help explain a single, often a static, concept (e.g., number, mathematical function, limit of function, continuity of function) in terms of a human motion. In mathematics, many mathematical concepts, such as function and continuity, are described in terms of graphical representations. Although these graphical representations are static, they can be transformed into motion events and understood as motions by motion-based metaphors. This can be done by either using a hand gesture to depict the graphical representation of the target concept or by mentally simulating hand movements that depict the graphical representation. By employing these mechanisms, the motor system becomes engaged in the process of aiding the learner to understand static mathematical concepts (concepts that are defined in terms of non-moving mathematical objects), acting as a cognitive resource to ground and understand non-motion mathematical concepts. In this paper we theorize that visual representations of mathematical concepts have varying degrees of what we term to be "motor strength" whereby, for example, curves of functions may be either strongly or weakly motoric depending on the degree to which they aid in the development of associated deep mathematical thinking.

Keywords: Graphical representation; Mathematical concepts; Motion-based metaphor

1. Introduction

Our daily communicative interactions often involve the use of metaphors where human motion is used to describe static concepts, what we term as motion-based metaphors. In a motion-based metaphor, the target of the metaphor is often a static concept, or even an abstract concept, which is understood as a motion concept. Such metaphors have been studied in many languages of the world, including English (Matlock, 2004, 2010; Langacker, 1999; Talmy, 1996), Japanese (Matsumoto, 1996), Spanish (Rojo & Valenzuela, 2003), Hindi (Mishra, 2009), and Danish (Wallentin et al., 2005). A metaphorical description of a static concept in terms of a motion concept can be done through the use of a verbal motion-based metaphor, a gesture, or both. The sentence *we are approaching the election day* is an example of a verbal motion-based metaphor. Using a forward hand movement to refer to the abstract concept of *time* as a moving object is the gestural representation of the metaphor *time is a moving object*. As mentioned, motion-based metaphors can appear in a written, verbal or gestural form. In mathematics, the focus study context of this paper, motion-based mathematical metaphors are seen as describing static mathematical concepts

(concepts that are defined in terms of non-moving mathematical objects) in terms of motion events. In fact, fundamental mathematical concepts such as function, limit, continuity, derivative, and integral are generally understood as motions (Lakoff & Núñez, 2000; Marghetis & Núñez, 2013; Núñez, 2005, 2008; Núñez & Lakoff, 1998). A given abstract mathematical concept may be embodied and represented in a variety of modes (Reys, 1972). Embodiment of mathematical knowledge is a way of conceptualizing abstract mathematical concepts as bodily-perceivable entities. Describing and understanding abstract mathematical concepts in terms of motion concepts is a particular way of embodying these concepts.

Even when an abstract mathematical concept is represented and understood in terms of a static concrete concept, motions may come into play. This happens particularly when the static concrete concept is depicted by the trace of hand movements not simply by the shape of hands. In such cases, the trace of hand movements shows the contour of a graphic representation as a mathematical concept. Although the concept and the contour of its graphical representation are static, the hand movement that depicts the contour is essentially a motion event. Therefore, the gestural representation of an abstract mathematical concept may be considered a motion-based metaphor. In fact, any gestural representation which depicts an abstract concept in terms of a spatio-motoric representation can be seen as a motion-based metaphor. In this paper, our aim is to show how motion-based metaphors and their gestural representations could enhance the processes involved in understanding mathematical concepts.

2. Embodiment theories and motion-based metaphors

According to embodiment theories, conceptual representations are totally or partly based on sensory-motor processes. The so-called "stronger" versions of embodiment hold that conceptual representations are primarily grounded through sensory-motor processes (e.g., Connell & Lynott, 2012; Gallese & Lakoff, 2005; Glenberg, et al., 2008; Lakoff, 2008; Shapiro, 2019). On the other hand, the weaker versions assume that conceptual representations are partly grounded through sensory-motor and emotional systems as well as, crucially, modality-independent systems which play a role in conceptual processing (e.g., Binder & Desai, 2011; Hauk & Tschentscher, 2013; Khatin-Zadeh et al., 2021a; Kiefer & Pulvermüller, 2011; Lambon-Ralph, 2013; Zwaan, 2014). A critical assumption of stronger versions of embodiment (Gallese & Lakoff, 2005) is that in the understanding of a metaphor, the same sensory-motor networks that are recruited to process the

