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Exploring algebraic expressions using the hundreds chart

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For many, algebra has been viewed as the "manipulation of symbols according to tightly prescribed rules" (Serow, Callingham, & Muir, 2019, p. 142). This view of algebra can limit student success in mathematics, particularly in algebraic thinking. Student often encounter difficulties with algebra. For example, Day, Stephens, and Horne (2017) reported upper primary and lower secondary students find "generalisations and explanation of algebraic thinking quite difficult" (p. 658). Serow et al. (2019) explains that "generalisations requires an idea of a variable" which are often "represented by letters" and "the use of letters to represent variable develops often slowly" (p. 142). To support students in explaining their mathematical thinking, students need a variety of experiences with the same conceptual focus. This can be achieved through the use of open-ended tasks.

Open ended tasks "provide students with opportunities for extending their mathematical thinking, exploring a range of options and making decisions and generalisations as they work through the tasks" (Reys et al., 2020, p. 54). Further, open ended tasks provide flexible learning opportunities for students, accommodating different student abilities as advocated by McKnight and Muligan (2010)

In this article, open-ended tasks are used to demonstrate a possible sequence of learning that can be utilised to develop algebraic concepts to achieve a range of learning outcomes including the development of generalisations. The use of open-ended tasks allows the integration of "a range of mathematical concepts in an efficient way" (McKnight & Muligan, 2010, p. 9). The algebraic concepts that are specifically developed by engaging in these open-ended tasks are variables, algebraic expression, pronumerals, constant, and substituting.

The aspects of working and thinking mathematically that are fostered include using multiple methods to find a solution which can then be used to check the reasonableness of derived answers; identifying and describing number patterns and relationships; using correct mathematical notation and conventions to write an algebraic expression making connections between mathematical concepts, specifically connecting patterns on the hundreds charts to algebraic expressions and arithmetic properties and laws;
Other connections are also developed by engaging in the task. By the time a student is in year 7, the hundreds chart should be a familiar tool. In using the hundreds chart, the transition between the representation tool and the abstract is facilitated.

Curriculum Connections

The Australian Curriculum: Mathematics content descriptors that the tasks address in this article are documented in Table 1. This task is designed for year 7 students. However, the task can be modified to be explored with younger students. An example of a possible modification is described in Table 1.

Table 1: Curriculum content and connection to task with the Australian Curriculum: Mathematics

<table>
<tr><th>Year Level</th><th>Content Descriptor</th><th>Connection to task</th></tr>
<tr><td rowspan="3">Year 7</td><td>Introduce the concept of variables as a way of representing numbers using letters (ACARA, 2020, ACMNA175)</td><td>Introduce variables in the form of pronumerals as a substitute for a constant. For example, let x represent 39.</td></tr>
<tr><td>Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)</td><td>Determine an expression that represents the sum of a set of numbers and then evaluate the expression by substituting the number that relates to the expression.</td></tr>
<tr><td>Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)</td><td>Extend the laws and properties of arithmetic by understanding that these laws can be applied to algebraic expressions.
Use mathematical conventions such as order of operations to write algebraic expressions and to evaluate expressions by substitution. Apply the communicative property, $a+b=b+a$; associative laws, $a+(b+c)=(a+b)+c$ or $a\times(b\times c)=(a\times b)\times c$ and distributive laws, e.g. $a\times(b+c)=(a\times b)+(a\times c)$.</td></tr>
<tr><td>Year 6</td><td>Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133)</td><td>Focusing on whole numbers, students select at least three numbers that form a row on the hundreds chart and then describe the rule that connects these number. For example,
<table><tr><td>84</td><td>85</td><td>86</td></tr></table>
A rule that connects the above three numbers is “add 1”.

Or
<table><tr><td></td><td></td><td>20</td></tr><tr><td></td><td>29</td><td></td></tr><tr><td>38</td><td></td><td></td></tr></table>A rule that connects the above three numbers is “subtract 9”.</td></tr>
</table>

The Tasks

The following three tasks show a developmental progression of learning to develop students’ skills, understanding and confidence in generalising and using algebraic expressions. All tasks require the use of a hundreds chart and follow the same seven step procedure. The tasks provide rich opportunities for effective teacher questioning that can be used to assess conceptual understanding. For example:

- What is the sum of the three numbers?
- What is the relationship between the three numbers? Can you describe how the three numbers are related?

The three tasks are described with examples over the next few pages.

Task 1:

 *Figure 1: Step 1 - highlight three numbers that form a row or column*	**Step 1:** Highlight three numbers that form a row or column **Step 2:** Take the numbers 12, 22 and 32. The sum of the three numbers is? (66) **Step 3:** What is the relationship between 12 and 22 or 32 and 22? In this example, 12 is ten less than 22 and 32 is ten more than 22. How can the two relationships be represented as two number sentences? $22-10$ 22 $22+10$ **Step 4:** Now, have a look at what is common in all three statements above. In this example, we can 22 in each three statements. If we let $x=22$, write an expression that connects 22 to 12 and 22 to 32. $x-10$ x $x+10$ **Step 5:** Write an expression to add all three numbers: $x-10+x+x+10$ **Step 6:** Simplify the expression $x+x+x=3x$ and $-10+10=0$ so the simplified expression is $3x$. **Step 7:** What happens when we let $x=22$? $3\times 22=66$ What do you notice about the two methods? What do you wonder? What do you appreciate? What scenario could you develop to represent the three numbers?

Students continue to explore patterns and relationships involving number sequences of three numbers, before progressing to larger rectangles. What generalisations can be made about the expression. That is, how does the coefficient of the pronumeral change as the size of the rectangle (or shape) increases? What will the coefficient of the pronumeral be for a rectangle that has six numbers, ten numbers or larger? When assigning a pronumeral to the middle number of a rectangle, what happens to the constants? That is, can children describe the relationship of adding and subtracting the same number from the algebraic expression?

Once students are familiar with the process of adding three connected numbers, students are encouraged to explore different relationships and shapes. For example, squares or triangles

Task 2:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

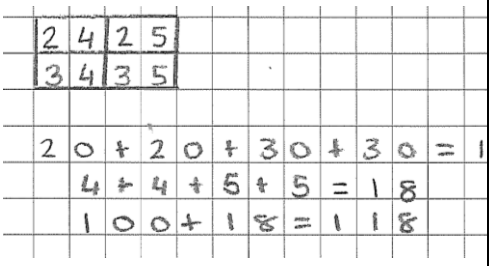
Figure 2: Step 1 - create a shape on the hundreds chart

Procedure:

1. Highlight a group of numbers on the hundreds chart
2. What is the sum of the numbers that form the shape?
3. Select one number as the starting point and describe the relationship between the remaining numbers
4. Assign a pronumeral to represent this number.
5. Substitute the pronumeral for the selected number
6. write an algebraic expression to represent the sum of the numbers
7. Check the sum of by substituting the original number into the expression.

Example 2:

1. Blue Square
2. Find the sum of the numbers



3. Starting number is 24

24	24 + 1
24 + 10	24 + 11

4. $x = 24$
5.

x	$x+1$
$x+10$	$x+11$

6. $x + x + 1 + x + 10 + x + 11$

which simplifies to

$4x + 22$

7. Let $x = 24$

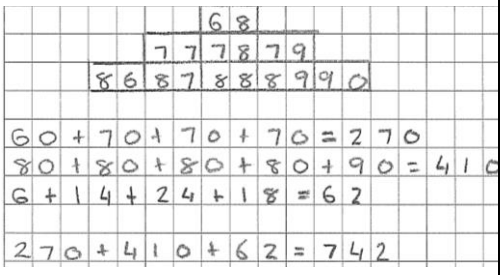
Then $4 \times 24 + 22$ or $4 \times 20 + 4 \times 4 + 22$

$= 80 + 16 + 22$

$= 118$.

Example 3

1. Green triangle(pyramid)
2. Find the sum of the numbers



3. Starting number is 79

		$79-11$		
	$79-2$	$79-1$	79	
$79+7$	$79+8$	$79+9$	$79+10$	$79+11$

4. $x = 79$
5.

		$x-11$		
	$x-2$	$x-1$	x	
$x+7$	$x+8$	$x+9$	$x+10$	$x+11$

6. $x + x - 11 + x - 2 + x - 1 + x + 7 + x + 8 + x + 9 + x + 10 + x + 11$

which simplifies to $9x + 31$

7. Let $x = 79$

Then $9 \times 79 + 31$ or $9 \times 80 - 9 \times 1 + 31$

$= 720 - 9 + 31$

$= 742$

Using the above two examples, students can generalise the pattern that they make by exploring relationships with the size of the shape and the coefficient. The more complex shapes reinforce properties of number, including additive and multiplicative thinking. That is, students are able to manipulate numbers by joining, separating and comparing while engaging in flexible mathematical reasoning. In turn, helping children to make generalisations about the algebraic expression that they are formed. The next step would be to introduce irregular shapes such as pentominoes, letters or numbers.