base of the metaphor are also employed to process the target concept. That is, when the metaphor *X is a Y* is processed, the same sensory-motor networks that are involved in the processing of *Y* are employed to process *X*. From this perspective, processing the metaphorical phrase *pushes the argument* (i.e. supports the argument) activates neural networks involved in the actual action of pushing an object (e.g., Gallese, 2003; Gallese & Lakoff, 2005; Gibbs, 2006, 2013; Lakoff, 2008, 2014). Here, *supporting an argument* (target of the metaphor) is described in terms of *pushing an object* (base of the metaphor). Similar ideas have been discussed in the cognitive science literature where processing motion-based metaphors involves the activation of the motor system (e.g., Feldman & Narayanan, 2004; Glenberg & Kaschak, 2002; Pulvermüller, 2005; Rizzolatti & Sinigaglia, 2008). Furthermore, findings of some studies have suggested that linguistic processing of static events can involve embodied simulation in both literal (e.g., Masson et al., 2008) and metaphorical language (Al-Azary & Katz, 2021). Motion-based metaphors that describe a static concept in terms of a motion concept are similar to metaphors that describe an abstract concept in terms of a static concrete concept but use the trace of body movements to show the contour of the static base concept. The activation of the motor system during the processing of first type of motion-based metaphors is confirmed by study findings which support the strong versions of embodiments (e.g., Gallese, 2003; Gallese & Lakoff, 2005; Gibbs, 2006). The activation of the motor system during the processing of second type of motion-based metaphors is relatively straightforward, as the movements of body parts that depict the trace of the contour of a base concept are controlled by the direct involvement of the motor system. The activation of the motor system associated with processing motion-based metaphors has been confirmed by neuroimaging evidence. One study suggested that MT+, which is a brain region that responds selectively to perceived motion, is activated during the processing of motion-based metaphors (Saygin et al., 2010). Another study indicated that parahippocampal gyrus, an area that is involved in spatial and motion processing, is engaged when comprehending motion-based metaphors (Yang & Shu, 2016). Furthermore, findings of a number of behavioral studies indicate that fictive motion metaphors, a kind of motion-based metaphors, are understood through a simulation of fictive motions (for a review, see Matlock, 2010).

3. Motion-based metaphor and the motor system recruitment

Motion concepts underlie many metaphorical descriptions of mathematical concepts (Khatin-Zadeh et al., 2021b; Lakoff & Núñez, 2000; Marghetis, & Núñez, 2013; Núñez, 2005, 2008; Núñez & Lakoff, 1998). One example of a conceptual motion-based metaphors is *numbers are locations in space* (Alibali & Nathan, 2012). This conceptual metaphor has many verbal realizations in mathematics discussions. The metaphor *as x approaches infinity, f(x) approaches 0* is one realization of this conceptual mathematical metaphor. Such motion-based metaphors are usually accompanied by gestures when these concepts are discussed (Alibali & Nathan, 2012; Khatin-Zadeh, 2021a; Núñez, 2005, 2008). In such cases, gestures offer a visual depiction of abstract concepts. This metaphoric description is spatio-motoric, as it involves space, bodily movements, and possibly the sensory-motor system. A large number of studies have demonstrated the effectiveness of using gestures in enhancing the process of acquiring mathematical knowledge (e.g., Goldin-Meadow et al., 2008; Johnson-Glenberg, & Megowan-Romanowicz, 2017; Macedonia, 2019; Nathan & Walkington, 2017; Radford, 2009; Singer et al., 2008; Yeo et al., 2017). In mathematics education, while mathematical concepts are taught using written symbols, images, and speech, embodiment approaches to mathematics education have broadened the scope of pedagogical resources to include new modalities such as gesture and other bodily movements (Edwards, 2019; Edwards et al., 2014; Hall & Nemirovsky, 2012).