Task 3:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 3: Create an irregular shape

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 4: Create a letter

Giving students the choice of shape, starting number makes this open-ended task differentiatable. Further, students could develop stories for the selected numbers, using the hundreds chart and algebraic expressions to formalise their story. Students have the freedom of choice and have the hundreds chart as an enabling tool that they could use to describe the relationship between the highlighted numbers. Further, by adding the numbers first using any known strategy, students have a point of reference that they can use to check their answer resulting from the algebraic expression.

Student misconceptions

The progression of learning and conceptual development provides a systematic approach to understanding algebraic expressions. A possible misconception can develop when starting with the three numbers in a row to the letter (task 1), missing out on the squares and irregular shapes (task 2) and progressing to task 3. That is, in the case of the letter G shown in Figure 4 where the sum of the 18 numbers is 909. Students may make the assumption that the simplification of the algebraic expression (step 6) will be $18x$ as there are 18 numbers highlighted. That is students missing steps 3 – 5, resulting in an incorrect expression and causing an error in step 7.

Developing understanding of generalisations

Day et al. (2017) developed learning progressions for algebraic reasoning exploring three areas: Pattern and Function; Equivalence and Generalisation. The generalisation learning progression has been used to evaluate the tasks outlined in this paper. The evaluation of the tasks using the generalisation learning progression indicates that the task has potential to develop students understanding of generalisations.

Table 2:Evaluation of tasks using the Learning Progressions for Generalisation

Zone	Descriptor	Evaluation		
1	Explain a generalisation of a simple physical situation	✓	Yes	Tasks 1 – 3
2	Explore and conjecture about patterns in the structure of number, identifying numbers that change and numbers that can vary	✓	Yes	Task 1 – 3
3	Explain generalisations by telling stories in words, with materials and using symbols.	✓	Yes	Step 7
4	Explain generalisations using symbols and explore relationships using technology	✓	Yes	Steps 4 – 7 A calculator could be used to identify relationships and aid the facilitation of simplifying the algebraic expressions;
5	Follow, compare and explain rules for linking successive terms in a sequence or pair quantities using one or two operations	✓	Yes	Step 3
6	Use and interpret basic algebraic conventions for representing situations involving a variable quantity	✓	Yes	Steps 4 – 7
7	Use and interpret algebraic conventions for representing generality and relationships between variables and establish equivalence using the distributive property and inverses of addition and subtraction	✓	Yes	Step 6
8	Combine facility with symbolic representation and understanding of algebraic concepts to represent and explain mathematical situations	✓	Yes	Task 1 – 3

Conclusions

The three tasks outlined in this paper aim to develops students' understanding and skills in generalising and conceptual knowledge of algebraic expressions. The hundreds chart was used to facilitate the development of generalising, providing a necessary link between the abstract and representational aspects of mathematics. The approach documented in this paper has an emphasis on "generalised numbers and exploring structured situations" and a functional approach which uses variables to describe patterns (Linsell, Cavanagh, & Tahir, 2013, p. 34). Students have the opportunity to create stories around their expressions, situating the learning and task into a meaningful context.

Learning algebra can be difficult at the best of times. The progressions of the three tasks is key in developing students' ability to generalise and to avoid possible misconceptions which can impact on their future learning in mathematics, particularly algebra.

References

Day, L., Stephens, M., & Horne, M. (2017). *Developing learning progressions to support mathematical reasoning in the middle years: Algebraic reasoning.* Paper presented at the 40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia, Melbourne.

Linsell, C., Cavanagh, M., & Tahir, S. (2013). Using meaningful contexts to promote understanding of pronumerals. *Australian Mathematics Teacher, 69*(1), 33-40.

McKnight, A., & Muligan, J. (2010). Teaching early mathematics 'smarter not harder': Using ppen-ended tasks to build models and construct patterns. *Australian Primary Mathematics Classroom, 15*(3), 4-9.

Reys, R. E., Rogers, A., Bennett, S., Cooke, A., Robson, K., Ewing, B., & West, J. (2020). *Helping children learn mathematics* (3rd Australian Edition ed.): Melbourne: Wiley.

Serow, P., Callingham, R., & Muir, T. (2019). *Primary Mathematics: Integrating Theory with Practice* (3rd ed.). Port Melbourne: Cambridge University Press.