Looking at another example, the graphical representation of a mathematical function in terms of a curve in a Cartesian coordinate system is not inherently a motion-based metaphor, as the curve is a static image. However, when the trace of gesture is used to depict this curve, this metaphor is transformed into a motion-based metaphor. Even a mental simulation of hand movements that depicts the curve in the absence of any real or concrete gesture can transform this metaphor into a motion-based metaphor. In fact, this visual non-motion-based metaphor may be transformed into a motion-based metaphor in two ways: firstly, by using the trace of a gesture to depict the graphical representation of the function; secondly, by mental simulation of hand movements that depict the graphical representation of that function even in the absence of a real gesture. In both cases, the transformation of a non-motion-based metaphor into a motion-based metaphor involves engaging the motor system. In fact, the result of this transformational process is to recruit more cognitive resources, including the motor system. The recruitment of further

resources is seen to enhance and facilitate the process of understanding and acquiring new knowledge (see, for example, Tall, 2011). The idea that static mathematical graphs can be processed via motoric simulations is supported by some empirical evidence. For example, Longcamp et al. (2003, 2011) and Wamain et al (2012) found that visual processing of hand-written letters activates left primary motor cortex and the supplementary motor region of the brain. One possible explanation for the activation of motor-related areas during processing static hand-written letters is that hand movements that produce the letters are simulated in the mind of the observer. Another study conducted by Umilta' et al. (2012) found that cortical motor system is activated during observing static abstract works of art. Results of a similar experiment conducted by Sbriscia-Fioretti et al. (2013) showed that premotor and motor cortical areas are activated when an observer looks at images of abstract works of art with marked traces of brushstrokes. If visual processing of static hand-written letters and paintings could activate the motor areas of the brain, this could happen for mathematical graphs as well. It means that when an abstract mathematical concept is described and represented in terms of a static graph, the motor system can be employed as an extra cognitive resource to process that concept. This could explain why body movements are so effective in enhancing students' learning in mathematics. In recent two decades, a large body of works on embodied cognition has emphasized the effective role of body, motion, and gesture in enhancing science and mathematics learning (e.g., Abrahamson et al., 2020; Khatin-Zadeh et al., 2022; Schaefer, 2019; Shvarts et al., 2021; Zona et al., 2019).

According to Lakoff and Núñez, (2000), describing a mathematical function in terms of a motion using a motion-based metaphor allows us to conceptualize that function as something that moves in a certain direction. Presenting a rigorous definition of the mathematical concept of continuity by precise mathematical terms is difficult, although it seems to be straightforward in common non-expert terms. Continuity can be understood as the preservation of closeness near a location (Núñez & Lakoff, 1998). This conceptualization of continuity involves the static concepts of location, landmark, reference point, and distance (Marghetis & Núñez, 2013). Inherently, this concept has nothing to do with motion-related concepts such as trajectory, path, directionality, etc. However, the static concept of continuity can be well understood in terms of motion-related concepts and can be grounded in gestures. In fact, gestures could function as a mediatory channel for grounding this concept through the sensory-motor system. The key feature of gesture is that it cross-modally describes the concept of continuity. That is, gesture converts the abstract

representation of the concept of continuity into a representation that uses new modalities, such as vision. By transforming the mode of representation, gestures visually show the meaning of continuity. The base domain of this metaphor has a strong perceptual (visual) and spatio-motoric dimensions. In this way, the highly abstract concept of continuity is understood in terms of a domain which is visual with strong spatio-motoric elements.

4. Motor strength of the base domain of a motion-based metaphor

In recent years, a number of studies have investigated the degree of perceptual strength of concepts in various languages (e.g., Chen et al., 2019; Filipović Đurđević et al., 2016; Miklashevsky, 2018; Speed & Majid, 2017). The aim of these studies has been to obtain a value that could show degree of perceptual strength for each of the five senses (i.e. vision, hearing, touch, smell, and taste) for various concepts. For example, while the concept of *sun* has a strong visual perceptual strength, it has weak auditory and haptic strength. On the other hand, while the concept of *scream* has a strong auditory strength, it has weak visual and olfactory strength. A more recent comprehensive study examined degree of perceptual and action strength of 40000 words in English (Lynott et al., 2019). The values that were obtained in this study could show the extent to which each concept is grounded through the sensory-motor system. However, the values that have been obtained in this study are largely dependent on literal and associative meanings of words. These values may not apply to metaphorical descriptions of concepts such as motion-based metaphors. For example, *mountain range* is a stationary concept. However, in the metaphorical sentence, *the mountain range passes through the country* is understood as a moving object. Therefore, the motor system may play at least a partial role in the way humans process this concept. In other words, the degree of involvement of the motor system in the processing of concepts depends on a variety of factors, including whether the word is understood in a literal or metaphorical sense, what the physical and sentential contexts are, as well as many other potential factors.

When abstract mathematical concepts are metaphorically understood in terms of motion-based metaphors, the base concepts may have a variety of degrees of motor strength. While some base concepts may be strongly motoric, others may have weaker degrees of motor strength. The idea of motor strength of concepts or their visual representation is supported by the findings of studies that have found observing static pictorial stimuli containing implied motion and even observing abstract works of art can activate the motor regions (e.g., Khatin-Zadeh, 2021b; Kim &

Blake, 2007; Kourtzi & Kanwisher, 2000; Lorteije et al., 2011; Osaka et al., 2010; Senior et al., 2000; Umilta' et al., 2012; Winwar et al., 2008; Williams & Wright, 2009).

Processing those motion-based metaphors in which a strongly-motoric concept is used as the base concept involves strong motor system engagement. That is, processing such metaphors leads to a significant activation of the motor system. When such metaphors are used, the motor system plays an active role in the grounding of the target concept. This is even the case with a metaphorical description of a mathematical function in terms of a visual representation, such as a curve. As suggested before, although a curve is a static visual representation, it could have some degree of what we term as "motor strength". This strength may be dependent on several factors. One possible factor is the extent to which that curve could be depicted through the trajectory of hand movements. In other words, those curves (and more generally those graphical representations) that can easily be depicted through the trajectory of hand movements have a high degree of motor strength. In such cases, source of the graph production may play an important role, with hand-produced graphs having a higher motor strength than computer-generated graphs. Therefore, the hand-produced graph of a mathematical function may have a higher motor strength than the computer-generated graph of the same function. For example, Figure 1 shows the hand-produced version of a mathematical function. Figure 2 shows the computer-generated version of a mathematical function. These two graphs are similar in many respects, and the hand-produced graph can be considered as an approximation of the computer-generated graph. However, the hand-produced graph may have a higher motor strength, as hand movements involved in the production of the hand-produced version may be simulated during visual perception.

Figure 1. Hand-produced version of the graph of a mathematical function

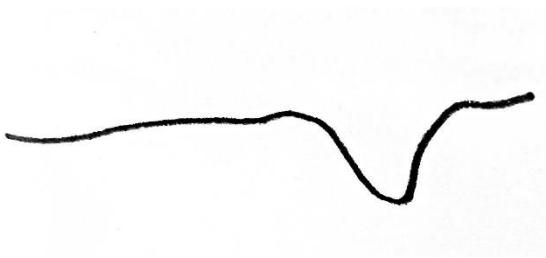
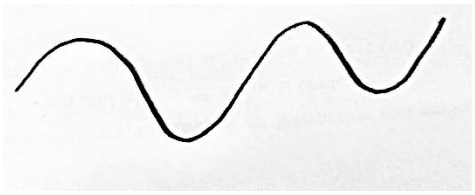


Figure 2. Computer-generated version of the graph of a mathematical function

Another possible factor that could affect motor strength of a curve is the extent to which the shape of the curve is associated with past motion-related experiences of the individual (Khatin-Zadeh, 2021b). The shapes of some curves are strongly associated with motion-related experiences. For example, many of us have had the experience of seeing the oscillating and parabolic movements of objects. Therefore, the graphical representations of functions such as $f(x)=Sin\ x$ (trace of an oscillating movement) and $f(x)= -x^2$ (trace of a parabolic movement) have a high degree of motor strength (Figure 3 and Figure 4). When an observer visually processes such curves, the past experiences that are associated with the shapes of these curves are activated, leading to the activation of the motor system. It should be noted that the hand-produced version of each graph could have a higher motor strength than the computer-generated version.

Figure 3. Hand-produced and computer-generated versions of the graphical representation of the function $f(x)=Sin\ x$



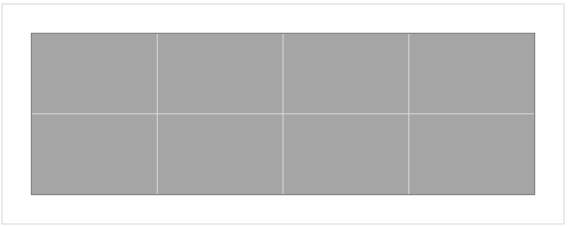
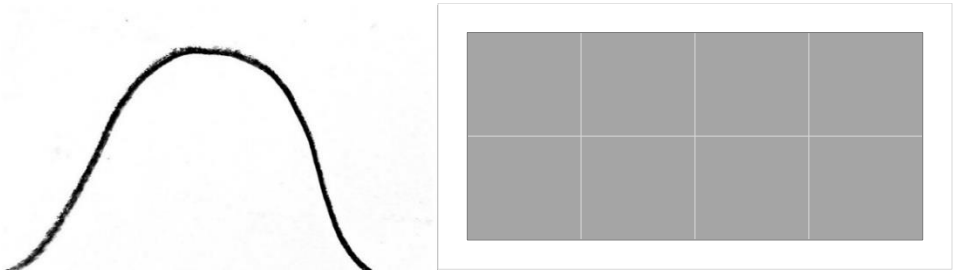


Figure 4. Hand-produced and computer-generated versions of the graphical representation of the function $f(x)= - x^2$



Graphical representations such as curves can easily be depicted. This can be seen as some kind of affordance of graphical representations, a notion initially introduced by Gibson (1979) and defined as action possibilities that are offered by an object. In contrast, for Reed (1996), affordances are properties of an object or the environment relative to an agent. A cup can be grasped by hands. This is an affordance of the cup. According to Gallese and Lakoff (2005), seeing a graspable object activates those neural substrates that are involved in actually grasping that object. When a mathematical function is described in terms of a graphical representation, such as a curve, the graphical representation has the affordance of being easily depicted by hands. Therefore, that graphical representation may have high motor strength.

5. Embodied and dis-embodied modes of mathematical thought

Throughout this paper, we took a weak embodiment perspective. Although the important role of embodiment in mathematics learning was emphasized, the role of a dis-embodied mode of thinking was not rejected. Graphical representations, motions, and motor strength of graphical representations are the embodied aspects of some mathematical concepts. But, they do not constitute the whole story. A question that may be raised is how mathematical concept can be processed abstractly without embodied simulation. Is embodiment necessary for mathematics learning and reasoning? How can dis-embodied computers outperform humans in mathematics if embodiment is so crucial for mathematical reasoning? In order to answer such questions, it should be noted that mathematical thought is a combination of embodied and dis-embodied modes of thinking. The embodied mode of thinking helps the individual to ground mathematical concepts

in the concrete environment. Once these concepts have been grounded in the concrete environment and established in the cognitive system of the individual, they can be represented by abstract mathematical symbols. In such conditions, the individual can perform mathematical operations with these symbols without activating the embodied representations of these symbols. Here, just a set of previously-established rules of operation among abstract mathematical symbols is applied. Such operations do not need an embodied mode of thinking. These are the operations that are performed by computers. Since computers are less affected by some factors that restrict human thought processes (such as memory limitation, cognitive overload, fatigue), they outperform humans in this dis-embodied mode of operation. That is why computers outperform humans in performing complex arithmetic operations. Such operations are performed on the basis of a set of previously-established rules that involve only abstract symbols (symbols that represent numbers). On the other hand, creative aspects of mathematical thought are highly reliant on an embodied mode of thinking. That is why humans outperform computers in this area. To summarize, embodied and dis-embodied thinking and reasoning are two modes of thought that are necessary for mathematics. Each one deals with specific aspects of mathematical operations.

6. Summary

Motion-based metaphors provide a mechanism for grounding abstract concepts in terms of a motion concept by the support of the motor system. In mathematics, many concepts are metaphorically described and understood in terms of graphical representations such as curves. Although these graphical representations are static images, they can be transformed into motion-based metaphors by using the trace of a gesture to depict the graphical representation or by mentally simulating hand movements that depict the graphical representation even in the absence of a real gesture. It is suggested that the visual representation of a mathematical concept such as the curve of a function has a certain degree of motor strength. While some curves may be strongly motoric, others may have a weaker motor strength. Degree of motor strength of a concept may be partly dependent on the affordances of that curve. If a curve (or other visual representations) has the affordance of being easily depicted or the individual has had the experience of depicting that curve, it will have a high degree of motor strength for that individual. In this way, the motor system can help that individual to ground that concept through the curve (or other visual representations). Identifying other possible factors that could determine degree of motor strength of a curve or more

generally degree of motor strength of a visual representation is a question that remains to be addressed in future studies.

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